Information and Bargaining Design

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Motivation

- A letter from a law firm from Wagga Wagga, NSW, informs you that the will of a third cousin of yours has made you the beneficiary of a parcel of land in the Riverina region
- A Chinese investor, who is known to have interest in buying land in the region, has also been sent a letter informing her about the opportunity that has arisen
- The law firm proposes to design a bargaining procedure that selects the information you and the prospective buyer collect, together with the price at which a sale might take place, with the goal of maximizing the gains from trade
- The law firm is highly reputable and has succesfully conducted many information and bargaining design deals in the past
- Should you, and the investor, accept the law firm proposal?
- Should you and the investor demand to be fully informed?
- If both of you accept, what procedure should the law firm design to maximize efficiency?

Information and Bargaining Design

- A buyer and a seller can use an intermediary to bargain about the price and the information on value and cost that they should acquire
- When buyer and seller have full, private information about their value and cost no Bayesian mechanism achieves efficiency (Myerson and Satterthwaite, 1983)
- Robustness: require *ex post equilibrium* and posterior individual rationality; then the bargaining mechanism must be a posted price (Hagerty and Rogerson, 1987; Copic and Ponsati, 2016)
- We foucus on robust mechanisms, but add the information design problem to the classic mechanism design problem

The Plan

- Static Information Disclosure and Trading Mechanisms
  - Intermediary maximizes the gains from trade
  - ex post equilibrium
- Sequential Information Disclosure and Trading Mechanisms
  - Intermediary maximizes the gains from trade
  - ex post perfect equilibrium
- Extensions
  - Profit Maximizing Intermediary
  - Dynamic Information Disclosure and Trading Mechanisms

Values and Costs

- The buyer's value is $v \in [0, 1]$
- The seller's cost is $c \in [0, 1]$
- $F_0(v) = \Pr_0(\text{value} \leq v)$ and $G_0(c) = \Pr_0(\text{cost} \leq c)$ are the true distributions from which value and cost are drawn. Assume they have no atoms.

The expectations according to these distributions are:

$$v_0 = \int_0^1 v dF_0(v)$$

and

$$c_0 = \int_0^1 c dG_0(c)$$

Feasible Distributions...

- Buyer and seller receive a signal, which can be interpreted as an unbiased estimate of value or cost
- No restrictions on signals. The feasible signal distributions are all the distributions of which the priors are mean preserving spreads:

$$\mathcal{F} = \left\{ F : \int_0^1 v dF(v) = v_0 \ \& \int_0^x F(v) dv \leq \int_0^x F_0(v) dv \ \ \forall x \in [0,1] \right\}$$

$$\mathcal{G} = \left\{ G : \int_0^1 c dG(c) = c_0 \ \& \int_0^x G(c) dc \leq \int_0^x G_0(c) dc \ \ \forall x \in [0,1] \right\}$$

...Distributions

- For the buyer:
  - Acquiring no information corresponds to the signal distributions that puts an atom of mass one on v_0
  - Full information acquisition (i.e., discovering the item's value) corresponds to the signal distribution $F_0(v)$

- For the seller:
  - Acquiring no information corresponds to the signal distributions that puts an atom of mass one on c_0
  - Full information acquisition (i.e., discovering the item's cost) corresponds to the signal distribution $G_0(c)$

Static Information Disclosure and Trading Mechanisms

- The designer simultaneously chooses price p and signal distributions F and G for buyer and sellers so as to maximize the gains from trade
- Interpretation: buyer and seller use an intermediary to bargain; they ask the intermediary to release information and select the trading price. The designer's problem is:

$$\max_{p\in[0,1],\, F\in\mathcal{F},\, G\in\mathcal{G}} \int_p^1 \int_0^p (v-c)dG(c)dF(v)$$

or, equivalently, letting $\mathbf{1}_F(p)$ be the mass (atom) that F puts on p:

$$\max_{p\in[0,1],\, F\in\mathcal{F},\, G\in\mathcal{G}} G(p)\int_p^1 vdF(v) - [1-F(p)+\mathbf{1}_F(p)]\int_0^p cdG(c)$$

or, letting E_F, E_G be the conditional expectation associated with F, G:

$$\max_{p\in[0,1],\, F\in\mathcal{F},\, G\in\mathcal{G}} \left(E_F\left[v|v\geq p\right]-E_G[c|c\leq p]\right)[1-F(p)+\mathbf{1}_F(p)]\,G(p) \tag{1}$$

Two-Point Discrete Distributions

Lemma

Given any solution to the maximization problem (1), there is a payoff equivalent solution in which the designer chooses two-point discrete distributions.

