Sectoral Shift, Job Mobility and Wage Inequality*

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Abstract

In the last few decades there is a clear shift of the U.S. economy from the non-service sector to the service sector. We document the patterns of changes in the employment share in services, the transition rates of workers between the two sectors and between different employment status, the relative wage income between the sectors, and wage inequality. To understand these changes jointly, we construct a dynamic equilibrium model of a two-sector economy where workers search both on the job and in unemployment. Assuming that the value-added per labor has been increasing in services relative to non-services, we estimate the model and make inferences on how the sectoral shift interacts with skill accumulation and labor market frictions.

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1. Introduction

In the last five decades, there has been a clear shift of the U.S. economy from the non-service sector to the service sector. In this paper, we first document these facts about the sectoral shift and then use these facts to estimate a dynamic model with labor market frictions.

The sectoral shift, described in detail in section 2, contains the following main facts. First, the employment share of the service sector in the economy has increased steadily from 51% in 1968 to 67% in 2010. Second, there are clear differences between the two sectors in the transition of workers. Not only is the transition rate of workers from unemployment to services higher than to non-services, but also the job-to-job transition rate from non-services to services is higher than the transition in the reverse direction. Third, the relative wage in the service sector to the non-service sector first fell until 1982 and then rose to a level in 2010 that was significantly higher than in 1968. Fourth, residual wage inequality has increased in both sectors, and the increase is larger in the service sector than in the non-service sector. Fifth, changes in the employment share, the relative wage and wage inequality are uneven across the three education groups of workers: college graduates, high-school graduates, and less-than high-school workers. In particular, the shift of employment to the service sector and the increase in the relative wage between the two sectors are weaker for high-school graduates than for the other two groups.

It is important to explain these facts jointly. The large sectoral shift of employment indicates that the US economy has been undergoing a structural change. Because this structural change is accompanied by a significant increase in the relative wage between the two sectors since the early 1980s, a natural hypothesis is that the structural change is driven by increases in the value added per effective labor input in the service sector relative to the non-service sector. This relative improvement in the service sector may be caused by a number of factors, such as an increase in the relative price and/or relative productivity in services to non-services or skill-biased technological progress that favors services. Taking an agnostic view of the cause of the structural change, we capture the change as a growing trend in the marginal revenue per effective worker in services relative to non-services. The objective is to use the above patterns of the sectoral shift to make inferences on how the sectoral shift interacts with skill accumulation and labor market frictions.
Search frictions are useful for understanding the patterns of the residual wage inequality and job-to-job transitions. Thus, we construct a model of a two-sector economy where the labor market has search frictions. The relative marginal revenue per effective labor input in the service sector to the non-service sector grows exogenously. A worker can work in either sector and there is a sector-specific component in a worker’s skill. More specifically, a worker’s skill is increased stochastically if the worker is employed continuously in the same sector, and the skill level has a stochastic reduction when a worker moves from one sector to the other. In addition, after a worker is matched with a firm, the pair draws a level of match-specific productivity. The effective labor input in a match is a product of the worker’s skill and the match-specific productivity. The level of output in a match is the product of the effective labor input and the sector’s marginal revenue per effective labor input.

In each period, a worker receives an opportunity to search in one of the two sectors. If the worker is already employed, he searches on the job. Search is directed in the sense that all searchers observe all offers before choosing which one to search for. We assume that an offer is a commitment to a wage share of output in the match which can be a function of the worker’s skill at the time of the match. The firms that make the same offer and the workers that search for the same offer can be regarded as a submarket. In each submarket, matches are generated according to a matching function that exhibits constant returns to scale. The matching probability in each submarket is a function of the tightness of the submarket, which is determined by free entry of vacancies. A firm is indifferent between a continuum of submarkets, because creating a vacancy in any of the submarkets generates the same expected surplus to the firm that is equal to the cost of a vacancy. In contrast, workers are not indifferent between the submarkets. Given the opportunity to search in a particular sector, there is a unique submarket that provides the worker with the optimal tradeoff between the gain in value from a match and the probability of getting the match. A worker always accepts a match that he searches for. If the worker is unemployed, the match generates a transition from unemployment to employment. If the worker is currently employed in the same sector where a new match is formed, the match generates a job-to-job transition within the same sector. If the worker is currently employed in a different sector from the one where a new match is formed, the match generates a job-to-job transition between the sectors.
Residual wage inequality arises in this model endogenously from search frictions and exogenously from match-specific productivity. Although a worker can observe all offers before choosing which one to apply for, the worker does not always get a match. Thus, identical workers can have different histories of search outcomes that induce them to search for different offers. For example, take two employed workers in the same sector who have the same skill, the same match-specific productivity and the same wage share in the current match. If they both receive the opportunity to search in a sector, they will choose to search for the same wage share. Suppose that one gets a new match in the period but the other does not. The worker who gets a new match will have a relatively higher value of employment, the gain to him from applying to any given offer in the future will be relatively smaller, and so he will search for a relatively higher wage share in the future. This process plays out over time to endogenously generate dispersion among workers’ wage shares. In addition, even when two workers are matched with firms that offer the same wage share, their wage levels can differ ex post if their matches draw different levels of match-specific productivity.

