Abstract

Since 2008, the central banks of advanced countries have borrowed trillions of dollars from their commercial banks in the form of interest-paying reserves and invested the proceeds in portfolios of risky assets. We investigate how this new style of central banking affects central banks’ solvency. After clarifying what is central bank solvency, and how it could be in danger, we characterize three sources of risk as applied to some central banks: interest-rate risk (the Federal Reserve), default risk (the European Central Bank), and exchange-rate risk (most small open economies). We find that a central bank that pays dividends equal to net income will always be solvent, but if the central bank is not recapitalized when net income is negative, or other precautions are put in place, then reserves will tend to grow more in crises than they shrink in normal times. We compute measures of the financial strength of central banks in 2014, and discuss how different institutions interact with quantitative easing policies to put these
banks in less or more danger of solvency. We conclude that the risks to solvency are real in theory, but remote in practice today.

JEL E31 E42 E58
The Bank of Japan in the 1990s, followed by the Federal Reserve, the European Central Bank, the Bank of England and the Swiss National Bank in the 2000s, made dramatic changes in the conduct of monetary policy in response to financial crises. They borrowed trillions of dollars from commercial banks by expanding reserves and paying interest on them, while using the proceeds to enlarge their holdings of government bonds and other risky investments. Critics of the new style of central-bank policy worry that, either once market interest rates begin to rise as the crisis recedes, or if there is a sovereign default on the bonds that the central bank is holding, or if the value of the exchange rate changes suddenly, then these central banks will come under stress. Might the double burden of capital losses on central banks’ portfolios and the rising cost of paying market interest on trillions of dollars of reserves threaten the banks’ solvency?

Studies of central bank solvency have so far fallen into two strands. One strand studies how concerns about central bank capital may affect monetary policy and inflation. Like most studies of monetary policy, it takes as given that the central bank is solvent, and explores the effect of different policies on inflation. Instead, we take as given the commitment to a target for inflation, and explore whether the central bank can stay solvent. We seek to lay the groundwork for future work to simultaneously determine solvency and inflation policy. Another strand statistically characterizes circumstances when the central bank capital can be negative, focussing on the peculiar accounting rules of central banks and on the items in their balance sheet. Because of this focus, that literature’s lessons apply only to the central bank under study, usually the Federal Reserve, but do not extend to other institutional setups. We provide an economic analysis on how different central banking rules and institutions will determine whether a central bank is insolvent. Our theory is simple but flexible enough to accommodate most central banks in the world. Our formal model of central bank risk, founded in modern finance theory, describes the evolution of a central bank’s financial position, and shows when can it become unsustainable. It allows us to at the same time spell out why central banks can go insolvent, how this can happen, and which institutions and policies make it more likely.

Our approach starts by laying out the resource constraint faced by the central bank, focussing on the path of reserves, since central banks are unique among financial institutions in their power to compel banks to make loans to the central bank in the form of reserves.  

1Appendix A provides a survey of the literature.
We introduce three forms of risk to the assets of the central bank: interest-rate risk that comes with the central bank holding long-term bonds that fall in value when interest rates rise; default risk that comes with holding bonds that may become impaired in some states of the world; and exchange-rate risk that arises when central banks hold foreign currency and gold in their assets.

Section 2 spells out the meaning of central bank solvency. We clarify that what makes the central bank special is its independence from the other branches of government, so that ultimately central bank solvency is always a political issue. We define solvency as the ability of a central bank to obey the dividend rule laid out in its statutes. Only then will the central bank not have to depend on other branches of the government to satisfy its own resource constraint. We show that if the central bank always pays dividends equal to net income, including when income is negative, reserves are stationary and the central bank is solvent. But, if the dividend rule rules out negative dividends, the central bank may resort to an inflationary policy—issuing reserves without paying high enough interest to maintain stable prices—to satisfy its resource constraint. A negative dividend is, in effect, a recapitalization of the central bank. We measure the excessive reserves that result as well as the potential expected increase in inflation and propose these as metrics of the financial strength of a central bank.

We apply these measures in section 3 to appraise the current financial strength of the Federal Reserve. We show that the new obligation to pay rising interest rates as the economy returns to normal has barely an effect on outstanding reserves and does not threaten solvency. However, quantitative easing, by expanding the Fed’s portfolio of government bonds and significantly lengthening its maturity, exposes the Fed to capital losses when interest rates rise. We discuss how different policies on managing the exit from quantitative easing affect the solvency of the Fed, and how its institutional practice of not marking losses to market ameliorates some concerns. We find that in almost all conditions the Fed would be able not only to pay a reserve interest rate high enough to keep inflation on target but also to continue paying a substantial positive dividend to the Treasury. The exception is early in the recovery, when capital losses on the bond portfolio cannot be covered by immediate earnings, but could easily be covered by later earnings.

We perform similar calculations for the ECB in section 5 focussing on the risk that some of the bonds on its portfolio may default. We find that only in the case of a comprehensive
financial meltdown would the ECB’s solvency be compromised. An important consideration for the ECB is what would happen under its repurchase agreements with banks in times of financial stress. Repos are short-term debt obligations of banks collateralized by government debt and some other types of securities. They are the institutional form by which the ECB conducts monetary policy. We show that this significantly limits the risk of solvency to the ECB since the ECB suffers capital losses only if the banks fail to repay the debt and the collateral loses value by more than the repo haircut. We then assess the recent announcement by the ECB of expanding its bond buying program and show how this European quantitative easing affects the solvency of the ECB.

Section 6 turns to exchange-rate risk. We discuss how exchange rate pegs and crises affect the central bank solvency, focussing both on the typical case of a central bank trying to defend a peg, as well as the more recent experience of the Swiss National Bank. Finally, we show that a central bank that undertakes a successful inflation stabilization program will often see its solvency compromised, and illustrate with the example of emerging markets. Institutionally, we discuss how provisions against future losses can lower the risk of insolvency.

New-style central banking, with large balance sheets and payment of interest on reserves, makes the topic of central bank solvency relevant and important. On a somber note, we conclude that central banks can go insolvent, and that this may happen under a variety of scenarios. On a brighter note, we conclude that the risk of this happening for the Fed, the ECB, and the SNB is remote right now and that losses can be easily managed by a temporary buildup of reserves. Constructively, we show how several institutions, including central bank deferred accounts, mark-to-market accounting, repurchase agreements, and exchange rate pegs can have a significant effect on the solvency of the central bank.

1 The resource constraint of the central bank

A central bank issues reserves, which are one-period debt claims on the central bank held by commercial banks. We denote their real value by \( V \) in units of output. Reserves have three important properties. First, they are always honored since the central bank does not default on reserves. Therefore, in an economy saturated with reserves, the rate on reserves is essentially the same as the market nominal rate on safe short-term debt. Second, the central bank uses these liabilities to acquire potentially risky assets, effectively buying them from
banks directly. Third, reserves are the economy’s unit of value. One unit of output costs $p$
units of reserves; $p$ is the price level. The real value of a unit of reserves is $1/p$.

### 1.1 The financial environment

We use the scalar $s$ to denote the state of the economy, which obeys a Markov process in the
set $\{1, \ldots, S\}$, and has a transition probability $\omega_{s,s'}$, where a prime denotes next period’s
value. We use the terminology in the rest of the paper that a variable is *state-dependent*
if it is a function of $s$ and possibly $s'$ as well. We let $m_{s,s'}$ denote the stochastic discount
factor for a real payoff. Then, the real present value of a random real payoff one period in
the future, $y_{s'}$, is

$$V(y_{s'}) = \sum_{s,s'} \omega_{s,s'} m_{s,s'} y_{s'}.$$  \(1\)

The standard assumption is that policy sets the nominal interest rate paid on holders of
reserves between this period and the next, using a state-dependent nominal rate $i_s$. In turn,
the safe market nominal rate is equal to:

$$\frac{1}{1 + i_s} = V\left(\frac{p}{p'}\right).$$  \(2\)

We assume that the result of the policy is a state-dependent inflation rate:

$$\pi_s = \frac{p'}{p} - 1.$$  \(3\)

We do not deal with the important issue of how inflation is kept on this target. We cannot do
justice here to the modern literature on price-level determination. The important assumption
that we are making is that $\pi_s$ is exogenous to the problems that we will be studying. Our
goal is to understand whether a particular inflation policy is consistent with keeping the
central bank solvent. If the answer is no, then one would like to know how the inflation
policy might have to change, but there is enough ground to cover on solvency alone before
one can get to that discussion.$^2$

$^2$An alternative central bank policy would be a state-dependent price-level policy, so that we could write
the price level as a function of the state of the economy, $p_s$. We do not pursue this approach because a
state-dependent price level implies essentially zero inflation—every time the economy returns to a given
state, the change in the price level since the first time it was in that state is exactly zero. No central bank
has an inflation target of zero. The announced targets of central banks in advanced countries cluster around
two percent per year. It is therefore more realistic to assume a state-dependent rate of inflation with all of
the $\pi_s$s non-negative and presumably near two percent.
equation (3) assumes that the central bank commits in the preceding period to achieve a future price $p'$. As a result, the safe real interest rate $r_s$ is:

$$1 + r_s = \frac{1}{V(1)} = \frac{1 + i_s}{1 + \pi_s}.$$  

(4)

This assumption makes the real interest rate not depend on the subsequent state $s'$, which is a minor convenience in the analysis. None of our results depend on this assumption in a significant way. An important assumption is instead that $r_s$ is also exogenous. Our analysis is not of the equilibrium in an economy where the central bank interacts with private agents to determine aggregate outcomes. We might want to end there, but before even starting we need to understand the constraints facing the central bank in isolation.

Finally, no theory of inflation is complete without a statement of how fiscal policy interacts with monetary policy. To focus on central-bank solvency, we must take a stand on the solvency of the overall government. We portray the central bank as an arm of a government that is committed to a fiscal policy that satisfies the intertemporal budget constraint by adjusting taxes and spending under all possible realizations of random events, without relying on inflationary finance from the central bank.\(^3\)

1.2 The central bank’s assets and their risks

Central banks typically hold debt instruments of different maturities as well as gold and foreign currencies. Before the 2008 financial crisis, the Federal Reserve almost only held Treasury bills and bonds, while the ECB had a significant amount of gold together with repo claims on commercial banks collateralized by short-term government debt. The Swiss National Bank, as is the case with most small open economies, held a significant amount of foreign currency reserves. With the crisis, all three central banks borrowed extensively from commercial banks to increase their holdings of debt instruments to include longer maturities, higher default risks, and more foreign currency.