Proof.

Suppose p^*, F^*, G^* are the maximizers of (1), then p^* and the following two point distributions are also maximizers. For buyers, take the distribution that puts mass $F^*(p^*) - \mathbf{1}_F(p^*)$ on $v_L = E_{F^*}[v|v < p^*]$ and mass $1 - F^*(p^*) + \mathbf{1}_F(p^*)$ on $v_H = E_{F^*}[v|v \geq p^*]$. For sellers, take the distribution that puts mass $G^*(p^*)$ on $c_L = E_{G^*}[c|c \leq p^*]$ and mass $1 - G^*(p^*)$ on $c_H = E_{G^*}[c|c > p^*]$. Note that if $F^* \in \mathcal{F}$ then the described two-point distribution for the buyer also belongs to $\mathcal{F}$, and similarly for the seller. $\square$

Feasible Two-Point Signal Distributions for the Buyer

For buyers, let $\{v_L, v_H\}$ be the set of possible signals with probabilities f_L, f_H. The following constraints must hold to guarantee that F_0 is a mean preserving spread of the signal distribution:

$$v_L f_L + v_H f_H = v_0 \tag{2}$$

$$(x - v_L)\, f_L \leq \int_0^x F_0(v)dv \quad \text{for } \; x \in [v_L, v_H) \tag{3}$$

$$(v_H - v_L)\, f_L + (x - v_H)\, 1 \leq \int_0^x F_0(v)dv \quad \text{for } \; x \in [v_H, 1] \tag{4}$$

We can use (2) to define v_L and replace it into the other two constraints; also using $f_H = 1 - f_L$, we obtain:

$$(x - v_H)f_L - v_0 + v_H - \int_0^x F_0(v)dv \leq 0 \quad \text{for } \; x \in [v_L, v_H)$$

$$x - v_0 - \int_0^x F_0(v)dv \leq 0 \quad \text{for } \; x \in [v_H, 1]$$

...Feasible Two-Point Buyer Signal Distributions

$$(x - v_H) f_L - v_0 + v_H - \int_0^x F_0(v)dv \leq 0 \quad \text{for } \; x \in [v_L, v_H) \tag{5}$$

$$x - v_0 - \int_0^x F_0(v)dv \leq 0 \quad \text{for } \; x \in [v_H, 1] \tag{6}$$

It is immediate to see that (6) holds since, integrating by parts,

$$\begin{aligned} x - v_0 - \int_0^x F_0(v)dv &= x - v_0 - xF_0(x) + \int_0^x vdF_0(v) \\ &= x[1 - F_0(x)] - \int_x^1 vdF_0(v) \leq 0 \end{aligned}$$

It is also immediate to see that the left hand side of (5) is a concave function of x which is maximized at x such that $f_L = F_0(x)$. Thus (5) holds if it holds for such x and we can write the constraint as:

$$(x - v_H)F_0(x) - v_0 + v_H - xF_0(x) + \int_0^x vdF_0(v) \leq 0 \quad \text{or,}$$

$$v_H \left[1 - F_0(x)\right] - \int_x^1 vdF_0(v) \leq 0$$

Feasible Two-Point Signal Distributions for the Seller

Repeating the same argument for the seller, using y instead of x, leads to the following constraint on the two-point distribution for the seller:

$$c_H \left[1 - G_0(y)\right] - \int_y^1 c dG_0(c) \leq 0 \quad \text{or,}$$

$$\int_0^y c dG_0(c) - c_L G_0(y) \leq 0$$

The Intermediary's Problem

Thus, we can write the intermediary problem as follows:

$$\max_{v_H, c_L, x, y} (v_H - c_L)\, G_0(y) \left[1 - F_0(x)\right] \quad \text{s.t.}$$

$$v_H \left[1 - F_0(x)\right] - \int_x^1 v dF_0(v) \leq 0$$

$$\int_0^y c dG_0(c) - c_L G_0(y) \leq 0$$

It is immediate to see that both constraints must bind, otherwise the designer would profit from raising v_H or lowering c_L. It follows that the designer's problem can be reduced to:

$$\max_{x,y} \int_x^1 v dF_0(v)\, G_0(y) - \int_0^y c dG_0(c) \left[1 - F_0(x)\right] \quad \text{or,}$$

$$\max_{x,y} \left(E_{F_0}[v | v \geq x] - E_{G_0}[c | c \leq y]\right) G_0(y) \left[1 - F_0(x)\right]$$