The model provides an intuitive explanation for the documented facts about the sectoral shift. First, because the marginal revenue per effective labor is increasing in the service sector relative to the non-service sector, there is a larger flow of workers from the non-service sector to the service sector than in the opposite direction, and the relative wage in the service sector to the non-service sector eventually increases. Second, wage inequality increases in both sectors, and it increases by more in the service sector than in the non-service sector. It is not surprising that wage inequality increases in the service sector, because the increase in the marginal revenue per effective labor in the service sector makes a large set of wages viable in the equilibrium. To compete against the service sector, the upper end of the wage distribution in the non-service sector thickens, and so wage inequality also increases in the non-service sector. Wage inequality increases by more in the service sector than in the non-service sector not only because there is more room for wages to grow at the top of the distribution in the service sector, but also because workers who search in the service sector are willing to accept lower wages initially in anticipation of faster growth in wages in the service sector. Third, the shift from the non-service sector to the service sector and the increase in the relative wage between the two sectors are likely to be uneven across the wage distribution. A worker near the top of the wage distribution in
the non-service sector is likely to switch into the service sector because staying in the non-service sector leaves very little room for further wage growth. A worker near the bottom of the wage distribution in the non-service sector is also likely to switch into the service sector because of the faster growth in wages in the service sector. In contrast, a worker in the middle of the wage distribution in the non-service sector has a relatively high value of staying in the sector. Because high-school graduates are likely to be in the middle of the wage distribution, the uneven pattern of transitions between the sectors implies that high-school graduates are less likely to switch from the non-service sector to the service sector than college graduates and the workers with less than high school education.

To quantify the implications of the model, we assume that the economy was in a steady state in 1968 and use the data to estimate the model parameters. Then, at the end of 1968, all individuals in the economy realize that the relative marginal revenue per effective labor in the service sector to the non-service sector would grow according to a deterministic trend toward the level in 2010. We compute the dynamic equilibrium and check whether the model produces the dynamic patterns of the employment share in services, the transition rates of workers, the relative wage, and the residual wage inequality.

In the literature on the sectoral shift, the paper most closely related to ours is Lee and Wolpin (2006). Lee and Wolpin construct a model where each worker faces a fixed cost of moving between the sectors and the labor market clears competitively. They estimate the model and compute it to show that the large sectoral shift of employment can be consistent with the behavior of the relative wage between the two sectors from 1968 to 2000. Our work contrasts with Lee and Wolpin’s in several aspects. First, the longer sample in our study shows different regularities from what Lee and Wolpin documented. The data in Lee and Wolpin (2006) ends in 2000, which shows that the relative wage between the sectors did not change significantly between the two ends of the sample period. Accordingly, their objective is to explain how the large sectoral shift in employment can be consistent with a constant relative wage between the two sectors. By extending the sample to 2010, we find that the relative wage increased significantly. Thus, one of our objectives is to explain why the large sectoral shift in employment is accompanied by an initial fall and an eventual increase in the relative wage between the two sectors. Second, we are interested in a broader set of facts than did Lee and Wolpin (2006). In addition to the shift in employment and the behavior of the relative wage between the two sectors, we document the patterns of
residual wage inequality and worker transitions both within each sector and between the two sectors. Third, to explain the patterns of residual wage inequality and parts of worker transitions, we build a model with search frictions in the labor market which are absent in Lee and Wolpin (2006).