We lump all central-bank assets into a single category called bonds, and denote by $B_s \geq 0$ the total number of bonds held by the central bank. In the model, a bond is an instrument that pays a real coupon $\theta c_s + (1 - \theta)e_{s'}$ next period, where $e_{s'}$ is the relative price change of the non-domestic assets held by the central bank. Since these assets in most central banks consist mostly of foreign currency, $e_{s'}$ is approximately the rate of depreciation of the

\(^3\)See Hall (2013) for evidence that the U.S. satisfied that hypothesis prior to the financial crisis starting in 2008.
domestic real exchange rate and $\theta$ is the fraction of domestic assets in the central bank’s portfolio. In the second period, the bond will pay a fraction of this payoff, and then $(1 - \delta)^2$ in its third period, and so on. We add up bonds in terms of the amount of output they will pay in the current period so, each period, the bonds inherited from the previous period shrink by the factor $1 - \delta$, and $1/\delta$ is the average maturity of the portfolio holdings. The bonds, which sell for price $q_s$, then give a payoff equal to the difference between the coupon payments and the repayment of the principal by a factor $\delta$.4

Central bankers pride themselves on holding conservative portfolios and, as a result, the solvency of the central bank is often taken for granted. Yet, the asset holdings of the central bank pose at least three distinct risks, captured by our peculiar bond, and which are especially acute during financial crises.

First, there is the risk that the bonds will not pay their full coupon, a rare but possible occurrence. We assume that the coupon on the bonds is one in all states apart from the financial crisis state $S$, when bonds suffer an impairment captured by $c_S \leq 1$. This possibility is pressing in Europe, where the ECB holds a significant amount of Greek debt that its government has shown the intention to default on.

The second risk comes with changes in asset prices. In our model, the price of the bonds $q_s$ satisfies the standard no-arbitrage asset-pricing recursion:

$$q_s = V(\theta c_s + (1 - \theta)c_s' + (1 - \delta)q_s').$$

(5)

If the central bank only holds domestic short-term bonds then the bonds are riskless, but if $\delta < 1$, changes in real interest rates will cause $q_s$ to change across states, especially as the economy enters and exits the crisis state. This is a particularly pressing issue for the United States today, as interest rates are expected to rise.

The third risk comes with changes in the price of gold and foreign reserves and is captured by the exchange-rate risk $e_s'$. In the SNB’s portfolio, where $\theta$ is close to zero, this is the main source of risk.

Therefore, through three parameters, $\delta, c_S$ and $\theta$ we can control different sources of risk: interest-rate risk, default risk and exchange rate. If all three parameters are equal to one, the bond is riskless and:

$$q_s = \frac{c_s}{1 + r_s}.$$  

(6)

Appendix B shows how to obtain this composite bond from mixing investments in short-run bonds, long-run bonds, gold and foreign currency, as we observe in central banks’ balance sheets.
1.3 Seignorage and inflation

The only source of income to a central bank outside of its portfolio of assets is seignorage income from printing currency. Modern central banks keep the values of reserves and currency at exact parity by trading one for the other. Therefore, the volume of currency outstanding is not a policy variable. There is nothing to the idea that central banks can expand their portfolios by “printing money.” Rather, they can issue reserves, so $V$ is the direct policy variable, not the issuance of currency. Moreover, the volume of currency tends to be stable in advanced countries. Central banks expand their portfolios to high levels almost entirely by borrowing from commercial banks at market interest rates.

We let $N_s$ be the public’s real holdings of currency in state $s$. The central bank’s real seignorage revenue from expanding the stock of outstanding currency is:

$$n_{s,s'} = \frac{p'N_{s'} - pN_s}{p'} = N_{s'} - \frac{N_s}{\pi_s}. \quad (7)$$

Although a central bank does not control seignorage directly, by using its policy rule to keep prices on target, it affects the demand for currency and thus the amount of seignorage. Real seignorage increases with inflation until it reaches a maximum and then declines. Seignorage is a form of taxation on the real currency held by the public, where inflation acts as the implicit tax, and there is a point beyond which higher taxes reduce revenue.\(^5\) We only assume that seignorage is state-dependent, and measure the relation between a higher inflation target and extra seignorage empirically.

1.4 The central bank’s resource constraint

The central bank’s outstanding real reserves follow the law of motion:

$$V' = (1 + r_s)V + q_{s'}[B_{s'} - (1 - \delta)B_s] - [\theta c_s + (1 - \theta)e_{s'}]B_s - n_{s,s'} + d_{s'}. \quad (8)$$

In a period when the economy is in state $s'$ and was previously in state $s$, the central bank’s new level of reserves covers the following: (1) the previous level of reserves times their real payment $(1 + r_s)V$, (2) net bond purchases at real cost $q_{s'}[B_{s'} - (1 - \delta)B_s]$, (3) receipts of bond interest this period on last period’s holdings and promised coupon rate

\(^5\)Cagan (1956) is the classic derivation of this function starting from the properties of the desire for real money balances and Hilscher, Raviv and Reis (In progress) provide recent estimates of the seignorage function.
\((\theta c_s + (1 - \theta) e_{s'})B_s\) units of output, (4) receipts of seignorage \(n_{s,s'}\), and (5) payment of dividends to the government \(d_{s'}\).

Equation (8) ignores the operating expenses of the central bank because these are usually small relative to its assets. The same applies to the revenue from direct loans to banks and to the mandatory statutory dividends that central banks are often committed to pay to the providers of their paid-in capital. We also ignore the use of resources for quasi-fiscal operations such as bailing out the banking system. These are common in developing countries, but not in advanced countries, which are the focus of our analysis.

The resource constraint of the central bank holds regardless of its monetary policy. The simplest case is the helicopter-drop model of a central bank that issues no reserves \((V = V' = 0)\) and holds no bonds \((B_s = B_{s'} = 0)\). In this case, equation (8) boils down to:

\[d_{s'} = n_{s,s'}\]  \hspace{1cm} (9)

In this case, the central bank transfers its seignorage revenue to the Treasury every period, as routinely assumed in simple models of monetary policy.

With outstanding reserves that pay interest, and a portfolio of bonds that fluctuates in value, the resource constraint of the central bank becomes more interesting. In principle, either reserves or bond holdings could adjust to make the constraint hold in every state of the world. During the recent financial crisis, both the Federal Reserve and the ECB altered their holdings of bonds to specifically intervene in selected financial markets, so in the next section we will take the choices of \(B_s\) as given and take \(V\) as the endogenous variable. In the following sections we will study how different policies for \(B_s\) affect the risks of solvency.

2 The Financial Stability of a Central Bank

The traditional measure of a corporation’s financial standing is the accounting measure variously termed capital, equity, or net worth. There are typically three reasons to measure it: to calculate the residual winding up value of the corporation, to assess its market value, and to ascertain the weight of equity versus credit in the firm’s funding.

None of these reasons applies to a central bank. Central banks cannot be liquidated because their creditors cannot demand to convert their credit into anything but what they already hold, currency and reserves. Therefore, there is no meaning to the residual value of a central bank because the central bank cannot be wound up. Central banks also do not
have a meaningful market value, since their goal is not profits, and shares in the bank are typically not traded. Finally, governments own the central bank, they often deposit funds with the central bank, and most of the assets of the central bank are government liabilities. The traditional distinction between equity-holders and credit-holders is blurry and confusing for a central bank.

Nonetheless, central banks may run into financial difficulties. If private agents refuse to hold its liabilities, the central bank can no longer operate and is effectively insolvent. This may come in the form of a hyperinflation or a currency reform even if there is no bankruptcy court for central banks. So far, actual insolvency of the central bank has occurred in some countries but never in an advanced county. Because the economy is saturated with reserves, and they are just another form of a government liability, agents will not allow the central bank to run a Ponzi scheme with reserves. Insolvency happens then if reserves become non-stationary and explode at a fast enough rate.

2.1 Dividends and central bank independence

Central banks pay dividends $d_s$ to their governments. If the central bank and the Treasury were perfectly integrated, the resource constraint in equation (8) would have little bite. The dividends $d_s$ would just record the transfers between a particular branch and the overall government, but these would all be subsumed within the operations of the government as a whole. It would make little sense to talk of the solvency of the central bank instead of the solvency of the government as a whole. The individual accounts of most government agencies do not merit any special attention.

However, central banks are different from government agencies. In most advanced countries, they are a separately organized entity, analogous to a wholly owned subsidiary of a corporation, with independence to pursue its mandate. There are many definitions and measures of how independent is a central bank, but a minimal one is that it need not ask the government for resources—see Cukierman’s (2008) survey. That is, a central bank is independent as long as its dividends do not deviate from the rule that was set out in the charters that gave it its independence. We propose to measure the financial strength of a central bank as its ability to stick to that rule. Unlike accounting measures, this definition is economically motivated. It makes clear that central bank solvency is always ultimately a political issue that is connected to the dividend rule for $d_s$. As we will proceed to show,
different rules specifying the amount the bank owes the government carry implications for what the central bank can achieve while still being implementable.

2.2 The net income rule

Most central banks have dividend rules based on a well-defined concept of net income. A bank must pay all of its net income to the Treasury as a dividend. A modern central bank earns a return on its bond portfolio, pays interest on the reserves it issues, and receives an inflow of seignorage. If its net income is calculated the way an economist would, in real terms with its portfolio marked to market, the dividend would obey the real mark-to-market dividend rule:

\[ d_s' = (\theta c_s + (1 - \theta) e_{s'} + q_{s'} - q_s - \delta q_{s'})B_s + n_{s,s'} - r_s V. \]

The dividend is the sum of (1) the return on the bond portfolio, the coupon payment \( \theta c_s \) plus the capital gain \( q_{s'} - q_s \) less the “depreciation” \( \delta q_{s'} \), all applied to the number of bonds; (2) the seignorage revenue from issuance of currency, \( n_{s,s'} \); and (3) the interest paid on reserves.

In this case, a striking result follows:

**Proposition 1** Under the real mark-to-market dividend rule in equation (10), the law of motion of reserves is:

\[ V_s = q_s B_s + V_0 - q_0 B_0. \]

Here \( V_0 - q_0 B_0 \) is an initial condition. Reserves are a function \( V_s \) of the current state.