The Optimal Static Mechanism

Proposition

Under the static information disclosure and trading mechanism that maximizes the gains from trade, the buyer observes whether the value is strictly below or at least as high as x and the seller observes whether the cost is strictly above or at least as low as y, where

$$E_{G_0}[c|c \leq y] = x \quad \textit{and}$$
$$E_{F_0}[v|v \geq x] = y$$

The trading price is any $p \in \left[E_{G_0}[c|c \leq y], E_{F_0}[v|v \geq x]\right] = [x, y]$.

Optimal Static Mechanism

Proof.

$$\max_{x,y} \int_x^1 v dF_0(v) G_0(y) - \int_0^y c dG_0(c) \left[1 - F_0(x)\right]$$

The foc's are:

$$-x f_0(x) G_0(y) + \int_0^y c dG_0(c) f_0(x) = 0 \quad \text{and}$$

$$\int_x^1 v dF_0(v) g_0(y) - y g_0(y) \left[1 - F_0(x)\right] = 0$$

The soc's are:

$$G_0(y) \left[1 - F_0(x)\right] f_0(x) g_0(y) \geq (y - x)^2 (f_0(x) g_0(y))^2 \quad \text{and}$$

$$1 \geq (y - x)^2 \frac{f_0(x)}{\left[1 - F_0(x)\right]} \frac{g_0(y)}{G_0(y)}$$

□

The Uniform Example

- Suppose the true distributions of values and costs F_0 and G_0 are uniform
- The solution to the foc's is: $x = 1/3$ and $y = 2/3$; the soc holds. The buyer observes whether the value is above or below $1/3$; the seller observes whether the cost is above or below $2/3$
- Any price $p \in [1/3, 2/3]$ is a solution
- Note that expected welfare is $\left(\frac{2}{3} - \frac{1}{3}\right) \frac{2}{3} \frac{2}{3} = \frac{4}{27}$ or 89% of the first best level (which is $\frac{1}{6}$)
- Interestingly, this is higher than the highest welfare achievable by a Bayesian mechanism when traders are fully informed. In that case trade takes place whenever $v \geq c + 1/4$ and expected welfare is $\frac{9}{64}$ or 84% of the first best level

Static Information Disclosure and Trading Mechanisms: Insight

- Full information is not optimal as it does not generate enough trade. Only buyers with value above p and sellers with cost below p trade; any efficient trade with either (i) $p > v > c$, or (ii) $v > c > p$ is lost
- The most valuable of such trades are those with (i) $v = p - \varepsilon_v$ and $c = \varepsilon_c$ and those with (ii) $v = 1 - \varepsilon_v$ and $c = p + \varepsilon_c$
- First effect of the optimal static information disclosure and trading mechanism: such valuable trades are completed
- Second effect: some inefficient trades are also completed, but the losses on those are smaller: trades with (i) $c = y - \varepsilon_c > v = x + \varepsilon_v$

What if the Seller is Fully Informed?

Proposition

In the static information disclosure and trading mechanism that maximizes the gains from trade when the seller is fully informed about her cost, the buyer obtains the same information than when the information disclosed to the seller is chosen by the intermediary. The gains from trade are also the same, but they now must be fully appropriated by the seller. That is, the buyer observes whether the value is strictly below or at least as high as x and the price is $p = y$, where

$$E_{G_0}[c|c \leq y] = x \quad \textit{and}$$
$$E_{F_0}[v|v \geq x] = y$$

- In the uniform model, $p = y = \frac{2}{3}$, $x = \frac{1}{3}$.

Sequential Information Disclosure and Trading Mechanisms

- The intermediary first discloses information to one of the traders (possibly randomly chosen). For concreteness, say information is disclosed to the seller
- The intermediary then asks the seller (the informed trader) to report her cost signal (i.e., the information observed)
- As a function of the reported cost signal the intermediary decides how much information to disclose to the buyer and the price at which trade may take place
- After observing her value signal, the buyer decides whether she wants to trade; the seller also decides whether she wants to trade
- Robustness: equilibrium notion is ex post perfect equilibrium

