The Politics of Attention

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March 2018

Hamilton vs. Madison

Coauthors of the federalist papers, which offered a justification and a marketing plan for the U.S. constitution

Then disagreed on Hamilton's economic policies as the Secretary of Treasury that featured national bank, national debt, policies favoring manufacturing and trade over agriculture

To "arouse and attract public attention," the two of them

- Founded political parties that adopted extreme and exaggerated positions
- Sponsored partisan newspapers

Madison vs. Hamilton (Cont'd)

Madison, who always believed that the country would have some manufacturing, trade and agriculture, said that "people need to look inwards to the center of the country, to farmers, and go back to the values that made American great, namely low taxes, agriculture and less trade..."

Hamilton responded to this by saying that "Madison's goal was to turn the United States into a primitive autarchy, self-reliant and completely ineffectual on the global scale..."

Reference: Feldman (2017)

The Downsian Doctrine

"In our model, as in the real world, political decisions are made when uncertainty exists and information is obtainable only at a cost. Thus a basic step towards understanding politics is analysis of the economics of being informed, i.e., the rational utilization of scarce resources to obtain data for decision-making."

- Anthony Downs, *An Economic Theory of Democracy (1957)*

Preview

A theory of how the need to capture voters' limited attention shape political behaviors and outcomes:

- Formalize Downs' thesis that voters are rationally inattentive
- Conduct equilibrium analysis in a generalized Downsian model of electoral competition
- Show that candidates differ on policy and issue positions in order to draw voters' attention to politics
- Comparative statics with respect to attention cost and news technology
- Historical relevance and modern implications

Agenda

1. Baseline model
2. Extensions
3. Discussion

Baseline Model

1. Setup
2. Optimal attention rule
3. Equilibrium analysis
4. Comparative statics and applications

Baseline Model

1. Setup
2. Optimal attention rule
3. Equilibrium analysis
4. Comparative statics and applications

Players

A unit mass of infinitesimal voters and two candidates α and β

Single-dimensional type capturing preferences for policies in $\Theta = [-1, 1]$

Camp α if type belongs to $\Theta_\alpha = [-1, 0]$ and camp β if type belongs to $\Theta_\beta = [0, 1]$

Type distributions:

- Voters: $t \sim P$ with support T and zero median
- Candidate c: $t_c \sim P_c$ with finite support $T_c \subset \Theta_c$

Policy and Payoff

Each candidate c can implement one of the finite policies in $A_c \subset \Theta_c$

In case candidate c assumes office and implements policy a:

- Voter t: $u(a, t)$
- Candidate c: $u_+ (a, t_c)$
- Candidate $-c$: $u_- (a, t_{-c})$

Assumption 1.

u is continuous. $u(\cdot, t)$ is strictly increasing on $[-1, t]$ and strictly decreasing on $[t, 1]$ for all t.

Timeline

1. Nature draws types
2. Candidate c observes t_c and proposes $a_c \in A_c$
3. The press releases news $\boldsymbol{\omega}$ about the policy state $\mathbf{a} = (a_\alpha, a_\beta)$
4. Voters attend to politics and cast votes
5. Winner is determined by simple majority rule with even tie breaking and implements his policy proposal in Step 2

Candidate's Strategy

Candidate c's strategy $\sigma_c : T_c \to \Delta\left(A_c\right)$

A stragtegy profile $\sigma = (\sigma_\alpha, \sigma_\beta)$ yields a random policy state with support $\mathrm{supp}(\sigma)$

Voter's Problem

1. Sincere voting, i.e., each voter perceives himself as the only decision maker and maximizes his expected utility
2. Attending to politics is costly and yields a signal that helps making decisions

Voter's Problem (Cont'd)

Assume $\boldsymbol{\omega} = \mathbf{a}$ in the baseline model

Voter t's attention rule is $m_t : \text{supp}(\sigma) \to [0,1]$:

- $m_t(\mathbf{a})$: prob. that voter t supports candidate β in state $\mathbf{a}$

Define

- $v(\mathbf{a}, t) = u(a_\beta, t) - u(a_\alpha, t)$
- $V_t(m_t, \sigma) = \mathbb{E}_\sigma [m_t(\tilde{\mathbf{a}}) v(\tilde{\mathbf{a}}, t)]$

Voter t's expected utility:

$$V_t(m_t, \sigma) - \mu_t \cdot I(m_t, \sigma)$$

Candidate's Problem

Winning probability:

- $\int_{t \in T} m_t(\mathbf{a})\, dP$: total votes for candidate β in state $\mathbf{a}$
- $w_\beta(\mathbf{a}) = w(\mathbf{a}) = \begin{cases} 0 & \text{if } \int_{t \in T} m_t(\mathbf{a})\, dP < \frac{1}{2} \\ \frac{1}{2} & \text{if } \int_{t \in T} m_t(\mathbf{a})\, dP = \frac{1}{2} \\ 1 & \text{if } \int_{t \in T} m_t(\mathbf{a})\, dP > \frac{1}{2} \end{cases}$
- $w_\alpha(\mathbf{a}) = 1 - w(\mathbf{a})$