**Proof:** Substitute equation (10) for \( d_s' \) in equation (8). The result simplifies to

\[ V' - q_{s'} B_{s'} = V - q_s B_s. \]

Therefore, \( V - q_s B_s \) is constant across periods. Letting the initial condition be \( V_0 - q_0 B_0 \) gives the result. □

During a crisis in state \( S \), a central bank whose policy assigns a high value to \( B_S \) will expand its bond holdings funded with added reserves. When the crisis is over and the bond-holding rule \( B_s \) assigns a normal, smaller level of bond holdings, reserves will contract as the bank sells bonds and uses the proceeds to pay off its debt to commercial banks. As the proposition shows, reserves are state dependent and the central bank is always able to honor...
its debts whatever is the payment on reserves that it chooses. The dividend rule may, in
exceptional conditions, call for negative dividends, that is, a government recapitalization of
the bank.

The formula in equation (10) is not how a central bank’s dividends are determined. Rather, dividends focus on net worth,

\[ W = q_s B_s - V - N_s, \]

with currency \( N_s \) treated as a liability. Net income is defined as the change in nominal net
worth that would occur if dividends were zero. Then, by definition, nominal net worth \( pW \) is
constant under a rule that sets dividends equal to that concept of net income. This dividend
rule also meshes well with the notion that central banks engage in open-market operations,
where either reserves or currency—the monetary base—move one-to-one with the purchase
or sale of bonds.

If the central bank pursued a state-dependent price-level policy, we could derive results
parallel to the ones for the real mark-to-market dividend rule—real reserves would be state
dependent and have a constant additive term interpreted as an initial condition. With the
more realistic state-dependent inflation policy, where all of the \( \pi_s \)'s are positive, there are
important differences. Combining the nominal payment-on-reserves rule, with the law of
motion for reserves in equation (8) and the condition that \( p'W' = pW \) with equation (13)
gives the \textit{nominal mark-to-market dividend rule}:

\[ d_s = \left( \theta c_s + (1 - \theta) c_s' + q_s' - \frac{q_s}{1 + \pi_s} - \delta q_s' \right) B_s - \frac{i_s V}{1 + \pi_s}. \]  

The first distinction from equation (10) is that net income is calculated in nominal terms,
so that higher inflation raises measured capital gains \( q_s' - q_s/(1 + \pi_s) \). The second distinction
is that central-bank accounting treats the growth of currency and resulting increase in bond
holdings as exactly offsetting, because currency is treated as a liability, even though the
present value of its future cash burden is zero. Given the accounting convention that the
income statement is the first difference of the balance sheet, nothing makes its way to the
income statement to reveal the income from seignorage.

The constancy of nominal net worth implies

\textbf{Proposition 2} \textit{Under the nominal mark-to-market dividend rule with } \( W_0 = 0 \), reserves
follow the state-dependent law of motion,

\[ V_s = q_s B_s - N_s. \]  

13
If $W_0 > 0$, beyond this state-dependent component, there is a time-dependent component which dies away if inflation is always positive and is equal to a constant if inflation is always zero.

**Proof:** The constancy of nominal net worth implies that, in time-dependent notation:

$$q_t B_t - V_t - N_t = \frac{p_0}{p_t} W_0,$$

so

$$V_t = q_t B_t - N_t - \prod_{\tau=0}^{t-1} \left( \frac{1}{1 + \pi_{\tau}} \right) W_0 \quad (17)$$

With $W_0 = 0$ we obtain equation (15), the state-dependent component. The last term on the right-hand side is: zero if $W_0 = 0$, depends on time and dies away if $W_0 > 0$ and $\pi_t > 0$, and is constant if $W_0 > 0$ and $\pi_t = 0$ for all $t$. □

In general, from an arbitrary starting point of real net worth $W_0$, reserves are not state dependent. Rather, they are the sum of a state-dependent component $q_t B_t - N_t$ and a negative component associated with the initial level of real net worth that dies away with time. The dying away would be at a constant rate if the rate of inflation were constant across all states.

Under this dividend rule, reserves fluctuate not only because of changes in bond holdings but also as the public varies its demand for currency. As before, the central bank is able to pay its reserves at whatever level it desires, and reserves during a crisis, $V_S$, may be high but they are sustainable. Again, the rule may call for negative dividends.

### 2.3 When is net income negative?

Before the crisis, the Federal Reserve’s portfolio consisted almost exclusively of short-term government bonds. Old-style central banking was characterized by $\delta = \theta = c_S = 1$. In this case, under the real mark-to-market dividend rule:

$$d_{s'} = n_{s,s'} + r_s(q_0 B_0 - V_0). \quad (18)$$

Historically, seignorage has almost always been positive, as have been real interest rates. Therefore, in the 100-year history of the Federal Reserve under old-style central banking, the Federal Reserve always paid a positive dividend to the Treasury.
Paying interest on reserves and holding investments with interest-rate, default and exchange-rate risk opens the possibility of negative dividends. Substituting the law of motion for reserves, equation (26), into the real mark-to-market dividend rule in equation (10) describes dividends in terms of exogenous variables and the initial condition:

\[
d_{s'} = n_{s,s'} + r_s(q_0B_0 - V_0) + [\theta c_s + (1 - \theta) e_{s'} - \delta q_{s'} - r_s q_s]B_s + (q_{s'} - q_s)B_s. \tag{19}
\]

New-style central banking comes with a new term beyond seignorage and the return on capital. It is the sum of (1) \((\theta c_s + (1 - \theta) e_{s'} - \delta q_{s'} - r_s q_s)B_s\), the difference between the coupon payment adjusted for bond depreciation less the real rate on reserves, and (2) \((q_{s'} - q_s)B_s\), the capital gain on the bond portfolio. If \(\delta = \theta = 1\) this second term is exactly zero. But for a central bank that holds long-term bonds and/or foreign assets, net income is more likely to be negative when the bond repayment is impaired, when the real exchange rate appreciates, or when there is a capital loss in the bond portfolio.

Using instead the nominal dividend rule in equation (14), we find, with the same substitutions:

\[
d_{s'} = \frac{i_s}{1 + \pi_s} (N_s + W_0) + \left[\theta c_s + (1 - \theta) e_{s'} + (1 - \delta) q_{s'} - \left(\frac{1 + i_s}{1 + \pi_s}\right) q_s\right]B_s. \tag{20}
\]

The first term is the inflation tax on holders of currency, who pay it by holding banknotes instead of interest-bearing assets. The quantity within the square brackets is the difference between the actual and the expected payment on the bonds. It is zero on average, but it may well be negative if coupons or future prices fall below average. Again, impaired coupons, real exchange rate appreciation, falls in bond prices, or rises in real interest rates all make it possible for net income to be negative.

Equation (20) holds whether interest is paid on reserves or not. Paying interest on reserves requires resources, but because it also lowers net income in the dividend rule, it has no effect on the evolution of reserves or on the equilibrium dividends paid. What distinguishes old-style from new-style central banking is instead the holding of long-term, non-domestic or not safe assets, increasing the risk from changes in real interest rates, coupons, exchange rates and bond prices. If \(\delta = \theta = c_s = 1\) and only one-period domestic safe bonds were held, then the term within square brackets would be identically zero, and the central bank always has positive dividends equal to the inflation tax.
2.4 Can the central bank pay a negative dividend?

While the Fed has used the net income rule to pay its dividends, there is no formal financial arrangement between it and the rest of the government. Paying negative dividends requires receiving a capital contribution from the Treasury. The appropriation of government funds would have to be approved by fiscal authorities, subject to the political process underlying it. This opens the door to political interference in monetary policy decisions. A central bank that relies on frequent recapitalizations from the Treasury will not be financially independent and will likely also not be able to determine monetary policy independently.

If the government resists paying into the central bank, on the grounds that it amounts to a bailout, then the central bank cannot sustain the rules in propositions 1 or 2. To see this, consider the scenario in which the central bank pays positive dividends equal to net income but never receives a government appropriation when net income is negative:

\[ d_s = \max(y_s, 0), \quad (21) \]

where \( y_s \) is the measure of net income otherwise used to set dividends. In this case, the central bank issues additional reserves to compensate for the foregone payment from the government, in the amount of the shortfall:

\[ z_s = d_s - y_s. \quad (22) \]

Consider first a central bank subject to the real mark-to-market dividend rule. An increase in reserves resulting from a positive shortfall \( z \) has the same permanent effect that we analyzed in connection with the initial condition in proposition 1. We let \( Z \) denote the cumulative sum of the \( z_s \), so \( Z' = Z + z_s' \). It is not a function of the current state alone. Rather, since equation (21) and equation (22) imply that \( z_s \) is always non-negative, as long as there is a positive probability of negative net income, then \( Z \) is a weakly increasing sequence over time that will tend to infinity. Retracing the steps that led to proposition 1 gives:

**Proposition 3** If the central bank pays real mark-to-market dividends according to the rule in equation (21), with net income given by equation (10), reserves are:

\[ V_t = q_t B_t + (V_0 - q_0 B_0) + Z_t, \quad (23) \]

so they drift upward without limit and are non-stationary.
At some point along this path where reserves run off to infinity, the central bank would likely be dissolved and a new currency introduced. From a different perspective, as reserves increase without bound, so does the interest expenses of the central bank, driving net income to unbounded negative realizations. The rule in equation (21) is unsustainable as it violates a no-Ponzi-scheme condition on central bank reserves.

The situation is somewhat different in the more realistic case of a central bank subject to a nominal mark-to-market rule and positive state-dependent inflation. Combining the rule in equation (14) with the constraint that dividends are non-negative in equation (22) into the law of motion for reserves in equation (8),

\[ W' = \frac{W}{1 + \pi_s} - z'. \] (24)

Whenever dividends are higher than net income, so \( z \) is positive, then net worth falls. In nominal terms, \( p'z' \) gives the the decline in nominal net worth this period.

Let \( pZ \) now denote the shortfall of nominal net worth from its initial level \( p_0W_0 \). With zero inflation, this is still the cumulative sum of the \( z \)s, but with positive inflation,

\[ Z' = \left( \frac{1}{1 + \pi_s} \right) Z + z'. \] (25)

Inflation makes the real shortfall in net worth fall over time if the \( \pi_s \)s are positive. In this case, \( Z \) obeys a stationary autoregressive time-series process.