Other examples of the contributions to the study of the sectoral shifts are Rogerson (1987), Ngai and Pissarides (2007), Kambourov (2009), Buera and Kaboski (2012). These papers have focused on the change in the employment share of services. In addition to this change, we examine the implications of the sectoral shift, including the relative wage, worker transitions and sector specific skill accumulation.

Our paper is also related to the large literature on changes in wage inequality in the last few decades. This literature is too large to be reviewed here adequately. Instead, we mention only a few papers that are tightly linked to the sectoral shift. As one of the facts documented here, we find that the shift from the non-service sector to the service sector is larger at the two ends of the wage distribution than at the middle of the distribution, which contributes to the hallowing-out in the middle of the wage distribution. This is related to the phenomena addressed by Acemoglu and Autor (2010) and Autor and Dorn (2011).

To generate residual wage inequality, our model relies heavily on the interaction between search frictions and on-the-job search. This interaction was first examined by Burdett and Mortensen (1998), who modeled search as an undirected process by which workers receive wage offers exogenously. In contrast, we model search as directed search by which workers choose which offer they apply to. Directed search has been formulated and explored in a growing literature, e.g. Peters (1991), Montgomery (1991), Moen (1997), Acemoglu and Shimer (1999), Burdett et al. (2001), Shi (2001), and Julien et al. (2000). There are two main reasons for using directed search in our study. First, it makes sense that workers intentionally choose the offers they want to apply for, rather than passively receive offers randomly. Second, with on-the-job search, undirected search models become intractable for incorporating dynamics, which are necessary for studying the sectoral shift. The source of the intractability is that the distribution of workers over wages is an aggregate state variable which can affect individuals’ decisions. In a dynamic equilibrium, this distribution changes endogenously as individuals move between jobs. Since the distribution can potentially have a large dimension, it is difficult to characterize the equilibrium interactions between
individuals’ decisions and the distribution. Directed search overcomes this difficulty.\footnote{Burdett and Coles (2003) extend the Burdett-Mortensen (1998) model to incorporate wage-tenure contracts. Postel-Vinay and Robin (2002) use a variation of the Burdett-Mortensen (1998) model to study worker transition and wage inequality in the steady state. Moscarini and Postel-Vinay (2009) have made progress in studying the dynamic equilibrium with undirected search on the job. However, it is difficult to make their model tractable for analyzing the model economy here which has ex ante and ex post heterogeneity and two sectors.}

The property of directed search that makes the dynamic analysis tractable is block recursivity. Namely, individuals’ decisions, their value functions and the market tightness function are all independent of the distribution of workers. This property is explored in a series of papers by Shi (2009), Menzio and Shi (2010a,b, 2011) and Gonzalez and Shi (2010), and it will be further explained in section 4.4. Relative to these papers with directed search, the current paper addresses a different issue, namely, how job mobility and wage dispersion respond to the sectoral shift. To address this issue, we develop a much richer model that incorporates two sectors, ex ante and ex post heterogeneity and the non-stationary equilibrium. Another contribution of our paper is to use the data to estimate key parameters associated with labor market frictions, such as those that affect matching rates, job-to-job transition rates and sector-specific skill accumulation.

\section*{2. Facts about the Sectoral Shift}

In this section, we document the main features of the shift from the non-service sector to the service sector in the United States. The main dataset used is the annual Current Population Survey (CPS) from 1968 to 2010, although we also use the quarterly CPS and the US Census data for some of the facts. The division of industries into services and non-services is similar to that used by Lee and Wolpin (2006). To focus on a set of robust facts, we include only the observations on male workers 25 to 60 years old who worked for 40 weeks or more in the previous year.\footnote{It is well-known that female labor force participation has steadily increased in the last few decades. Because female workers are more likely to enter the service sector than the non-service sector, their increased participation should partially crowd out male workers’ employment and wages in the service sector relative to the non-service sector. In this sense, excluding the observations on female workers strengthens the findings. In the model described later, we will allow for uneven entry of new workers into the two sectors in order to capture the general equilibrium effect of the increased labor market participation by female workers on male workers.} We document the facts in four groups: the employment share of the service sector, transition rates of workers, the relative wage between the two sectors, and wage inequality.
The employment share of the service sector is depicted in Figures 1.1 and 1.2. Figure 1.1 shows the pattern for the entire sample and for both the CPS and the Census. The Census data shows that the employment share of the service sector in the US economy has grown steadily in the last five decades. The CPS shows a similar pattern for the last four decades, with the service share of employment growing from 51% in 1968 to 67% in 2010. Figure 1.2 exhibits the change in the service share of employment in three education
groups and controlling for age.\textsuperscript{3} The shift of employment to the service sector is the largest for workers with the college degree. The shift is also stronger for workers without a high school degree than workers with a high school degree, at least in the 1970s through 1990s. In this sense, workers in the middle group has moved the least to the service sector.