Candidate c's expected utility:

$$V_c(m, \sigma) = \mathbb{E}_\sigma \left[w_c(\tilde{\mathbf{a}})\, u_+(\tilde{a}_c, \tilde{t}_c) + (1 - w_c(\tilde{\mathbf{a}}))\, u_-(\tilde{a}_{-c}, \tilde{t}_c) \right]$$

Equilibrium

A strategy profile (m^*, σ^*) is a BNE if

1. m_t^* maximizes voter t's expected utility, taking σ^* as given:

$$m_t^* \in \underset{m_t : \text{supp}(\sigma^*) \to [0,1]}{\arg\max} V_t(m_t, \sigma^*) - \mu_t \cdot I(m_t, \sigma^*)$$

2. σ_c^* maximizes candidate c's expected utility, taking m^* and σ_{-c}^* as given:

$$\sigma_c^* \in \underset{\sigma_c}{\arg\max}\, V_c(m^*, \sigma_c, \sigma_{-c}^*)$$

For now, suppose $\mathbf{a} \notin \text{supp}(\sigma^*)$ leads all players to pay a big penalty through, e.g., a complete voter abstention

The Downsian Doctrine

"In our model, as in the real world, political decisions are made when uncertainty exists and information is obtainable only at a cost. Thus a basic step towards understanding politics is analysis of the economics of being informed, i.e., the rational utilization of scarce resources to obtain data for decision-making."

- Anthony Downs, *An Economic Theory of Democracy (1957)*

The Downsian Doctrine (Cont'd)

Evidence:

- Voters are poorly informed, hold sticky party images and seek informational shortcuts, e.g., party identity, personal traits
- Consume soft news, e.g., the "Oprah effect"
- Rational attention allocation, e.g., farmers vs. laborers during Eisenhower's term, blacks vs. whites on civil rights issues

References: Campell et al. (1960); Popkins (1994); Baum and Jamison (2006); Vavreck (2009)

Theorization

Acquire any signal of the policy state at the following cost:

$$\underbrace{\mu_t}_{\text{marginal attention cost}} \cdot \underbrace{I\left(m_t, \sigma\right)}_{\text{mutual information}}$$

Marginal Attention Cost

Rich heterogeneity arising from age, gender, ideology, income, race, etc.

Major cost shifters:

- Improvements in education and printing technology
- Intensified competition for consumer eyeballs
- Opportunities to entertain and to socialize, made accessible through cable TV, internet and digital media

References: Campell et al. (1960); Popkins (1994); Baum and Kernel (1999); Gentzkow et al. (2004); Teixeira (2014); Prior (2005); Dunaway (2016); Perez (2017)

Mutual Information

Suffice to consider binary signals that induce obedient behaviors on the voter's part

Then

$$I(m_t, \sigma) = H(\sigma) - H(\sigma \mid m_t)$$

where

$$H(\sigma) = - \sum_{\mathbf{a} \in \text{supp}(\sigma)} \sigma(\mathbf{a}) \log(\sigma(\mathbf{a}))$$

is the entropy of the policy state, and $H(\sigma \mid m_t)$ is the conditional entropy of the policy state

Reduction in Uncertainty

Extreme cases:

1. Most informative voting ($I = +\infty$):

$$m_t(\mathbf{a}) = \begin{cases} 1 & \text{if } v(\mathbf{a}, t) > 0 \\ 0 & \text{otherwise} \end{cases}$$

2. Least informative voting ($I = 0$): decision and policy state are independently distributed

Information Theoretic Foundation

Shannon (1948)'s fundamental problem of communication:

$$\text{Source data} \xrightarrow{\text{channel}} \text{received signal}$$

Here, $\text{policy state} \xrightarrow{\text{attention}} \text{voting decision}$

Shannon's entropy determines the minimum channel capacity to losslessly transmit source data as encoded in binary digits

If voters ask yes-no questions at a fixed unit cost, then the expected cost is entropy-based

Baseline Model

1. Setup
2. Optimal attention rule
3. Equilibrium analysis
4. Comparative statics and applications

Optimal Attention Rule

Lemma 1.

Fix any σ and t. For any $\mathbf{a} \in \text{supp}(\sigma)$,

$$m_t^*(\mathbf{a}) \begin{cases} = 0 & \text{if } \mathbb{E}_\sigma\left[\exp\left(\mu_t^{-1} v(\tilde{\mathbf{a}}, t)\right)\right] \leq 1, \\ = 1 & \text{if } \mathbb{E}_\sigma\left[\exp\left(-\mu_t^{-1} v(\tilde{\mathbf{a}}, t)\right)\right] \leq 1, \\ \in (0,1) & \text{otherwise}, \end{cases}$$

and the following condition holds true in the last case:

$$v(\mathbf{a}, t) = \mu_t \cdot \log\left(\frac{m_t^*(\mathbf{a})}{1 - m_t^*(\mathbf{a})} \cdot \frac{1 - \mathbb{E}_\sigma[m_t^*(\mathbf{a})]}{\mathbb{E}_\sigma[m_t^*(\mathbf{a})]}\right).$$

Optimal Attention Allocation

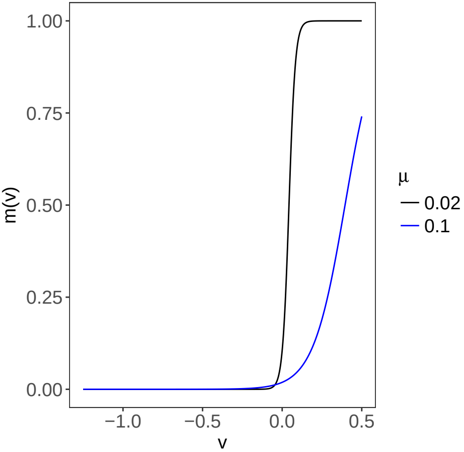

Figure 1: Plot $m_t^*(\mathbf{a})$ against $v(\mathbf{a}, t)$ for $t = -.25$: a_c is uniformly distributed on Θ_c and $u(a,t) = -|t-a|$.