Given the law of motion for net worth, the same steps that led to proposition 2 now show the following result:

**Proposition 4** If the central bank pays nominal mark-to-market dividends according to the rule in equation (21), with net income given by equation (14), reserves are:

\[ V_t = q_tB_t - N_t - \prod_{\tau=0}^{t-1} \left( \frac{1}{1 + \pi_\tau} \right) W_0 + Z_t. \] (26)

The term \( q_tB_t - N_t \) is state-dependent. If inflation is positive, the term in \( W_0 \) dies away and \( Z_t \) is stationary, so reserves \( V \) are stationary. If inflation is always zero, the term in \( W_0 \) is constant, and \( Z_t \) drifts upwards without limit, so reserves \( V \) are non-stationary.

A corollary of the proposition is that nominal reserves \( pV \) are non-stationary and track \( pZ \). Inflation therefore plays an important role with a nominal mark-to-market dividends rule. If inflation is zero or, similarly, if the price-level is state-dependent, then reserves again
shoot to infinity as a result of the Treasury not paying into the central bank. With positive inflation though, real reserves are stationary. A policy that tries to keep nominal net worth constant leads to falling real net worth when inflation is positive. As a result, when the central bank fails to receive funds from the Treasury, inflation over time erodes this real shortfall and keeps real reserves stationary.

2.5 Deferred assets and retaining earnings

Most advanced central banks have some discretion over their dividends—they are able to recover from the issuance of reserves in lieu of recapitalization by paying subsequent dividends that are less than net income. At the same time, the spirit of the rules in the charter of most central banks prevents the central bank from accumulating a large surplus on the government, and gives the Treasury discretion to reclaim this surplus through statutory transfers.

We model the relationship by hypothesizing an account balance $D$ that measures the backlog of negative net income realizations that the Treasury did not cover by recapitalizing the central bank. This is a deferred asset capturing the claim that the central bank has on its own future remittances to the Treasury. According to the accounting rules of the Federal Reserve it would appear as a negative liability. When $D$ is positive, the central bank can draw it down by paying a dividend $d'$ less than net income $y'$. On the other hand, when $y'$ is negative, the balance $D$ rises by $-y'$. The dividend rule is

$$d' = \max(y' - D, 0).$$

We also assume that an upper limit $\bar{D}$ applies to the balance. Central-bank charters are vague about this limit but they allow the Treasury to, at its discretion, reclaim the surplus accumulated by the central bank. It is quite plausible that if the balance in $D$ was high the Treasury would perceive the payments associated with this debt to the central bank as an effective recapitalization. A balance above $\bar{D}$ would put in question the independence of the central bank in the same way that we argued before when we set this limit to zero. At the limit, $\bar{D}$ must be below the present value of seignorage, otherwise the central bank would need to be recapitalized in present value.

Balances in the $D$ account decline with inflation, matching the decline that occurs in the extra component of reserves, $Z$. Putting all of these elements together, we get the law of motion,

$$D' = \min \left( \bar{D}, \frac{1}{1 + \pi_s} (D - \max(y' - d', 0) + \max(-y', 0)) \right).$$

(28)
As before, the law of motion of the bulge in reserves caused by missed recapitalizations, $Z$, is

$$Z' = \frac{1}{1 + \pi_s} Z + d' - y'.$$

(29)

This provision, with a reasonably generous value of $\bar{D}$, will cancel a bulge of reserve issuance following an episode of negative net income by cutting subsequent dividends and using the funds to pay off the bulge of reserves. The degree of protection depends on $\bar{D}$ and the frequency and magnitude of negative incomes compared to offsetting positive incomes in normal times. But in an infinite lifetime, the buildup of reserves $Z$ will rise above any finite level with positive probability at some time. Allowing the central bank to pay low dividends to recover from a buildup lowers the probability that reserves will reach a high level but does not prevent it.

The message from these results is that, whatever the dividend rule or the inflation policy, negative net income poses a challenge. If the Treasury does not recapitalize the central bank, reserves will have to rise. In all cases, reserves could with positive probability become arbitrarily high, and in some cases may even become non-stationary.

### 2.6 Quantitative measures of the financial strength of a central bank

Facing a shortfall in its capital caused by negative net income, a central bank can only obtain funds to prevent reserves from jumping to high levels from two sources: the Treasury or seignorage. Either of them puts in question the central bank’s ability to pursue its mandate, the first by jeopardizing its independence, and the second by requiring inflation above target. These two sources of the resources needed by the central bank provide two different measures of its financial strength.

The first measure is the value of $Z$. This is the shortfall in its capital at a date in time, and also the resulting excess reserves outstanding. After this amount is paid into the central bank, reserves can revert to their state-dependent levels in propositions 1 and 2. A central bank relying on a large discretionary intervention of the Treasury is financially weak, and it is weaker the larger is the required amount $Z$. While we started this section by arguing that the usual arguments for the usefulness of accounting capital or equity do not apply to the central bank, under the nominal mark-to-market rule, $Z$ is precisely the shortfall in the net
worth of the central bank. Our economic analysis therefore provides a justification for why central bank capital matters.

Instead of relying on the Treasury, the central bank can raise seignorage by increasing its inflation target. This generates the resources to raise net income to either bring $Z$ down or prevent negative net income from occurring in the first place. A second measure of the financial weakness of a central bank is the extra inflation it needs to bring about in order to maintain the stationarity of reserves.

3 The Federal Reserve

The main lesson from the previous section is that the solvency of a central bank depends crucially on what happens to its dividends when net income is negative. Prior to 2008, the Fed held mostly safe short-term Treasury debt with small potential capital losses and it never paid interest on reserves. As a result, net income was always positive. During and after the crisis, the Fed has functioned as a highly profitable hedge fund, borrowing at low rates in the short-term market to fund holdings of higher-yield longer-term debt generating substantial positive net income.

The solvency of the Fed is at stake when the recovery occurs. At that time, interest rates increase, which both raises the payment on reserves and lowers the value of the Fed’s portfolio of longer-term bonds. In this section, we simulate what would happen to reserves and dividends under different scenarios for the recovery. Any answer to the question of how the federal government would deal with a situation calling for recapitalization of the Fed is strictly conjectural. Our approach is to take as given real interest rates and the inflation target, and to study how different institutions for calculating dividends and for managing the exit from the crisis would affect its solvency. But first we describe how our stylized model approximates the Federal Reserve and how we measure the relevant variables and calibrate the model.

3.1 The Fed and the model

A central bank’s institutions in our model are captured by the triple $(\delta, c_S, \theta)$ and by a dividend rule. The Fed barely has any foreign investments and it holds almost exclusively securities issued by the Treasury or agency securities guaranteed by it. For the Fed, $c_s = \theta = 1$ in our model.
Of course the Fed does not hold delta bonds, but rather Treasury bills, notes, and bonds of various maturities. All actual Treasury instruments pay off face value at maturity, rather than melting away like a delta bond, and notes and bonds make periodic coupon payments as well. There is no computational obstacle to building a model that keeps track of the historical path of purchases and sales and record gains and losses only upon sale. But the complexity of the model would stand in the way of the points we want to make in this paper. Instead, we use the data in the annual report of the Federal Reserve on the value and maturity of the Treasury securities it holds, to calculate the value-weighted average maturity of the Fed’s financial assets. The parameter $\delta$ is the reciprocal of the average maturity, and between 2009 and 2013, $\delta$ is on average 0.128.\footnote{We keep $\delta$ constant throughout, but historically $\delta$ was larger, and looking forward perhaps the Fed will reduce the maturity of its portfolio as it exits the crisis. Variations in $\delta$ by state, though feasible, would much complicate the model. Our procedure is reasonably accurate, because the central risk to the Fed occurs upon exiting the crisis state, when the portfolio is large. Prior to the crisis, the portfolio is smaller, so our exaggeration of the capital gain at the beginning of the crisis is not large.}

We describe the Fed’s financial stability under the hypothesis that it pays dividends according to the nominal mark-to-market rule discussed in the previous section. In fact, the Federal Reserve does not mark its bond portfolio to market, so unrealized capital gains do not enter into its calculations of net income and are not paid out as dividends. We discuss this practice and how it affects the results in section 3.7.

### 3.2 The states of the economy

Our annual data cover the period from 1954 through 2013. To define the states of the U.S. economy, we use the realized real T-bill rates, measured as the nominal rate on one-year bills minus the increase in the Consumer Price Index. We allow for 5 states, where the first four correspond to the behavior of the interest rate until 2008, and the real rates for these states are the medians within the quartiles of the distribution of the rate over that period. Since then, the economy has been in state 5, the crisis state, with a real rate essentially minus the inflation rate, because the nominal rate has been close to zero. The transition into state 5 from 2007 to 2008 occurred out of state 1.

Our estimates imply that the economy has a financial crisis with a probability of 1.7%. Because the economy has yet to transit out of the crisis state, we took the exit rate to be 20 percent per year as a rough estimate, and assumed the transition was into state 3, the median state. We therefore assumed that a crisis has an expected life of 5 years.
estimated transition matrix appears in Table 3 in Appendix C, together with the implied stationary distribution across states.

### 3.3 The inputs of the model

We use data from the annual financial statements of the Federal Reserve System to fill in the variables in the model. Table 1 shows each of these variables at each state of the economy.

We measure the holdings of bonds \( q_s B_s \) as the total U.S. Treasury and agency securities held by the Federal Reserve system. To calculate currency holdings \( N_s \), we use currency in circulation. The difference between the two is close to the reserves outstanding, as the Fed’s accounting net worth is relatively small. We divide all nominal variables by GDP to express them in real terms as a percentage of GDP, and use the GDP deflator as our measure of inflation \( \pi_s \). Note that in the crisis state, the bond portfolio of the Fed is more than twice as large in the other four states.

The last column of the table shows asset prices of our artificial delta-bond. Recall that the price of delta-bonds is given by

\[
q_s = \mathbb{V}(c_s + (1 - \delta)q_{s'}).
\]

We use this delta-bond price to account for capital gains and losses on the portfolio of the Fed. To apply the \( \mathbb{V} \) operator, we need to have the stochastic discount factor \( m_{s,s'} \). A
A reasonable class of SDFs satisfies
\[ m_{s,s'} = \beta \frac{\mu_{s'}}{\mu_s}. \]  
(30)

For example, in the consumption CAPM with time-separable expected utility, \( \mu_s \) is marginal utility. Under this assumption, the definition for the real interest rate gives a condition that the SDF must satisfy:
\[ \beta (1 + r_s) \sum_{s'} \omega_{s,s'} \mu_{s'} - \mu_s = 0. \]  
(31)

With the normalization \( \mu_1 = 1 \), this is a system of \( N \) equations in \( N \) unknowns, \( \beta, \mu_2, \ldots, \mu_N \) that takes as inputs the data on \( r_s \). Though nonlinear, the system solves easily by standard methods. Bond prices then solve the linear pricing recursion. Appendix C shows the auxiliary parameters that come out of applying the formula.\(^7\)

### 3.4 The Fed in crisis and recovery

We begin with the case where the Treasury recapitalizes the Fed whenever net income is negative, so that proposition 2 holds and there are no concerns about solvency. In this case, reserves are state-dependent (we assume any initial effect \( W_0 \) has long since melted away on account of positive chronic inflation).