\textbf{Figure 2.1}

\textbf{Figure 2.2}

\textsuperscript{3}We regress the explanatory variables on a 4\textit{th} order polynomial of age and time dummy. The exhibited patterns are the effect of the time dummy.
To get a more detailed view of the shift in employment to the service sector, we depict the flows of workers between the two sectors and between different labor market status, using the CPS. Figure 2.1 shows the transition rates of employed workers between the two sectors. The transition rate from non-services to services is uniformly higher than the reverse flow. Figure 2.2 depicts the transition rates from each of the two sectors into unemployment. The transition rate from non-services into unemployment is uniformly higher than the transition rate from services into unemployment. Figure 2.3 depicts the transition rates of workers from unemployment into each of the two sectors. The transition rate into services is higher than the rate into non-services, at least since the beginning of 1980s. Thus, if a worker is unemployed, the worker is more like to find employment in services than in non-services; a worker is less likely to become unemployed if the worker is employed in services than in non-services; and an employed worker is more likely to change the sector if the worker is employed in non-services than in services. These differences between the two sectors in the transition rates all contribute to the growth of employment in services relative to non-services.

The relative growth of employment in services has been accompanied by non-monotonic changes in the relative wage income in services to non-services. Figures 3.1 and 3.2 depict this relative wage income in log terms, where wage income is deflated with the CPI. Relative wage income was roughly equal to one in 1968. It fell in 1970s, bottomed out around 1982,
and then increased until 2005 before falling mildly near the end of the sample. Controlling for age and depicting the change for the three education groups, Figure 3.2 shows that the fall in 1970s in relative wage income in services was similar for the three education groups. However, the three groups experienced different changes in relative wage income since the beginning of 1980s. The gain in relative wage income in services is clear for workers with the college degree, but not so for workers with and without the high school degree. In fact, relative wage income appears to be roughly constant since 1980s for workers with the high school degree.

Figure 3.1

Figure 3.2
The sectoral shift is also accompanied with changes in wage inequality. We measure wage inequality by the standard deviation of log wage income.\textsuperscript{4} As shown in Figure 4.1, wage inequality has increased in both sectors, but the increase is more pronounced in the service sector than in the non-service sector. Figures 4.2 - 4.4 exhibit changes in residual wage inequality, after controlling for education and age. In both sectors, the rise in residual wage inequality is the strongest among workers with the college degree. In contrast, among workers without the high school degree, the residual wage inequality increased only slightly in the service sector and even decreased in the non-service sector.

\textsuperscript{4}The dynamic patterns of wage inequality are similar if log wages, rather than log wage income, are used, although the latter is more noisy. Also, wage inequality can be measured in terms of the 90-10 percentage wage differential. The broad patterns are similar, although there are some discrepancies.
3. The Model Environment