Endogenous Confirmatory Bias

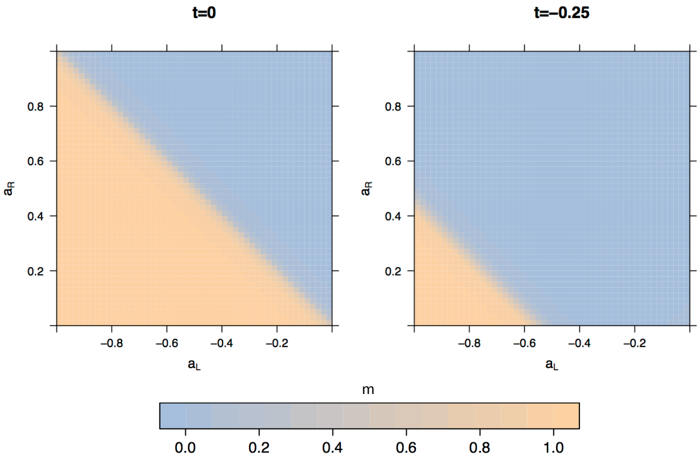

Figure 2: Plot $m_t^*(\mathbf{a})$ against $\mathbf{a}$ for $t = 0$ and $-1/4$: a_c is uniformly distributed on Θ_c, $u(a,t) = -|t-a|$ and $\mu = .02$.

Baseline Model

1. Setup
2. Optimal attention rule
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Symmetry

Assumption 2.

For all a and t, $\tilde{u}(a,t) = \tilde{u}(-a,-t)$ for all $\tilde{u} \in \{u, u_+, u_-\}$, $\mu_t = \mu_{-t}$, $P(t) = P(-t)$ and $P_c(t) = P_{-c}(-t)$.

Symmetric equilibrium:

1. $\sigma_c(a \mid t) = \sigma_{-c}(-a \mid -t)$ for all a and t
2. $m_t(-a, a') = 1 - m_{-t}(-a', a)$ for all t and $(-a, a') \in \mathrm{supp}(\sigma)$

Example

Candidates can be either centrist ($t = \pm\frac{1}{4}$) or extreme ($t = \pm\frac{3}{4}$) with prob. $\frac{1}{2}$

Payoff functions are $u(a,t) = -|t-a|$, $u_+(a,t) = R - \alpha_+|t-a|$, $u_-(a,t) = -\alpha_-|t-a|$

Differing types of candidate adopt different pure strategies

Policies are $-a_2 < -a_1 \leq 0 \leq a_1 < a_2$, each realized with prob. $\frac{1}{2}$

Example (Cont'd)

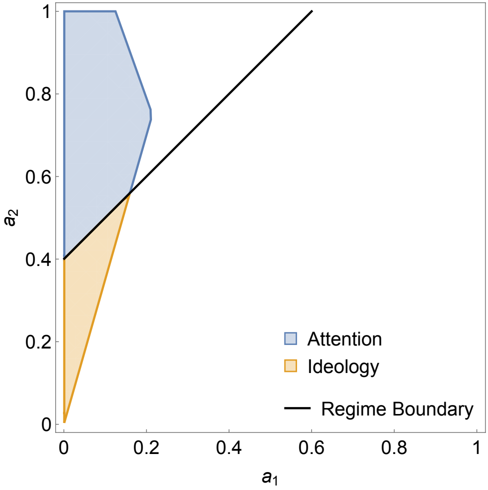

Figure 3: Equilibrium outcomes: $\alpha_+ = 9$, $\alpha_- = 1$, $R = 6$. The regime boundary is drawn for voter $t = -.001$.

Matrix Representation

Let $-a_N < \cdots < -a_1 \leq 0 \leq a_1 < \cdots < a_N$ be policy proposals

Let $\mathbf{A}$, $\mathbf{\Sigma}$ and $\mathbf{W}$ be $N \times N$ matrices:

- $\mathbf{A}$: policy matrix, $\mathbf{a}_{ij} = (-a_i, a_j)$
- $\mathbf{\Sigma}$: probability matrix, $\sigma_{ij} \geq 0$, $\sum_{i,j} \sigma_{ij} = 1$
- $\mathbf{W}$: winning probability matrix, $w_{ij} \in \{0, 1/2, 1\}$

E.g., $\mathbf{\Sigma} = \frac{1}{4}\mathbf{J}_2$ in our example, and winner is determined as in the case of complete information if

$$w_{ij} = \begin{cases} 0 & \text{if } a_i > a_j \\ \frac{1}{2} & \text{if } a_i = a_j \\ 1 & \text{if } a_i < a_j \end{cases}$$

Matrix Representation (Cont'd)

Definition 1.