In the ergodic distribution of the economy, the Fed earns a positive net income in every single pairs of succeeding states but for one. Only when the economy exits from the crisis, does the Fed incur losses. They are a substantial 0.8% of GDP but only happen once every 59 years in the ergodic distribution. Moreover, when the Fed enters a crisis, its dividends are unusually positive at 0.8% of GDP, and while the crisis lasts, net income is 0.3% of GDP. Therefore, on average, the Fed is solvent under new-style central banking.

It is only during a crisis that the Fed’s solvency may be at stake. We proceed by tracking the Fed’s finances during an extreme but still plausible worst-case scenario, where the economy moves over a 15-year period from state 1 to a crisis in state 5 for five years and a recovery in state 3 for the remaining 9 years. Throughout, we assume a 2-percent inflation target and set the initial conditions to match the level of reserves in 2007.

At the outset of the crisis, the Fed borrows $1.21 trillion from the banking system to buy bonds. High levels of bond holdings financed by high levels of reserve borrowing continue until the end of the crisis, when the economy shifts to state 3. The Fed sells about $1.28 trillion in bonds during the recovery period.

---

\(^7\)This approach to asset pricing was first taken by Hall (2013), and is closely related to the approach later taken by Ross (2015).
trillion in bonds and uses the funds to retire a similar volume of reserves, which decline to almost their initial level. Reserves are elevated during the crisis state, but are otherwise low and stable during the other states. In turn, the nominal interest rate starts at a fairly high level prior to the crisis, plunges to zero for the crisis years, and then resumes a moderate positive level for the remaining time in state 3. The bond price follows the price level on a trend, but rises at the beginning of the crisis and falls when the crisis ends. Therefore, the Fed faces significant risk during the recovery, when the fall in bond prices causes capital losses in its large portfolio, and the rising interest rate raise the payment on reserves.

Figure 1 shows the Fed’s dividend to the Treasury and the flows that determine it. The least important determinant is the payment on reserves, which appears negatively because it is a deduction from the dividend. During the crisis the interest rate paid is zero, and outside of the crisis reserves are small so the interest paid has a barely visible negative effect. The coupon earnings from the bond portfolio less depreciation are likewise of little importance. Deducting depreciation is the equivalent of not counting the return of principal as part of the earnings from a bond. The most important determinant of dividends is the capital gain on the bonds. The Fed records a capital gain upon entering the crisis state, with negative real interest, and pays a corresponding dividend. Upon exiting the crisis state, the Fed suffers a capital loss. The loss is larger than the gain because it applies to the large portfolio acquired during the crisis.

Figure 2 shows the flows that raise or reduce reserves. Interest on reserves is a minor factor contributing to growth and seignorage is a minor factor contributing to shrinkage. The big factor is purchasing and selling bonds. When the crisis strikes, the Fed expands reserves to buy bonds; when it ends, the Fed sells a large volume of bonds and pays down reserves. Dividend payments, which add to reserves, also have noticeable roles at the beginning of the crisis—when capital gains from the lower interest rate accrue and are paid to the Treasury—and at the end of the crisis—when capital losses from the higher interest rate accrue and the Treasury bails the Fed out. The negative dividend payment at that time is effectively a recapitalization of the Fed.

3.5 The solvency of the Fed

If the Treasury has a policy of no recapitalization, the Fed must borrow from the banking system by issuing reserves to cover negative income. The adoption of a dividend rule that
Figure 1: Components of the Fed’s Dividend to the Treasury

Figure 2: Flows Into and Out of Reserves
causes the Fed’s debt to the banking system to rise to ever-higher levels in each crisis could compromise the Fed’s solvency.

The Treasury allows the Fed to retire the extra reserves through the $D$ account we described earlier. We take the upper limit to be $D = 0.02$ or 2 percent of GDP, a limit that is not binding in our scenario. The result is that the balance $D$ is the same as the extra reserves $Z$, which is our dollar measure of the financial weakness of the Federal Reserve.

Figure 3 shows the operation of the $D$ account in our earlier scenario. Until the crisis ends, dividends equal net income, resulting in a large payout to the Treasury when the Fed’s bond portfolio appreciates at the onset of the crisis. When the economy recovers, net income is negative for a year. The Fed issues extra reserves to cover the absence of recapitalization. The balance in the $D$ account rises by $119$ billion, along with a bulge in reserves of the same amount. For the next 5 years, the Fed pays the Treasury zero dividends and gradually works of the balance in $D$ and the extra reserves that had been issued when the crisis ended. Thus, the Treasury does not make a cash payment to the Fed, but the equivalent happens over time, as the Treasury foregoes dividends in equivalent amount. The following year, the Fed pays a positive dividend but less than net income. In the succeeding years, $D = Z = 0$ and reserves are back to their state-dependent normal values.

An alternative measure of the solvency of the central bank is by how much inflation would have to be raised forever when the Fed has negative net income to pay off so that there is no balance in the $D$ account. Providing an accurate estimate would require having reliable estimates of the seignorage function. Instead, we provide a lower bound by assuming that the higher inflation would have no effect on the real holdings of currency thus maximizing the seignorage that the central bank could generate. This measure is 15.2% extra inflation in this case, a large amount reflecting the limited ability to raise seignorage via higher inflation.

3.6 Managing the balance sheet: speed and timing of the exit

Our conclusion that the danger to the Fed’s solvency comes from the capital losses when exiting the crisis, suggests that anything that lowers the fall in bond prices reduces the danger of insolvency. To investigate this conclusion, first we make the exit from the crisis occur to state 1, rather than 3. The fall in bond prices when exiting the crisis is now higher and so are the capital losses. Figure 4 shows that the amount accumulated in the $D$ account is now much larger, verifying that capital losses are indeed the key danger to the Fed. Our
conclusion that the Federal Reserve is still at little risk is also approximately unchanged. Now, the maximum recapitalization the Fed would need is $176 billion. It takes now 10 years, instead of 6, reclaiming dividends from the Treasury to bring this amount to zero.

How can the Fed lower this solvency risk? One possibility is to manage the exit from the crisis differently. In particular, we assumed that the Fed shrinks its balance sheet back to pre-crisis levels right as the recession ends. Because it sells a large share of its portfolio at the same time as real interest rates rise and bond prices fall, the Fed realizes a large loss. The Fed has suggested instead selling its crisis portfolio slowly. Figure 4 shows what would happen to the Fed’s solvency if it took 5 years to gradually sell the bonds bought during the crisis. This “slow winding down” policy makes almost no difference to the realized net income.

If shrinking the balance sheet slowly makes little difference, perhaps doing the opposite helps. The opposite, in this case, consists of reversing quantitative easing before interest rates rise. We add a sixth state and repeat our simulations to calculate the net income and solvency of the Fed if in that sixth state, the balance sheet falls to its pre-crisis size, but the nominal interest rate stays at zero. The following period, the crisis is over as we transition
Figure 4: The financial strength of the Fed under alternative policies and institutions back to state 3. Figure 4 shows that selling before would lower the risks to the Fed’s solvency. But still, the $D$ account stays positive for two years.

A more prolonged and slower exit from quantitative easing therefore raises the risks of solvency. But managing the balance sheet does not eliminate the risk to the Fed’s solvency. As long as the Fed marks its portfolio to market, it will recognize the losses at the end of the crisis, whether it sells the bonds faster or slower.

3.7 Institutional remedies: marking the Fed’s portfolio

All the policy counterfactuals so far assumed that the Fed’s bond holdings were marked to market each year in determining net income $y$. The Fed is more likely to have trouble under this dividend rule than under its actual dividend rule, because it earns capital gains on its portfolio upon entering the crisis state, which it would pay out immediately to the Treasury. In actuality, the Fed retains those funds unless it sells the bonds. It computes net income as if the bond were invariably worth its nominal purchase price, which is usually quite close to the nominal principal returned at maturity (we skip over the intricacies of accounting for a difference between the purchase price and the principal amount).
It is difficult to capture book to market accounting accurately in our setup. But, if the Fed sells all of its bonds when interest rates rises, as in our baseline case, then this would make little difference. When the crisis ends, the fall in bond prices would coincide with the Fed bringing the size of its assets back to its pre-crisis level. Therefore, the Fed would sell most of its bonds and realize the capital losses, just as in our mark-to-market benchmark.

Instead, if the Fed slowly winds down its portfolio, there would be a noticeable difference. Because most of the bonds the Fed bought during the crisis would mature in the post-crisis years, their losses would take the form, not of the immediate capital loss that we record, but rather of a stream of coupon payments at rates lower than the current market rate during the crisis years. By recording the bonds at an artificially high price on its accounts, the Fed would record lower returns on them than the market interest rate that it pays on reserves. The interest paid on reserves now shows up as a leading source of losses, for several years after the crisis, instead of the immediate capital loss at the time of the exit. Importantly, the present value of dividends is the same: accounting rules do not offer free lunches.

Figure 4 shows that path of the $D$ account with a slow winding down, and spreading down the capital losses over the 5 years of the winding down. Now, when the economy exits the crisis state, the Fed’s dividend records a small loss only associated with the selling of one fifth of the Fed’s bonds. In the following years though, the payment of interest on reserves leads to the Fed making persistent losses for 5 years. When the crisis ends and the Fed’s income is low, both because of low coupons on its holdings of bonds purchased during the crisis years and because of paying now-higher rates on reserves, the Fed does through a long period of losses. The amount in the $D$ account actually rises over time.