Time is discrete and lasts forever. The economy has two sectors, indexed by \( i \in \{1, 2\} \). Sector 1 is the service sector and sector 2 is the non-service sector. Workers and firms are risk neutral and discount future with the factor \( \beta \in (0, 1) \). The measure of firms in each sector is determined by free entry of vacancies. A firm can create vacancies in either sector, and the cost of maintaining a vacancy for a period in sector \( i \) is \( k_i > 0 \). The measure of workers in the economy is fixed at one. Each worker can supply one unit of
labor inelastically in a period. Workers are heterogeneous ex ante in the skill level and ex post in match-specific productivity. A worker’s skill level, denoted $h$, is publicly observable and distributed over the set $H = \{h_1, h_2, \ldots, h_{N(h)}\}$, where $N(h) \geq 1$ and $h_{N(h)} > h_{N(h)-1} > \ldots > h_1 > 0$. The skill distribution is endogenously determined by the processes described below for skill accumulation and skill losses. Match-specific productivity $z$ is drawn after a worker is matched with a firm and it is permanent in the match. The unconditional mean of $z$ is $Ez = 1$, and the distribution of $z$ has the density function $\phi(z)$ over the support $Z = \{z_1, z_2, \ldots, z_{N(z)}\}$, where $N(z) \geq 1$ and $z_{N(z)} > z_{N(z)-1} > \ldots > z_1 > 0$.\footnote{Match-specific productivity is introduced for two reasons. First, it helps to capture wage inequality among workers with the same skill and the same employment history. Second, it allows the possibility that wage may decrease in some of the job-to-job transitions, as observed in the data.}

In any given period, a worker is either employed or unemployed. When unemployed, a worker’s home production has the value $b_t(h) \geq 0$, and we set the worker’s $z$ as $z = 0$. When employed at a job with $(z, h)$ in sector $i$, the revenue produced in the match in period $t$ is $Y_{1,t}(z, h)$. We normalize $Y_{2,t}(z, h) = Y_2(z, h)$ for all $t$ and so, for any given $(z, h)$, the variations in $Y_{1,t}(z, h)$ over time are changes in the revenue of a match in sector 1 relative to sector 2. As such, they include changes in not only the relative productivity but also the relative price of the product in the service sector to the non-service sector.

A worker’s skill can change in three ways. First, skills can be accumulated on the job. For a worker with skill $h_n \in H$ who is employed in sector $i$, after production in the current period, the worker’s skill in sector $i$ changes to $h_{n+w}$ with probability $A_i(h_n, h_{n+w})$. If the current skill levels are put as the rows and the new skill levels as the columns, the matrix $A_i$ is an upper triangular matrix to represent skill accumulation. Because this process is intended to capture skill accumulation on the job, an unemployed worker’s skill is not subject to this stochastic change.\footnote{When a worker first becomes unemployment, there is a skill loss according to a process described later. However, to simplify the analysis, we assume that an unemployed worker’s skill remains constant over the duration of unemployment as long as the worker stays in the same sector.} Second, a worker may lose part of the skill when the worker becomes unemployed. When separating into unemployment, a worker’s skill falls from the current level $h$ to $h_1$ with probability $ch_1/h$ and stays at $h$ with probability $1 - ch_1/h$, where $c \in (0, 1)$ is a constant that captures the magnitude of skill loss in the probabilistic fashion. Third, a worker may lose part of the skill when the worker switches sectors. In each period, a worker in sector $i$ with skill $h_n$, employed or unemployed, draws a potential skill level that he will have in the other sector $i' \neq i$. The realization of this draw
is equal to \( h_{n-n'} \) with probability \( D_i(h_n, h_{n-n'}) \). To represent a potential skill loss, the matrix \( D_i \) is a lower triangular matrix with the current skills as the rows and the potential skill in the other sector as the columns. Thus, skills have a sector-specific component.

Let us clarify several aspects of the random draw according to \( D_i \). First, the draw takes places regardless of whether a worker intends to switch sectors. The phrase “potential skill” emphasizes the fact that this skill level will be applicable only when the worker takes up a job in the other sector, in which case the loss of sector-specific skills becomes irreversible. If the worker does not take up any job in sector \( i' \), instead, his skill in sector \( i \) stays at the level determined by \( A_i \).\(^7\) Second, the draw takes place before a worker searches in the other sector. This assumption simplifies the analysis by maintaining the structure that a worker’s skill level will be known before the worker searches. Third, the draws according to \( D_i \) are independent among the workers and over time. Specifically, if a worker does not take up a job in the other sector in the current period, then the worker’s potential skill in the other sector in the next period will be a new draw according to the distribution \( D_i \).