Sequential Information Disclosure and Tradining Mechanisms

- Let $\widehat{c}$ be the seller's cost report
- Let $p\left(\widehat{c}\right)$ be the price chosen by the intermediary
- Since the intermediary wants to maximize the gains from trade, our previous argument suggests that the optimal buyer disclosure policy is to give the buyer a two-point, discrete, signal distribution, letting her know whether her value is above or below $\widehat{c}$
- We want to induce the seller to report her true signal (a revelation principle argument). This imposes an incentive constraint
- The buyer accepting to trade when her value is above the reported cost imposes the posterior individual rationality constraint:

$$p\left(\widehat{c}\right) \leq E_{F_0}\left[v \middle| v \geq \widehat{c}\right]$$

- We also want the truthtelling seller to accept to trade; this imposes the posterior individual rationality constraint:

$$p\left(\widehat{c}\right) \geq \widehat{c}$$

The Seller's Incentive Constraint

- Since the intermediary wants to maximize the gains from trade, try giving full information to the seller
- If the seller truthfully reports her cost, the proposed sequential disclosure and trading mechanism is *ex post efficient*
- The payoff of a seller who reports $\widehat{c}$ while her true cost is c is:

$$u_S\left(\widehat{c}; c\right) = \left[p\left(\widehat{c}\right) - c\right]\left[1 - F_0\left(\widehat{c}\right)\right]$$

- Let $u_S\left(c\right) = u_S\left(c; c\right)$; by the envelope theorem:

$$u_S'\left(c\right) = -\left[1 - F_0\left(c\right)\right]$$

or, integrating and using the boundary condition $p\left(1\right) = 1$:

$$\left[p\left(c\right) - c\right]\left[1 - F_0\left(c\right)\right] = u_S\left(c\right) = \int_c^1 \left[1 - F_0\left(\widetilde{c}\right)\right] d\widetilde{c}$$

integrating by parts and rearranging we obtain

$$p\left(c\right) = \int_c^1 \frac{\widetilde{c}}{1 - F_0\left(c\right)} dF_0\left(\widetilde{c}\right) = E_{F_0}\left[v | v \geq c\right]$$

- Price function satisfies posterior IR constraints of buyer and seller

The Optimal Sequential Mechanism

Proposition

Under the sequential information disclosure and trading mechanism that maximizes the gains from trade, one trader, say the seller, obtains full information about her type and reports it to the intermediary. The intermediary chooses an information policy that lets the buyer (the other trader) observe whether her value is above or below the reported type (cost) of the seller. As a function of the report $\widehat{c}$*, the intermediary chooses a price function* $p(\widehat{c})$ *that gives all the gains from trade to the seller (the fully informed trader):*

$$p\left(\widehat{c}\right) = E_{F_0}[v|v \geq \widehat{c}]$$

Efficient trading results as an ex post perfect equilibrium of the mechanism.

Sequential Information Disclosure and Trading Mechanisms: Insight

- Informing first one trader and asking her to report her information allows the intermediary to condition the information received by the other trader and the price on the first trader's type
- The second trader can be given the minimum information needed to implement ex post efficient trading, conditional on the report
- Choosing a price that gives the fully informed trader all the gains from trade induces her to report truthfully, as it aligns her private incentives with the social goal
- It is less obvious, but perhaps not surprising, that no other price function would work

What if the Seller is Fully Informed?

- It is immediate that the same sequential information disclosure and trading mechanism (except that the seller needs not to be informed) maximizes the gains from trade when the seller is already fully informed about her cost

Extension I: Profit Maximizing Intermediary

Static Information Disclosure and Trading Mechanisms

Proposition

Under the static information disclosure and trading mechanism that maximizes the intermediary's profit the buyer observes whether the value is strictly below or at least as high as x and the seller observes whether the cost is strictly above or at least as low as y, where

$$E_{G_0}[c|c \leq y] = x \quad \textit{and}$$
$$E_{F_0}[v|v \geq x] = y$$

The price charged to the buyer in order to trade is $p^B = E_{F_0}[v|v \geq x] = y$. The price paid to the seller when trading is $p^S = E_{G_0}[c|c \leq y] = x$.

Proof.

- The chosen information disclosure policy is the same as when the gains from trade are maximized
- The chosen prices guarantee that all the generated surplus accrues as a profit to the intermediary

□

Profit Maximizing Intermediary

Sequential Information Disclosure and Trading Mechanisms

Proposition

There is no sequential information disclosure and trading mechanism that is ex post efficient and gives all the gains from trade to the intermediary as profit.