$[\mathbf{A}, \boldsymbol{\Sigma}]$ *is* $\mathbf{W}$*-incentive compatible if there exists* σ *such that*

1. *the probabilities of policy states under* σ *are given by* $\boldsymbol{\Sigma}$*, i.e.,*

$$\sigma(\mathbf{a}_{ij}) = \sigma_{ij} \; \forall i, j;$$

2. *each* σ_c *maximizes candidate* c*'s expected utility, taking the winning probability matrix* $\mathbf{W}$ *and the other candidate's strategy* σ_{-c} *as given, i.e.,*

$$\sigma_c \in \arg\max_{\sigma'_c} V_c\left(\mathbf{W}, \sigma'_c, \sigma_{-c}\right).$$

Matrix Representation (Cont'd)

Definition 2.

$\mathbf{W}$ *is* $[\mathbf{A}, \boldsymbol{\Sigma}]$*-rationalizable if*

$$w_{ij} = w(\mathbf{a}_{ij}) \ \forall i, j,$$

where $w(\mathbf{a}_{ij})$ *can be obtained from plugging* $m_t^*(\mathbf{a}_{ij})$, $t \in T$ *under* $[\mathbf{A}, \boldsymbol{\Sigma}]$ *into function* w.

Equilibrium Policy

For any prob. matrix $\mathbf{\Sigma}$, define

$$\mathcal{E}(\mathbf{\Sigma}) = \left\{ \mathbf{A} : \exists \mathbf{W} \text{ s.t. } \begin{array}{c} [\mathbf{A}, \mathbf{\Sigma}] \text{ is } \mathbf{W} - \text{IC} \\ \mathbf{W} \text{ is } [\mathbf{A}, \mathbf{\Sigma}] - \text{rationalizable} \end{array} \right\}$$

Assumption 3.

$u(a, t)$ is concave in a for all t.

Equilibrium Policy (Cont'd)

Theorem 1.

Assume Assumptions 2 and 3. Then for any integer N and any $N \times N$ probability matrix $\mathbf{\Sigma}$,

$$\mathcal{E}(\mathbf{\Sigma}) = \left\{ \mathbf{A} : [\mathbf{A}, \mathbf{\Sigma}] \text{ is } \widehat{\mathbf{W}}_N - IC \right\},$$

where $\widehat{\mathbf{W}}_N$ is an $N \times N$ matrix whose ij^{th} entry is

$$\widehat{w}_{ij} = \begin{cases} 0 & \text{if } a_i > a_j \\ \frac{1}{2} & \text{if } a_i = a_j \\ 1 & \text{if } a_i < a_j \end{cases}$$

Implications

Winner is determined the same way as in the case of complete information

Voter characteristics are irrelevant to the determination of equilibrium policies

Simplify the problem of finding fixed points

Proof Sketch

Step 1 By symmetry,

$$\begin{aligned}\int m_t^*(\mathbf{a}_{ij})\,dP(t) &= \int_{t<0} 1 - m_{-t}^*(\mathbf{a}_{ji})\,dP(t) \\ &\quad + \int_{t>0} m_t^*(\mathbf{a}_{ij})\,dP(t) \\ &= \int_{t>0} m_t^*(\mathbf{a}_{ij}) - m_t^*(\mathbf{a}_{ji})\,dP(t) + \frac{1}{2}\end{aligned}$$

Step 2 By FOC,

$$\operatorname{sgn} m_t^*(\mathbf{a}_{ij}) - m_t^*(\mathbf{a}_{ji}) = \operatorname{sgn} v(\mathbf{a}_{ij}, t) - v(\mathbf{a}_{ji}, t)$$

Proof Sketch (Cont'd)

Step 3 By concavity, the following holds true for all $i > j$ and t:

$$\begin{aligned} & v\left(\mathbf{a}_{ij}, t\right) - v\left(\mathbf{a}_{ji}, t\right) \\ & = u\left(a_j, t\right) + u\left(-a_j, t\right) - \left[u\left(a_i, t\right) + u\left(-a_i, t\right)\right] \\ & \geq 0 \end{aligned}$$

Combining Steps 1-3 yields $\int m_t^*\left(\mathbf{a}_{ij}\right) dP(t) \geq \frac{1}{2}$ for all $i > j$

Step 4 Show that $m_t^* \in (0, 1)$ in a neighborhood of the median voter in case $N \geq 2$

Baseline Model

1. Setup
2. Optimal attention rule
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Attention vs. Ideology

Definition 3.

Voter t acts based on ideology if

$$m_t^* = \begin{cases} 0 & \text{if } t < 0, \\ 1 & \text{if } t > 0, \end{cases}$$

and he pays active attention to politics if $m_t^* \in (0, 1)$.