The highest skill level is an absorbing state of the skill accumulation process. To eliminate the uninteresting outcome that all workers eventually reach the highest skill in the steady state, we assume that a worker exits the labor force in each period exogenously with probability, \( 1 - \rho \in (0, 1) \). When a worker exits the labor force, a new worker with skill \( h_1 \) enters the labor market through unemployment. A new worker enters sector 1 with probability \( \zeta_t \) and sector 2 with probability \( 1 - \zeta_t \). We allow \( \zeta_t \) to vary over time in order to capture the effects of uneven changes over time in female participation between the two sectors. Although we focus on male workers, such uneven changes in female participation can exert general equilibrium effects on the sectoral shift of male workers. The time-varying \( \zeta_t \) mimicks these effects as increased entry of new male workers in one sector relative to the other sector.

All workers have the possibility to search for jobs. In each period, an unemployed worker in sector \( i \) receives an opportunity to search (off the job) with probability \( \lambda_{ui} \), and a worker employed in sector \( i \) receives an opportunity to search (on the job) with probability \( \lambda_{ei} \), where \( \lambda_{ui}, \lambda_{ei} \in (0, 1] \). Conditional on having the search opportunity, a worker can search in his incumbent sector with probability \( \alpha_i \) and in the other sector with probability \( (1 - \alpha_i) \).

\(^7\)This assumption is reasonable. Just searching in the other sector does not make one lose his skill in the current sector.
where $\alpha_i \in (0, 1)$. The parameter $\alpha$ captures the fact that an employed worker’s search opportunity may not be symmetric between the two sectors, possibly because a worker in one sector knows relatively less about the other sector than the incumbent sector and/or because changing sectors has a fixed cost that we do not model explicitly. Both $\lambda$ and $\alpha$ are indexed by a worker’s incumbent sector, in general. Note that the processes with $(\lambda, \alpha)$ describe only the opportunities to search, not the actual turnover of workers. The latter depends on workers’ search decisions and the equilibrium distribution of offers.

In this specification, which sector a worker can search is determined exogenously. The purpose of making this assumption is to generate a significant overlap between the supports of equilibrium distributions of worker values in the two sectors. If workers can choose which sector to search, instead, a significant fixed cost to switching sectors is needed to generate the overlap between the supports. Our specification is simpler, and the exogenous parameters $(\alpha_1, \alpha_2)$ can be interpreted as an implicit way to capture this fixed cost.

The labor market in each sector is organized into a continuum of submarkets, indexed by $(x, h)$, where $h$ is a job seeker’s skill level at the time of hiring and $x$ is the expected sum of discounted utilities offered to a worker in the submarket. Firms choose which sector and submarket to post vacancies. As said earlier, the cost of posting a vacancy in sector $i$ for one period is $k_i$. Workers observe the offers in all submarkets before choosing which submarket to search and, in this sense, search is directed. Matching inside each submarket is random. Let $\theta$ be the tightness of a submarket, i.e., the ratio of vacancies to searching workers in the submarket. The matching probability in a submarket with tightness $\theta$ is $p(\theta)$ for a searching worker and $q(\theta)$ for a vacancy. The matching technology that generates these matching probabilities has constant returns to scale, and so $q(\theta) = p(\theta)/\theta$. We impose the standard assumptions: $p(\theta) \in [0, 1]$ for all $\theta$; $0 < p'(\theta) < p(\theta)/\theta$ for all $0 < \theta < \infty$, $p''(\theta) < 0$, $p(0) = 0$ and $p(\infty) = 1$. If a worker gets a match after search, the worker quits the current job (if he is employed) and takes up the new match.\(^8\)

A firm can use a lottery to deliver the expected value $x$ to a worker. Lotteries are useful for convexifying the set of payoffs to the firm because, in the absence of lotteries, a worker’s unilateral decision to quit into unemployment can make the firm’s payoff a non-monotonic

\(^8\)As is standard in the literature, we assume that a worker’s incumbent firm cannot make counter offers to the worker after the worker finds a new match. For justifications, see Burdett and Mortensen (1998), Burdett and Coles (2003), and Shi (2009). For (undirected) search models where firms can make counter offers, see Postel-Vinay and Robin (2002).
function of the offer. Denote the lottery in submarket \((x, h)\) as \((x_\ell, \pi_\ell)_{\ell=1,2}\), where \((x_1, x_2)\) are the prizes of the lottery, with \(x_2 \geq x_1\), and \((\pi_1, \pi_2)\) are the associated probabilities. A prize of the lottery, \(x_\ell\), is an expected value to the worker over match-specific productivity. To deliver the prize \(x_\ell\), the firm gives the worker a share of output in each period, denoted \(\omega_\ell \in [0,1]\). Equivalently, an offer can be referred to as \((x_\ell, \pi_\ell)_{\ell=1,2}\) or \((\omega_\ell, \pi_\ell)_{\ell=1,2}\), which depends on \((x, h)\). We assume that a job seeker must accept an offer before the lottery is played and match-specific productivity is realized for the new job. If the realization of the lottery or match-specific productivity is unfavorable, the worker can choose to quit after working for one period.