- Either seller or buyer must have full information to achieve ex post efficiency
- Trader with full information must obtain an information rent

Proposition

The profit maximizing sequential information disclosure and trading mechanism yields the intermediary a profit at least as high as the profit maximizing static information disclosure and trading mechanism (or is it exactly as high?).

Extension II: Dynamic Information Disclosure and Trading Mechanisms...

- To achieve ex post efficiency a sequential information disclosure and trading mechanism must give all the surplus to the agent who is the first to obtain information (e.g., the seller)

- Is it possible to design information disclosure and trading mechanisms that are ex post efficient but in which the gains from trade are shared (ex post)?

- The answer is yes, by using dynamic information disclosure and trading mechanisms

...Dynamic Information Disclosure and Trading Mechanisms...

- One trader, the seller for concreteness, is informed over time. Let $t \in [0, 1]$ be time
- At time t the seller is informed whether her cost is $1 - t$ and is also asked whether or not she wants to collect more information
- If the seller chooses the stopping time τ, then the buyer observes whether her value is above or below $\widehat{c}$ where $\widehat{c}$ is defined by $\widehat{c} = 1 - \tau$.
- The intermediary chooses the same price $p(\widehat{c})$ for buyer and seller
- $p(\widehat{c})$ is chosen so that $\widehat{c} = c$; that is, it is optimal for the seller to stop collecting information when she discovers her true cost

...Dynamic Information Disclosure and Trading Mechanisms...

Assumption

The function $p(\widehat{c})$ is continuous and differentiable almost everywhere and it satisfies the following properties:

1. $p(\widehat{c}) \geq \widehat{c}$
2. $p(\widehat{c}) \leq \mathbb{E}_{F_0}\left[v \mid v \geq \widehat{c}\right]$
3. $p'(\widehat{c}) \geq \left[p(\widehat{c}) - \widehat{c}\right] \dfrac{f_0(\widehat{c})}{1 - F_0(\widehat{c})}$
4. $p'(\widehat{c}) \leq \left[p(\widehat{c}) - \mathbb{E}_{G_0}\left[c \mid c < \widehat{c}\right]\right] \dfrac{f_0(\widehat{c})}{1 - F_0(\widehat{c})}$

- By 1. the seller with cost $\widehat{c}$ is willing to sell at price $p(\widehat{c})$
- By 2. the buyer with value above $\widehat{c}$ is willing to buy
- By 3. the seller prefers to stop the discovery phase when she discovers her true cost
- By 4. the seller prefers to continue when she has not yet discovered her true cost.

...Dynamic Information Disclosure and Trading Mechanisms...

Proposition

Under any price rule satisfying the Assumption, in the ex post perfect equilibrium of the dynamic information disclosure and trading mechanism the allocation is ex post efficient. The seller stops the disclosure phase when she discovers her cost and the buyer accepts to trade at the selected price if and only her value is above the seller's cost.

The Price Solution

- Take any integrable function

$$\alpha(c) \in \left[\int_0^c \frac{\widetilde{c}}{G_0(\widetilde{c})} dG_0(\widetilde{c}), c\right]$$

- Set

$$p(\widehat{c}) = \int_{\widehat{c}}^1 \frac{\alpha(c)}{1 - F_0(c)} dF_0(c)$$

- For example, if F_0 and G_0 are uniform we can take:

$$\alpha(c) = \begin{cases} \frac{4}{5}c & \text{for} \quad c < \frac{2}{3} \\ 2c - 1 & \text{for} \quad c \geq \frac{2}{3} \end{cases}$$

  which gives

$$p(\widehat{c}) = \begin{cases} \frac{2}{5}(1 + \widehat{c}) & \text{for} \quad \widehat{c} < \frac{2}{3} \\ \widehat{c} & \text{for} \quad \widehat{c} \geq \frac{2}{3} \end{cases}$$

- Under this pricing rule in the uniform model the seller's ex ante payoff is $\frac{28}{270}$, while the buyer's ex ante expected payoff is $\frac{17}{270}$

Conclusions

- Classical Mechanism Design: Agents have full private information, the allocation (or social choice) rule must be decided
- Information Design (e.g., Bayesian Persuasion): The allocation mechanism is fixed, the information to be given to the agent must be chosen
- Information Disclosure and Mechanism Design (this paper): Both the allocation mechanism and the information to be given to the agents can be chosen
  - New insights can be gained, e.g., it is optimal to not give full information to both players, efficient bilateral training is possible with sequential or dynamic disclosure.