Attention Set

For any prob. matrix $\mathbf{\Sigma}$, $t < 0$ and $\mu > 0$, define

$$\mathcal{A}_t\left(\mathbf{\Sigma},\mu\right)=\left\{\mathbf{A}\in\mathcal{E}\left(\mathbf{\Sigma}\right):\mathbb{E}_{\left[\mathbf{A},\mathbf{\Sigma}\right]}\left[\exp\left(\mu^{-1}v\left(\tilde{\mathbf{a}},t\right)\right)\right]>1\right\}$$

Then

$$\mathcal{E}\mathcal{A}_t\left(\mathbf{\Sigma},\mu\right)=\mathcal{A}_t\left(\mathbf{\Sigma},\mu\right)\cap\mathcal{E}\left(\mathbf{\Sigma}\right)$$

is the set of equilibrium policy matrices that draws t's attention to politics, and

$$\mathcal{E}\mathcal{I}_t\left(\mathbf{\Sigma},\mu\right)=\mathcal{A}_t^c\left(\mathbf{\Sigma},\mu\right)\cap\mathcal{E}\left(\mathbf{\Sigma}\right)$$

is the set of equilibrium policy matrices that leads t to act based on ideology

Regularity Conditions

Assumption 4.

$u(a', t) - u(a, t)$ is strictly increasing in t for all a, a'.

Assumption 5.

There exist $t < 0$ and $\kappa > 0$ such that $|v(\mathbf{a}, t) - v(\mathbf{a}, 0)| > \kappa |t|$ for all $\mathbf{a}$.

Attention-Driven Polarization

Theorem 2.

Assume Assumption 1-4. Fix any probability matrix $\mathbf{\Sigma}$ *such that* $N \geq 2$ *and* $\mathcal{E}(\mathbf{\Sigma}) \neq \emptyset$.

Then for any $t < 0$ *satisfying Assumption 5 and* $v(\mathbf{a}, t) > 0$ *for some* $\mathbf{a} \in \mathbf{A} \in \mathcal{E}(\mathbf{\Sigma})$, *as* μ *increases from zero to infinity,*

1. $\mathcal{EI}_t(\mathbf{\Sigma}, \mu)$ *expands and* $\mathcal{EA}_t(\mathbf{\Sigma}, \mu)$ *shrinks;*
2. $\min\{u(a_1, 0) - u(a_N, 0) : \mathbf{A} \in \mathcal{EA}_t(\mathbf{\Sigma}, \mu)\}$ *is increasing in* μ;
3. *the above stated variables do not always stay constant.*

Attention-Driven Polarization (Cont'd)

Policy divergence is instrumental for capturing voter's attention

Evidence:

- Hamilton vs. Madison
- The conformity in the 50's led to the failure of many voters to perceive any party difference on critical issues
- Bush vs. Reagan on women's rights in 1984 Republican primary

Example: A Revisit

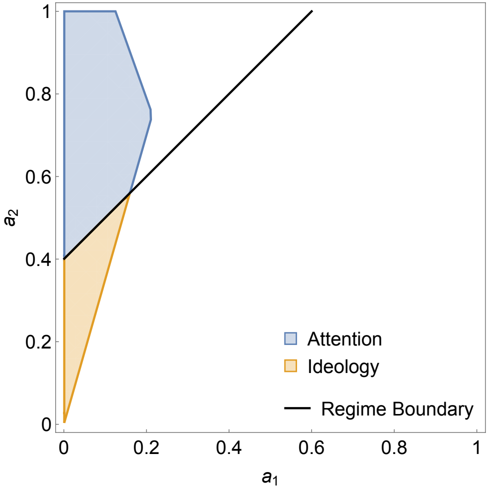

Figure 4: Equilibrium outcomes: $\alpha + = 9$, $\alpha_{-} = 1$, $R = 6$. The regime boundary is drawn for $t = -.001$.

Voter Uncertainty

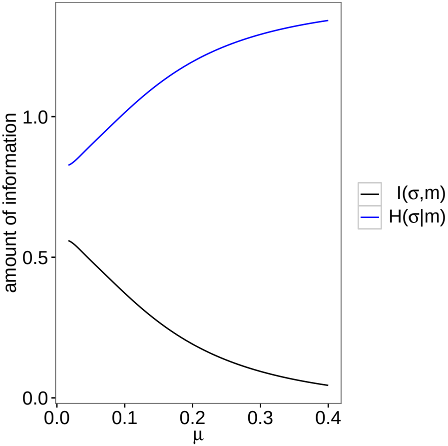

Figure 5: Plot mutual information and conditional entropy against μ for $t = -.05$: policies equal $\pm\frac{1}{4}$ and $\pm\frac{3}{4}$ with equal probability and $u(a, t) = -|t - a|$.

Opinion Polarization

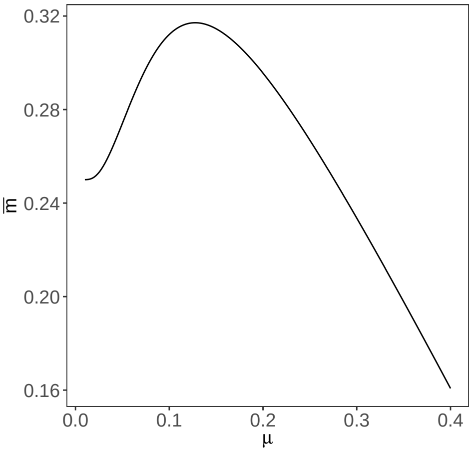

Figure 6: Plot $\mathbb{E}_{\sigma^*}\left[m_t^*(\mathbf{a})\right]$ against μ for $t = -.05$: policies equal $\pm\frac{1}{4}$ and $\pm\frac{3}{4}$ with equal probability, $u(a, t) = -|t - a|$.