The actual policy therefore has the effect of smoothing the Fed’s dividend relative to our assumption of marking to market. Actual practice avoids a big payment at the beginning of the crisis followed by a recapitalization by the Treasury upon recovery. The difference that marking to book, instead of marking to market makes, is to the timing of dividends. Instead of recording a large bulge of income in one year, the same bulge gets spread over several years. Insofar as it is the maximum amount carried in the deferred account that threatens the Fed’s independence, then this accounting convention is effective at keep in the $D$ account below its $\bar{D}$ upper bound. Insofar as it is the number of years with a positive balance in the $D$ account that matters, then the risk to solvency ois the same as under our baseline.
4 The European Central Bank

The European Central Bank is the coordinating agency of the Eurosystem, comprising the ECB and the national central banks of the euro countries. Throughout our discussion, we use the name ECB, though our law of motion for reserves describes the Eurosystem, while our analysis of solvency relates more closely to the ECB. Insofar as no country leaves the euro-area, this distinction is unimportant.

4.1 The ECB and the model

Since late 2008, the ECB has pursued policies similar to those of the Federal Reserve. Its assets have risen substantially funded by borrowing from banks by issuing reserves, which pay close-to-market interest rates. In terms of the maturity of these assets, it is hard to choose $\delta$ in our model, because the ECB does not publish detailed data on the securities that it holds. We set $\delta = 0.128$, the value that we estimated for the United States.

Like most central banks, but unlike the Fed, the ECB has traditionally held a large share of its assets as gold and foreign reserves. While the expansion of the balance sheet after 2008 involved almost only European securities, therefore reducing the share of gold and foreign reserves, they were still on average 0.538 of the direct holdings of assets by the ECB after 2008. We set $\theta$ to this number.

More interesting, the ECB faces significant default risk on its holdings of sovereign bonds from countries in the periphery. In the model, the ECB suffers a default loss of $1 - c_S$ of its coupon expectation every period that it remains the financial crisis state. Note that the bond-pricing equation assumes that all bonds resume paying full coupon rates once the economy exits state $S$—the bank does not lose a fraction $1 - c_S$ of the value of its bond holdings. In our scenario, the economy spends 5 consecutive years in the crisis state, which is also the expected duration of crises in general. A shortfall in the coupon of one percent ($c_S = 0.99$) for one period lowers the value of a bond by 1/6 of a percent, because the bond sells for about 6 units of output. Thus a loss of one percent for 5 consecutive years lowers the value by about $5/6 = 0.83$ of a percent. As an extreme but not implausible scenario, we consider the possibility that the ECB’s bond holdings of Greek, Irish, Italian, Portuguese and Spanish bonds all go into default and that the haircut is 65%, as happened during the Greek bailout of 2012. This case corresponds to $c_S = 0.884$. 
Table 2: Inputs

<table>
<thead>
<tr>
<th>State number</th>
<th>Safe rate, $r$</th>
<th>Direct bond holdings, $q_{Bd}$</th>
<th>Repos, $q_{Br}$</th>
<th>Currency, $N$</th>
<th>Inflation, $p'/p-1$</th>
<th>Coupon on foreign assets, $e$</th>
<th>Anticipated default</th>
<th>Unanticipated default</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.276</td>
<td>6.074</td>
<td>4.867</td>
<td>6.288</td>
<td>0.063</td>
<td>4.786</td>
<td>7.096</td>
<td>7.133</td>
</tr>
<tr>
<td>2</td>
<td>-0.235</td>
<td>12.209</td>
<td>8.393</td>
<td>9.374</td>
<td>0.094</td>
<td>6.139</td>
<td>7.070</td>
<td>7.468</td>
</tr>
</tbody>
</table>

4.2 Data and calibration for the euro area

Our data for Europe cover the period from 2000 through 2013. We take the exogenous real interest rates to be the 1-year Euribor rate minus inflation measured by the euro-area Harmonized Index of Consumer Prices. We define two states, one between 2000 and 2008, and the crisis covering 2009-13. With only one observation of each, it is impossible to estimate a transition matrix. Instead, we take the probabilities associated with a crisis to be the same as for the United States—the euro-area economy enters a financial crisis with probability 0.016 and the stationary probability of a crisis is 0.081. Our source for the ECB’s finances is its last weekly financial statement published each year. For the depreciation in the holdings of foreign assets, we take a weighted average of the change in the price of gold and in the real effective exchange rate, with weights given by the shares of gold and foreign securities in the ECB’s portfolio.

Table 2 shows the key exogenous variables for the ECB. As in the United States, the crisis corresponds to the nominal interest rate falling to zero, and the size of asset holding by the ECB more than doubling. Asset prices increase during the crisis, and the table shows the change in asset prices under two circumstances: if markets perceive the possibility of a default, or if they believe instead that $c_S = 1$. In a crisis, the lower real interest rates raise asset prices, whilst the impaired coupon lowers them. Therefore, when there is default, the change in asset prices getting into and out of the crisis is smaller, and so will be the capital gains and losses.

4.3 Default and the ECB’s solvency risk with direct holdings

To save space, we do not repeat the same set of figures for the ECB that we presented for the Federal Reserve, but they are available from appendix D. As in the United States, a crisis
comes with an increase in bond prices and a fall in nominal interest rates, and an expansion in bond holdings together with reserves. Looking at the components of the dividend paid, they are dominated by capital gains and losses, while the evolution of reserves is likewise dominated by the cost of buying and selling bonds and collecting coupons while suffering depreciation.

Figure 5 instead shows the predicted path for net income and for the balance in the $D$ account after a 5-year crisis as in the last section. The figure shows three cases. In the first case, there is no default ($c_S = 1$) in the crisis, whereas in the other two, coupons are impaired. The difference between them is that, in one, which we call “unanticipated default”, the prices of the bonds do not reflect the possibility of a default. In the second case, of “anticipated default”, the prices of the bonds in both periods reflect the rational expectation that the economy may enter state 2 and default happen. The first case tries to capture the possibility that the default comes as a surprise to the markets, that bond prices did not anticipate, whereas in the second case the central bank would have bought the bonds that fueled the expansion in its balance sheet at a low price, given their eminent default.
Like the Fed, the ECB faces a risk to solvency when exiting the crisis, as asset prices fall lowering net income. However, if the default is anticipated, income stays positive throughout. On the one hand, the flow of coupon income is lower than before, and no longer covers the depreciation of the bond as before, lowering net income. But, on the other hand, the lower coupon implies that bond prices during the crisis are lower.

Only when default is not expected to happen do asset prices fall sufficiently when exiting the crisis so that the capital loss offsets the other sources of regular income of the ECB. In the anticipated default case, the ECB is actually better off than if there is no default. The larger fall in asset prices in the latter case more than offsets the loss from the impaired coupons. This confirms our conclusion that capital gains and losses in the central bank’s portfolio due to asset price movements is the main source of solvency risk.

Still, even as the ECB sells €665 billion of its assets, it only loses at most −€31 million, which is repaid within three years. The loss is smaller than for the Fed, and there is an outstanding balance in the D account for fewer years. The main reason is that our calibration points to a smaller change in asset prices when the economy exits the crisis.

4.4 Institutional remedies: repo holdings

The baseline scenario considered only the direct holding of securities by the ECB. However, the ECB has traditionally conducted its monetary policy using repurchase agreements with banks. The repos are effectively term lending to commercial banks collateralized by government bonds or government-guaranteed securities. The ECB requires collateral in excess of the value of the loan, by as much as 15 percent (the haircut), and frequently revalues its collateral using market prices and requires extra collateral to be posted to lower its credit risk. A credit loss from a repo involves both default by the borrowing bank and an impairment of the value of the collateral in excess of the haircut. Default would result in reorganization of the bank and its exclusion from future borrowing from the ECB. The ECB would suffer credit losses on its repos only in the event of a general financial meltdown in the euro countries.

Repos have floating rates indexed to the one-week repo rate, so the relevant interest rate is still the short-term safe rate \( r_s \). We let \( q_s B_s^d \) denote the real value of the securities held directly and \( q_s B_s^r \) denote the real value of repos, so the total assets of the ECB are \( B_s = B_s^d + B_s^r \). If a meltdown has low probability, we can treat repos as safe loans, with
no risk of capital gains or losses to the ECB. In that case, repos have the same financial character as negative reserves. Issuing reserves and investing the proceeds in repos has a neutral effect on solvency because this asset and liability earns exactly the same interest rate. Likewise, any coupon payments or capital gains and losses stay with the bank, not the ECB. Therefore, all the formulas from section 2 still apply as long as we replace $B_s$ with $B^d_s$ and $V$ with $V - q_s B^r_s$.

At the other extreme, with a significant probability of meltdown, the ECB faces the possibility of capital losses, when euro banks become insolvent and the ECB’s collateral loses value by more than the haircuts. In this case, we need to take into account the asymmetry inherent in ownership of any debt claim, that the owner will never receive more than face value but may receive less. Letting $I$ be an indicator for the bank’s failure to pay off, net income under the nominal mark-to-market rule becomes:

$$d_{s'} = \left( \theta c_s + (1 - \theta)e_{s'} + (1 - \delta)q_{s'} - \frac{q_s}{1 + \pi_s} \right) B^d_s - \frac{i_s (V - B^r_s)}{1 + \pi_s} + I \left( \theta c_s + (1 - \theta)e_{s'} + (1 - \delta)q_{s'} - \frac{(1 + i_s)q_s}{1 + \pi_s} \right) B^r_s$$

The term preceded by $I$ says the ECB suffers the capital loss if the bank fails to pay off the repo, but also no longer pays the interest rate due on the repo contract. Our earlier analysis of the conditions when reserves are stationary continues to apply. But in terms of our simulations that follow, the change when banks fail to pay off on repos on impaired assets can be substantial.

Before 2008, the duration of the repos was mostly one week, with the longest being 3 months, so it would take extremely wide price movements in a short period of time for a bank to become insolvent and not honor its repos with the ECB. Since 2008, 6-month and 12-month repos became dominant, and in December 2011 and February 2012, the ECB lent €1 trillion through 3-year long-term refunding operations, effectively three-year repos. Taking a worst case scenario, we assume that $I = 1$ if the return from holding the bonds is negative. That is, banks do not pay off the repo when the economy exits the crisis. Figure 6 shows the net income and solvency of the ECB under different perspectives on the repos. In the first case, ‘safe repos”, we take the view that only bonds held directly lead to risk, as in the previous figure. This scenario corresponds to $I = 0$ in the law of motion. In the second case, we take the other extreme view that all of the bonds held by the ECB are direct holdings, whether they are collateral for repos or not, in the same way we did for the
Federal Reserve. Finally, in the third scenario, more realistically, we take the asymmetric case with \( I = 1 \). The ECB bears capital losses in excess of haircuts, but not gains, on the bonds that it holds as collateral. For all cases, we assume the case of unanticipated default, since this was the one for which losses were larger in our previous experiment.