Agenda

1. Baseline model
2. Extensions
3. Discussions

Equilibrium Purification

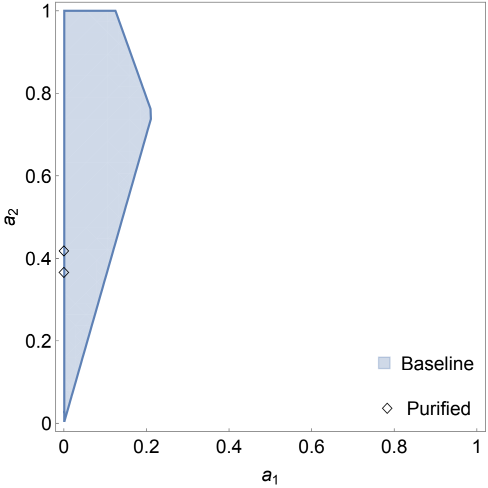

Figure 7: Equilibrium policies before and after purification: $\alpha_+ = 9$, $\alpha_- = 1$, $R = 6$.

Limited Commitment and Campaign Messages

Winning candidate fulfils campain promise with prob. γ and adopts his most preferred policy with prob. $1 - \gamma$

Evidence:

- The 1976 campaign portrayed Carter as being "outside and honest," though subsequent conversation between Humphrey revealed that he was closer to the party's "default value"
- Gary Hart's "new ideas" and Mondale's criticism of "where is beef?"
- Roger Ailes described his role as Bush's strategist against Dukakis: "every single thing I did was designed to push the two candidates further apart..."

Multiple Issues

Two issues a and b, both take values in Θ; pareto frontier $\mathcal{B}(a)$: $\mathcal{B}' < 0$, $\mathcal{B}'' < 0$, $\lim_{a \to -1} \mathcal{B}'(a) = 0$ and $\lim_{a \to 1} \mathcal{B}'(a) = +\infty$

A unit mass of infinitesimal voters and two candidates

Single-dimensional type representing preference weight on b; pro-a types belong to $\Theta_a = [-1, 0]$ and pro-b types $\Theta_b = [0, 1]$

Payoffs are $u(a, b, t)$, $u_+(a, b, t)$ and $u_-(a, b, t)$, all strictly increasing and smooth in (a, b)

Assumption 6.

$u(a, b, t)$ is strictly concave in (a, b) and $-\frac{u_a(a,b,t)}{u_b(a,b,t)}$ is increasing in t for all (a, b).

Multiple Issues (Cont'd)

Define $\widehat{u}\left(a,t\right) = u\left(a, \mathcal{B}(a), t\right)$, $\widehat{u}_{+}\left(a,t\right) = u_{+}\left(a, \mathcal{B}(a), t\right)$ and $\widehat{u}_{-}\left(a,t\right) = u_{-}\left(a, \mathcal{B}(a), t\right)$

Corollary 1.

The augmented economy satisfies Theorem 1 under Assumptions 2 and 6, as well as Theorem 2 under Assumptions 2, 4, 5 and 6.

Issue ownership, e.g., inflation vs. unemployment, economy vs. defense (see Petrocik (1996))

Noisy News

$\boldsymbol{\omega} = (\omega_\alpha, \omega_\beta) \sim f(\cdot \mid \mathbf{a})$ with support Ω:

- $\Omega = \Omega_\alpha \times \Omega_\beta$
- $f(\boldsymbol{\omega} \mid \mathbf{a}) = f_\alpha(\omega_\alpha \mid a_\alpha) \times f_\beta(\omega_\beta \mid a_\beta)$

Timeline:

1. Nature draws types
2. Candidate c observes t_c and proposes $a_c \in A_c$
3. The press draws $\boldsymbol{\omega}$ from Ω according to $f(\cdot \mid \mathbf{a})$
4. Voters attend to politics and cast votes
5. Winner is determined by simple majority rule with even tie breaking and implements his policy proposal in Step 2

Attention Rule and Expected Utilities

$m_t : \Omega \to [0,1]$: prob. that voter t supports candidate β in each news state

For given $x = (f, \sigma)$, define

- $\nu_x(\boldsymbol{\omega}, t) = \mathbb{E}_x[v(\tilde{\mathbf{a}}, t) \mid \boldsymbol{\omega}]$
- $V_t(m_t, x) = \mathbb{E}_x[m_t(\tilde{\boldsymbol{\omega}})\, \nu_x(\tilde{\boldsymbol{\omega}}, t)]$

Voter t's expected utility is $V_t(m_t, x) - \mu_t \cdot I(m_t, x)$

Candidate c's expected utility is

$$V_c(m, x) = \mathbb{E}_x[w_c(\tilde{\boldsymbol{\omega}})\, u_+(\tilde{a}_c, \tilde{t}_c) + (1 - w_c(\tilde{\boldsymbol{\omega}}))\, u_-(\tilde{a}_{-c}, \tilde{t}_c)]$$

Symmetry

Assumption 7.