If banks always honor their repurchase agreement, the ECB is in less danger than the Fed, as we discussed before. If instead the repos are not honored, then when asset prices fall, the holders of the repurchase agreements hand in the collateral to the ECB. The losses rise to €52 billion. Compared to the case were the ECB held all securities directly, with the repos, the ECB earns less while entering and staying in the crisis because the positive capital gains on the bonds stay with the banks. But, when there are large capital losses, then the possibility that repos are honored significantly lowers the risk to the ECB’s solvency.

4.5 Policy options: the risks of quantitative easing

Initially, in response to the crisis, the ECB expanded its repo program by both extending its maturity and increasing its size. These long-term repurchase obligations (LTRO) extended the usual procedures of operation of the ECB although, by continuing to accept government
securities that most credit agencies rated as junk and doing so for longer periods of time, some of these new repos were farther from the "safe repo" case and closer to "risky repos". At the same time, the ECB also bought securities directly through its Outright Monetary Transactions program (OMT), accounting for the increase in securities directly held in Table 6.

At the start of 2015, the ECB announced a new policy that, the Public Sector Purchase Programme (PSPP), whereby it would buy €1080 billion in government securities. This policy of quantitative easing will roughly double the value of securities held directly by the ECB.

To investigate the effect of this policy in the solvency of the ECB, we add a third state of the world. It is in all identical to the crisis state, but in which direct bond holdings double relative to what they have been. This state lasts for two years, before the economy finally exists the crisis, back into state 1. Figure 7 considers the effect of this policy in the solvency of the ECB.

If there is no default, so that the securities that the ECB plans to buy turn out to be safe, the larger balance sheet implies that capital losses at the time of the exit are now significantly
larger. As a result, the ECB accumulates a balance in its $D$ account that only disappears 4 years after the exit. If coupons turn out to be unexpectedly impaired, the losses are much larger peaking at €73 billions and take a long time to recover. Engaging in quantitative easing raises the risk of solvency relative to enlarging the repo program. Of course, this may be more than offset by potential benefits of this policy in pursuing the ECB's policy goals.

5 The central bank of a small open economy

For most central banks around the world, the balance sheet has neither repo contracts, nor holdings of long-term bonds with losses that are accounted in peculiar ways. The risk of default on those assets is negligible, and the maturity is so low that the interest-rate risk is mute. Rather, the balance sheet of central banks of small open economies has currency and reserves as its liabilities, and gold and foreign exchange as its dominant assets. For these banks, $\delta = c_s = 1$ but the share of foreign-denominated assets is very large, so $\theta$ is close to zero.

Relative to the Fed and the ECB, the typical central bank’s accounts are also easier to read. Because assets made of foreign reserves are easy to value, the way in which net income is assessed is close to our mark-to-market rule. Because monetary operations involve exchanging foreign for domestic currency, their assets are directly held, without the complications of repo contracts. Therefore, the net income for a typical, small open-economy central bank is given simply by:

$$d_s' = n_{s,s'} + r_s(V(e_{s'})B_s - V_s) + (e_{s'} - (1 + r_s)V(e_{s'}))B_s.$$ (33)

Aside from seignorage and the return on its initial capital, the central bank earns a return on its portfolio of foreign assets. It is positive if the exchange rate depreciation ($e_{s'}$) is larger than was expected.

When the foreign exchange holdings ($B_s$) are small, net income is close to seignorage, which is typically positive. Starting with the Asian financial crisis of the 1990s, many central banks started accumulating large amounts of foreign reserves, and this hoarding accentuated after the 2008 financial crisis. Moreover, many countries continue to peg their exchange rate. The combination of pegged exchange rates and large balance sheets in the form of foreign reserves constitutes the new style of central banking for most open small open economies. This leads to a few potential dangers to the central bank solvency.
Insofar as a peg unexpectedly succeeds for a period of time, the exchange rate depreciates by less that was expected by markets. Real interest rates rise as the central bank defends the peg, and these higher expenses in paying the return on reserves are not offset by gains in the foreign currency portfolio since the exchange rate does not change, so $e_{s'} - (1 + r_s)\mathbb{V}(e_{s'}) < 0$. The central bank realizes a loss just as its balance sheet $B_s$ is large as a result of trying to defend the peg. When the peg is finally abandoned, the large depreciation of the currency would lead to large capital gains. But, by then, typically the central bank is left with reserves close to zero, so the gain ends up being minimal.

At the other extreme, the Swiss National Bank saw its balance sheet expand to almost the size of the GDP of the country since 2010. The SNB imposed a ceiling on the exchange rate and recorded a larger than usual net income for a few years. At the start of 2015, it unexpectedly announced it was letting go of the ceiling. The swiss franc appreciated quickly, and the SNB realized a large loss. If it had abandoned the peg later, and the balance sheet had grown even more, it would have recorded an even larger loss, which in the case of the SNB could have exceeded the budget of the federal government.

A final interesting case arises when a small open economy in a developing country successfully engages in an inflation stabilization program. If this was partly unexpected, it leads to an appreciation of the exchange rate. Our formula predicts that losses result. Therefore, our model explains the common, but perverse, occurrence where central banks that were successful at stabilizing inflation operating with negative capital as a result of losses on their foreign reserves holdings, see Dalton and Dziobek (2005).

Because reserves are large and exchange rates move quickly and by large amounts, small open economies frequently experience negative net income and operate under negative balance-sheet capital. Our theory would predict both that these central banks sometimes become insolvent (they do), but also that they are more likely to take precautions to avoid it. The central banks of Mexico and Brazil, for instance, have discretion to not distribute dividends if they feel they need to retain earnings against possible future losses. Provisioning works similarly to the deferred account that we explained and reduces the chances of insolvency.

---

8Kluh and Stella (2002) discuss many of these cases of the past thirty years.
6 Concluding Remarks

We provided one of the first systematic analysis of central bank solvency, what it means, what affects it, and when is it in danger. Theoretically, our first key result is that if a central bank follows a market-to-market net income rule, then it is always solvent. While, under old-style central banking, net income was almost guaranteed to be positive, with new-style central banking we showed that interest-rate risk default risk, and exchange-rate risk can all lead to negative net income. Our third result was that if negative income doesn’t comes with fiscal support, in the form of recapitalizations from the fiscal authority, then reserves drift to infinity and the central bank is insolvent. Our fourth result was that using a deferred account, whereby past losses must be offset before new gains are distributed as dividends, significantly lowered this risk.

Applied to the United States, we find that the Fed is in little danger of failing to meets its obligations or requiring recapitalization in an economy subject to occasional crises resembling the one that began in 2008. The answer to the question posed at the outset of this paper—is the Fed at risk for losing the ability to stabilize prices because it becomes insolvent?—is an unambiguous no. Given the historical volatility of interest rates and bond prices, the Fed is quite safe under a fully effective price stabilization policy. The Fed is quite bulletproof as long as the Treasury does not increase its demands for current income, or does not allow it to run a deferred account for a few years following the exit.

We also find that, if its only concern was to minimize the risk of solvency, the Fed would want to unwind its QE policies before it starts raising interest rates. Moreover, we showed that the way in which the Fed marks its portfolio does not have an effect on the present value of its losses, but it does affect their time path. While marking to market would lead to large losses at the time of exit of the crisis, followed by gains that would bring the deferred account to zero, following instead the accounting conventions of the Fed will imply that it records losses for many years but that their cumulative amount is not as high at any particular date.

Applied to the Euro-area, we found that partly because of larger currency in circulation and so larger seignorage revenues, and partly because the fall in asset prices when exiting the crisis is not as large, the ECB is in less danger than the Fed. While the risk of default on some of the bonds that it holds leads to solvency concerns, insofar as this risk was priced in when the ECB bought them, the impact on net income is small.
Institutionally, we found that the ECB has an important potential advantage over the Fed because it holds a substantial fraction of its assets as collateralized loans to banks rather than as outright ownership of bonds. As long as the banks meet their obligations to repay the loans even when the collateral value declines, the ECB’s repo positions amount to safe short-term borrowing, insulated from interest-rate fluctuations. Whereas the Fed suffers capital losses on its entire portfolio when interest rates rise in a recovery, the ECB’s losses only occur among its direct holdings of bonds. The ECB is only at risk if the collateral in its repos loses substantial value and the counterparty banks are unable to repay.

However, by adopting quantitative easing, the ECB has both increased its direct holdings of bonds, as well as increased the size of its balance sheet. Both have increased the total potential losses when it exits the crisis.

Finally, we showed that for most of the remainder central bank, the risks come from fixing exchange rate and accumulating large foreign reserves. Defending the peg can come with potentially large losses. Institutionally, the ability of the central bank to retain earnings in those countries serves to reduce the heightened risk of insolvency.
References

Bassetto, Marco and Todd Messer, “Fiscal Consequences of Paying Interest on Reserves,”  
*Fiscal Studies*, December 2013, 34, 413–436.

Berriel, Tiago and Saroj Bhattarai, “Monetary Policy and Central Bank Balance Sheet  

Bhattarai, Saroj, Gauti Eggertsson, and Bulat Gafarov, “Time Consistency and the Dura-

University and Columbia University.

Cagan, Phillip, “The Monetary Dynamics of Hyperinflations,” in Milton Friedman, ed.,  
117.

Carpenter, Seth B., Jane E. Ihrig, Elizabeth C. Klee, Daniel W. Quinn, and Alexander H.  

Christensen, Jens, Jose Lopez, and Glenn Rudebusch, “A Probability-Based Stress Test of  

Cukierman, Alex, “Central bank independence and monetary policymaking institutions:  
736.

Dalton, John and Claudia Dziobek, “Central Bank Losses and Experiences in Selected  

DelNegro, Macro and Christopher A. Sims, “When Does a Central Banks Balance Sheet  

Goodfriend, Marvin, “Monetary Policy as a Carry Trade,” *Monetary and Economic Studies*,  
November 2014, 32, 29–44.

Greenlaw, David, James D. Hamilton, Peter Hooper, and Frederic S. Mishkin, “Crunch  
Times: Fiscal Crises and the Role of Monetary Policy,” 2013. Columbia University and  
UC San Diego.


Bibtex: @UNPUBLISHED{HallReis:FSCB,author = {Hall, Robert E. and Ricardo Reis},title = {Maintaining Central-Bank Solvency under New-Style Central Banking },note = {Hoover Institution, Stanford University, and Columbia University},month = {March},year = {2015}}
Appendices

A Appendix A: Survey of the literature

One strand of work has studied the accounting principles and the history of central banks around the world to document when the accounting capital of the central bank is negative. Milton and Sinclair (2010) provide a survey and two examples are Dalton and Dziobek (2005) and Leone (1994). However, this literature struggles to explain conceptually why central bank capital would matter and why so often central banks operate with negative capital. Our theoretical framework explains why balance-sheet capital per se does not determine the solvency of a central bank, but that we can use it as an imperfect measure of the amount that the central bank would have to be recapitalized by in order to return reserves to their stationary path. Moreover, our study of different institutions and our formal treatment of the determinants of central bank solvency complement this literature’s collection of case studies.

Stella (1997), Stella (2007), and Klüh and Stella (2002) discuss whether measures of central bank accounting capital constrain monetary policy actions. Berriel and Bhattarai (2009) formalize the effects that a concern for its capital will have on the interest rates chosen by a central bank, while Bhattarai, Eggertsson and Gafarov (2014) discuss how the maturity composition of the balance sheet affects the ability of the central bank to commit to future monetary policies. Our approach is the opposite, and as such very complementary with these papers. They take as given the solvency of the central bank and study its effects on interest rates and inflation. We take interest rates and inflation as given and study the implications for solvency.

We focus on the solvency of the central bank from the perspective of when will its net income become negative or the amount in the deferred account become too large. An alternative, complementary approach starting with Reis (2013) assumes that neither of these will generate the political problems that we foresee (this literature sprung after the first draft of our paper was circulated). With no constraint on the value of its net income at a particular date, the only constraint facing the central bank is that it satisfies its intertemporal budget constraint. That is, it assumes that the central bank will remain independent of the fiscal authorities as solvent, as long as the present value of its dividends is positive. Bassetto and Messer (2013) discuss how the payment of interest on reserves affects this intertemporal
constraint. DelNegro and Sims (2015) study how inflation may have to change in order to generate the seigniorage revenue that will keep the central bank solvent. Hilscher et al. (In progress) measure the present value of the central bank dividends and estimate by how much it changes with inflation.

A series of papers studied the particular risk for the net income of the Federal Reserve of its recent balance sheet. Carpenter, Ihrig, Klee, Quinn and Boote (2013) and Greenlaw, Hamilton, Hooper and Mishkin (2013) present statistical projections for components of the balance sheet of the Federal Reserve, while Christensen, Lopez and Rudebusch (2015) use a dynamic term structure model to obtain precise estimates of the interest rate risk faced by the Fed. Our two conclusions from section 3 agree with theirs: the main risk facing the Fed is interest-rate risk, and it materializes into lower net income when exiting the crisis. While our calculations there are imprecise and take a worst case scenario, these other authors carefully calculate plausible estimates. At the same time they do not answer why would negative net income matter, while we provide a theoretical analysis. Also, while they focus on the Federal Reserve, our theory applies to any central bank.

Very few papers have studied the institutions that may affect the solvency of the central bank. Goodfriend (2014) is an exception. Starting from our warning that the Fed will likely have negative net income, he suggests that it should provision its earnings. Carpenter et al. (2013) show that under its current rules, the Fed can approximately use of the deferred account that we propose.

\section*{Appendix B: Multiple central bank investments}

Consider a central bank that can split its investment at date $t$ between long-term infinitely-lived bonds $B^L_t$, short-term one-period bonds $B^S_t$, and foreign assets $B^F_t$. Their respective prices are $q^L_t, q^S_t$ and $\epsilon_t$ as the real exchange rate for the foreign bond. The total value of the portfolio today then is:
\[ q^L_t B^L_t + q^S_t B^S_t + \epsilon_t B^F_t = q_t B_t \]
where $q_t B_t$ is the defined total value of the portfolio.

The payoff of the portfolio one period later is:
\[ (\ddot{c}_t + q^L_{t+1}) B^L_t + B^S_t + \epsilon_{t+1} B^F_t. \]
This will match our formulation in equation (8) if it equals:

\[ q_{t+1}[\theta c_t + (1 - \theta)e_{t+1} - (1 - \delta)]B_t \]

One can verify that this is true as long as:

\[
\begin{align*}
\theta_t &= 1 - \frac{\epsilon_t B_t^F}{q_t B_t^S} \\
\delta_t &= \frac{q_t^S B_t^S}{\theta_t q_t B_t} \\
q_t &= q_t^L \\
e_{t+1} &= \frac{\epsilon_{t+1} q_t}{\epsilon_t} \\
c_t &= \tilde{c}_t (1 - \delta_t) + \frac{\delta_t q_t}{q_t^S}.
\end{align*}
\]

Therefore, the relevant assumption is that \( \theta_t \) and \( \delta_t \) are constant over time. Or, in other words, that the central bank keeps the portfolio shares of short-term, long-term and foreign bonds constant. If this is the case, then the artificial delta composite bond in the main text will accurately capture the payoffs or this more complicated and realistic portfolio.

C  Appendix C: the US data and simulations

The estimated transition matrix and the stationary distribution is in Table 3. Applying our procedure to determined bond prices gives the results in Table 4. Notice that marginal utility is quite low in the crisis state, 5, corresponding to a high level of consumption in that state. To rationalize the low real interest rate in the crisis state in terms of the consumption Euler equation, it must be the case that consumption is expected to fall when in the crisis. Accordingly, consumption in the crisis must be unusually high. Obviously aggregate consumption does not behave this way across our states. Hall (2011) discusses how low real rates in the crisis economy might be consistent with the consumption paths of a limited group of wealthier consumers who participate in securities markets. Most consumers, in that view, are at a corner in their intertemporal consumption problems, so the slope of their consumption profiles does not contribute to asset pricing. He points to data from the Survey of Consumer Finances to support this hypothesis. The subjective discount factor \( \beta \) is 0.985.

Turning to the ergodic steady state, Table 5 shows the net income in the ergodic steady state when the central bank gets recapitalized. We set them to zero when the probability of the adjacent states zero. Net income is only negative in the state (5,3) that
### Table 3: Transition Matrix and Stationary Distribution of States of the Economy

<table>
<thead>
<tr>
<th>From state number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Stationary probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.43</td>
<td>0.21</td>
<td>0.21</td>
<td>0.07</td>
<td>0.07</td>
<td>0.237</td>
</tr>
<tr>
<td>2</td>
<td>0.31</td>
<td>0.31</td>
<td>0.38</td>
<td>0.00</td>
<td>0.00</td>
<td>0.220</td>
</tr>
<tr>
<td>3</td>
<td>0.23</td>
<td>0.31</td>
<td>0.15</td>
<td>0.31</td>
<td>0.00</td>
<td>0.220</td>
</tr>
<tr>
<td>4</td>
<td>0.07</td>
<td>0.14</td>
<td>0.14</td>
<td>0.64</td>
<td>0.00</td>
<td>0.237</td>
</tr>
<tr>
<td>5</td>
<td>0.00</td>
<td>0.00</td>
<td>0.20</td>
<td>0.00</td>
<td>0.80</td>
<td>0.085</td>
</tr>
</tbody>
</table>

### Table 4: Prices and returns

<table>
<thead>
<tr>
<th>State</th>
<th>Safe rate, $r$</th>
<th>Coupon, $c$</th>
<th>Marginal utility, $\mu$</th>
<th>Bond price, $q$</th>
<th>Nominal rate, $i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.043</td>
<td>1.000</td>
<td>1.000</td>
<td>6.711</td>
<td>0.075</td>
</tr>
<tr>
<td>2</td>
<td>0.019</td>
<td>1.000</td>
<td>0.987</td>
<td>6.834</td>
<td>0.047</td>
</tr>
<tr>
<td>3</td>
<td>0.013</td>
<td>1.000</td>
<td>0.966</td>
<td>6.958</td>
<td>0.047</td>
</tr>
<tr>
<td>4</td>
<td>-0.006</td>
<td>1.000</td>
<td>0.926</td>
<td>7.225</td>
<td>0.059</td>
</tr>
<tr>
<td>5</td>
<td>-0.015</td>
<td>1.000</td>
<td>0.839</td>
<td>7.643</td>
<td>0.004</td>
</tr>
</tbody>
</table>

### Table 5: Dividends in the ergodic distribution

<table>
<thead>
<tr>
<th>From state number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.24%</td>
<td>0.32%</td>
<td>0.40%</td>
<td>0.56%</td>
<td>0.82%</td>
</tr>
<tr>
<td>2</td>
<td>0.16%</td>
<td>0.24%</td>
<td>0.33%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>3</td>
<td>0.09%</td>
<td>0.17%</td>
<td>0.25%</td>
<td>0.43%</td>
<td>0.00%</td>
</tr>
<tr>
<td>4</td>
<td>0.04%</td>
<td>0.13%</td>
<td>0.21%</td>
<td>0.39%</td>
<td>0.00%</td>
</tr>
<tr>
<td>5</td>
<td>0.00%</td>
<td>0.00%</td>
<td>-0.85%</td>
<td>0.00%</td>
<td>0.28%</td>
</tr>
</tbody>
</table>

Table 5: Dividends in the ergodic distribution
Finally, Figure 8 shows the nominal interest rate and the nominal bond price over our simulation period.

D Appendix D: the Euro-area data

Tables Table 6 and Table 7 and figures Figure 9, Figure 10, and Figure 11 show the omitted simulation results for the ECB. For the ECB, we set the inflation target to 0.5% so that the nominal interest rate is close to zero in the crisis state.
<table>
<thead>
<tr>
<th>From state number</th>
<th>To state:</th>
<th>Stationary probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>0.98 0.02</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.20 0.80</td>
</tr>
</tbody>
</table>

Table 6: Transition Matrix and Stationary Distribution for the Euro-area

<table>
<thead>
<tr>
<th>State</th>
<th>Safe rate, ( r )</th>
<th>Coupon, ( c )</th>
<th>Marginal utility, ( \mu )</th>
<th>Bond price, ( q )</th>
<th>Nominal rate, ( i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.013</td>
<td>1.000</td>
<td>1.000</td>
<td>7.096</td>
<td>0.034</td>
</tr>
<tr>
<td>2</td>
<td>-0.002</td>
<td>0.884</td>
<td>0.934</td>
<td>7.070</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Table 7: Prices and returns

Figure 9: Interest Rate and Bond Prices in Europe
Figure 10: Components of the ECB’s Dividend

Figure 11: Flows Into and Out of ECB Reserves