$f_c(\omega \mid a) = f_{-c}(-\omega \mid -a)$ *for all* a *and* ω*.*

Symmetric equilibria:

1. $\sigma_c(a \mid t) = \sigma_{-c}(-a \mid -t)$ for all a and t
2. $m_t(-\omega, \omega') = 1 - m_{-t}(-\omega', \omega)$ for all t and $(-\omega, \omega') \in \Omega$

Matrix Representation

Let $-\omega_K < \cdots < -\omega_1 < 0 < \omega_1 < \cdots < \omega_K$ be news signals, and write $\boldsymbol{\omega}_{mn} = (-\omega_m, \omega_n)$ for $m, n = 1, \cdots, K$

Let $\mathbf{A}$ and $\boldsymbol{\Sigma}$ be as above, and $\mathbf{W}$ be a $K \times K$ matrix whose mn^{th} entry $w_{mn} \in \{0, 1/2, 1\}$ represents candidate β's winning probability in news state $\boldsymbol{\omega}_{mn}$

$\langle \mathbf{A}, \boldsymbol{\Sigma}, \mathbf{W} \rangle$ can be attained in a symmetric equilibrium under f if

1. $[\mathbf{A}, \boldsymbol{\Sigma}]$ is $\langle f, \mathbf{W} \rangle$-IC
2. $\mathbf{W}$ is $\langle f, \mathbf{A}, \boldsymbol{\Sigma} \rangle$-rationalizable

Equilibrium Policy

For any prob. matrix $\mathbf{\Sigma}$ and f, define

$$\mathcal{E}(\mathbf{\Sigma}, f) = \left\{ \mathbf{A} : \exists \mathbf{W} \text{ s.t. } \begin{array}{c} [\mathbf{A}, \mathbf{\Sigma}] \text{ is } \langle f, \mathbf{W} \rangle - \text{IC} \\ \mathbf{W} \text{ is } \langle f, \mathbf{A}, \mathbf{\Sigma} \rangle - \text{rationalizable} \end{array} \right\}$$

Assumption 8.

For all c and all $a, a' \in A_c$ and $\omega, \omega' \in \Omega_c$ such that $a < a'$ and $\omega < \omega'$,

$$\frac{f_c(\omega' \mid a)}{f_c(\omega \mid a)} < \frac{f_c(\omega' \mid a')}{f_c(\omega \mid a')}.$$

Equilibrium Policy (Cont'd)

Theorem 3.

Assume Assumptions 2, 3, 7 and 8. Then for any probability matrix $\boldsymbol{\Sigma}$ with $N \geq 2$,

$$\mathcal{E}(\boldsymbol{\Sigma}, f) = \left\{ \mathbf{A} : [\mathbf{A}, \boldsymbol{\Sigma}] \text{ is } \langle f, \widehat{\mathbf{W}}_K \rangle - IC \right\}.$$

Implications:

- Voter characteristics are still irrelevant to the determination of eqm. policies
- News technology matters (and the effect is subtle)

Blackwell-Informativeness

Definition 4.

f is more Blackwell-informative than f' (f' is a garble of g, $f \succeq f'$) if there exists a Markov kenel ρ such that for all $\mathbf{a}$ and $\boldsymbol{\omega'}$,

$$f'\left(\omega' \mid \mathbf{a}\right) = \sum_{\omega \in \Omega} f\left(\omega \mid \mathbf{a}\right) \rho\left(\omega' \mid \omega\right).$$

Examples of garbling and degarbing:

- News papers became more informative and less partisan during 1870-1920 (Gentzkow et al. (2004))
- Rise of partisan media and fake news (Levendusky (2013); Barthel and Holcomb (2016); Lee and Kent (2017))

Media-Driven Polarization

For any prob. matrix $\mathbf{\Sigma}$, $t < 0$ and f, define

$$\mathcal{A}_t(\mathbf{\Sigma}, f) = \left\{\mathbf{A} \in \mathcal{E}(\mathbf{\Sigma}, f) : \mathbb{E}_{\langle f, [\mathbf{A}, \mathbf{\Sigma}]\rangle} \exp(\nu_x(\omega, t)) > 1\right\}$$

Theorem 4.

Assume Assumptions 1, 2, 5, 7 and 8. Fix any probability matrix $\mathbf{\Sigma}$ with $N \geq 2$ and any $f \succeq f'$.

Then for any $t < 0$ such that $\mathcal{A}_t(\mathbf{\Sigma}, f)$, $\mathcal{A}_t(\mathbf{\Sigma}, f') \neq \emptyset$,

1. $\mathcal{A}_t(\mathbf{\Sigma}, f') \subset \mathcal{A}_t(\mathbf{\Sigma}, f)$;
2. $\min_{\mathbf{A} \in \mathcal{A}_t(\mathbf{\Sigma}, f')} \nu_{\langle f'', \mathbf{A}, \mathbf{\Sigma}\rangle}(\omega_{K1}, 0) > \min_{\mathbf{A} \in \mathcal{A}_t(\mathbf{\Sigma}, f)} \nu_{\langle f'', \mathbf{A}, \mathbf{\Sigma}\rangle}(\omega_{K1}, 0)$ *for all $f'' = f, f'$.*

Interpreting Policy Divergence

$$\nu_{\langle f,\mathbf{A},\boldsymbol{\Sigma}\rangle}\left(\omega_{K1},0\right)=\underbrace{\mathbb{E}_{\langle f,\mathbf{A},\boldsymbol{\Sigma}\rangle}\left[u\left(\tilde{a}_{\beta},0\right)\mid\omega_{\beta}=\omega_{1}\right]}_{(1)}$$

$$-\underbrace{\mathbb{E}_{\langle f,\mathbf{A},\boldsymbol{\Sigma}\rangle}\left[u\left(\tilde{a}_{\beta},0\right)\mid\omega_{\beta}=\omega_{K}\right]}_{(2)}$$

where

$$(1)=\frac{\sum_{i=1}^{N}f_{\beta}\left(\omega_{1}\mid a_{i}\right)u(a_{i},0)\sigma_{i}}{\sum_{i=1}^{N}f_{\beta}\left(\omega_{1}\mid a_{i}\right)\sigma_{i}}$$

and

$$(2)=\frac{\sum_{i=1}^{N}f_{\beta}\left(\omega_{K}\mid a_{i}\right)u(a_{i},0)\sigma_{i}}{\sum_{i=1}^{N}f_{\beta}\left(\omega_{K}\mid a_{i}\right)\sigma_{i}}$$

Proof Sketch

Garbling adds mean-preserving spreads to voter's expected gain from choosing one candidate over another:

Lemma 2.

Fix any t, σ and $f \succeq f'$, and write $x = (f, \sigma)$ and $x' = (f', \sigma)$. Then,

(i) $\mathbb{E}_x [\nu_x (\omega, t)] = \mathbb{E}_{x'} [\nu_x (\omega', t)]$;

(ii) *for any $\omega' \in \Omega$, there exist probability weights $\{\pi (\omega', \omega)\}_{\omega \in \Omega}$ such that*

$$\nu_{x'}(\omega', t) = \sum_{\omega} \pi (\omega', \omega) \, \nu_x (\omega, t) \, .$$

Proof Sketch (Cont'd)

A policy matrix $\mathbf{A}$ belongs to the attention set only if

$$\nu_{\langle f,\mathbf{A},\boldsymbol{\Sigma}\rangle}(\boldsymbol{\omega}_{K1},0) \geq \text{ a constant independent of } f$$

Garbling reduces the informativeness of extreme signals and hence the left-hand side of the above inequality:

Lemma 3.

For any $f \succeq f'$ *and* $[\mathbf{A},\boldsymbol{\Sigma}]$, $\nu_{\langle f,\mathbf{A},\boldsymbol{\Sigma}\rangle}(\boldsymbol{\omega}_{K1},0) \geq \nu_{\langle f',\mathbf{A},\boldsymbol{\Sigma}\rangle}(\boldsymbol{\omega}_{K1},0)$.

Example

News report centrist ($\omega = \pm\omega_1$) or extreme ($\omega = \pm\omega_2$), where $0 < \omega_1 < \omega_2 < 1$

News technology $f_\xi = f_{\alpha,\xi} \times f_{\beta,\xi}$, where $f_{\beta,\xi}(\omega_2 \mid a) = a + \xi(1 - a)$

$\xi \in (0,1)$ degree of slanting, $f_\xi \succeq f_{\xi'}$ if $\xi < \xi'$

Example (Cont'd)

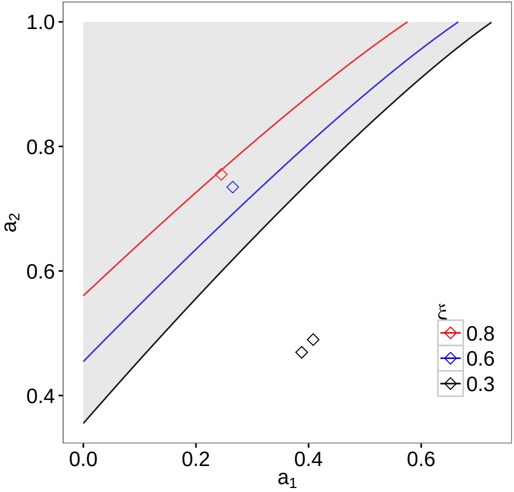

Figure 8: Equilibrium outcomes: $\alpha_+ = 3$, $\alpha_- = 1$, $R = 8$. Diamonds represent policy profiles, and shaded areas to the northwest of solid lines represent attention sets of voter $t = -.001$.

Agenda

1. Baseline model
2. Extension
3. Discussions

Literature

Rational inattention: Sims (1998, 2001); Woodford (2008); Matějka and McKay (2015); Yang (2016)

Voting with uncertainty:

- Probabilistic voting: Wittman (1983); Calvert (1985); Groseclose (2001); Martinelli (2001); Aragones and Palfrey (2002); Duggan (2005); Gul and Pesendorfer (2009)
- Signaling: Callander and Wilkie (2007); Kartik and McAfee (2007); Callander (2008)
- Bounded rationality: Yuksel (2014); Matějka and Tabellini (2015)

Literature (Cont'd)

Other voting models generating policy divergence:

- Entry deterrence: Palfrey (1984); Callander (2005); Callander and Wilson (2007)
- Citizen-candidate: Osborne and Slivinski (1996); Besley and Coate (1997); Grober and Palfrey (2014)
- ...

Monotone comparative statics: Milgrom (1981); Milgrom and Shannon (1994); Edlin and Shannon (1998)

Conclusion

Study how the need to capture voter's limited attention affect political outcomes and behaviors

Predict attention- and media-driven polarizations

Discuss historical accounts and modern implications