Let us make several clarifications about the submarkets. First, a firm commits to a wage share, rather than a wage level, to deliver the prize of a lottery. This specification is a convenient way to allow the wage level to vary with the worker’s future skill and the sector’s marginal value of output. In particular, for a worker with a skill level \(h\) who is newly matched with a firm in sector \(i\) at the wage share \(\omega(h)\), the wage level in a future period \(t\) will be \(\omega(h)Y_{i,t}(z, h_t)\), which depends on the worker’s future skill \(h_t\) and sector \(i\)’s relative value of output in the future. Second, we assume that the wage share is constant during a worker’s employment in a firm and, in particular, we abstract from wage-tenure contracts. This assumption is made for determining a worker’s wage profile in the duration of employment in a firm. Because workers and firms are risk neutral in our model, there can be a continuum of wage paths that deliver the same expected value to the worker. If workers are assumed to be risk averse, instead, then it is possible to uniquely determine the optimal wage-tenure contract that delivers an offer value (e.g., Burdett and Coles, 2003, Shi, 2009), but this extension complicates the analysis significantly. Third, it is straightforward analytically to allow the wage share to depend on match-specific productivity \(z\), as well as the worker’s skill \(h\). We do not pursue this extension because it does not change the numerical results significantly, and yet it increases the computation time substantially when the number of possible realizations of \(z\) is large.\(^9\)

\(^9\)The distinction of submarkets by \(h\) is entirely for the purpose of easing the description. The same equilibrium can be attained with a different description that does not distinguish submarkets by \(h\). The reason is that different workers will self-select into different offers under directed search (see Menzio and Shi, 2010b). To see this, suppose that each submarket in a sector is described by a vector \(x = (x_1, x_2, \ldots, x_N(h))\), with a tightness \(\theta(x)\). If a worker with skill \(h_n\) is matched with a firm in submarket \(x\), the offer to the worker is the \(n\)-th component of the vector \(x\). Knowing this, a worker of each particular skill-\(h\) will choose to only enter the market that the \(n\)-th component of the vector \(x\) and the associated matching probability
A match can be destroyed either exogenously or endogenously. Endogenous separation occurs in two ways. First, if an employed worker accepts a new match as a result of the on-the-job search, the worker’s current match is destroyed. Second, after production in each period, each worker employed in sector $i$ chooses the probability to break up the match and separate into unemployment, $d_i$. The lower bound on this choice is $\delta_i \in (0, 1)$, which is the probability that Nature destroys the match exogenously. The probability $d$ in excess of $\delta$ represents endogenous separation into unemployment. When separating into unemployment, a worker loses part of the skill according to the process described earlier. Allowing for endogenous separation into unemployment is important for workers to break up matches that have too low match-specific productivity. Also, it enables workers in the non-service sector to move to the service sector through unemployment, in addition to on-the-job search. Some matches in the non-service sector may be viable initially but, after some length of time, the match may no longer be as attractive to the worker as quitting into unemployment and searching for a job in the service sector.

Let us describe the timing of events in each period. A period is divided into four stages: production and skill accumulation, separation into unemployment, search, and the realization of $z$. These stages are described below:

(i) Production and skill accumulation: Output is produced and wages are paid. An unemployed worker receives the benefit $b$. Then, a worker exits the market exogenously with probability $1 - \rho$ and is replaced with new worker entering the labor market through unemployment. If a worker employed in sector $i$ survives, his skill level in sector $i$ is augmented according to the transition matrix $A_i$. An unemployed worker’s skill remains unchanged.

(ii) Separation into unemployment: A worker employed in sector $i$ chooses the probability $d_i$ to separate into unemployment. The lower bound on $d_i$ is $\delta_i$. When a worker separates into unemployment, the skill decreases probabilistically.

(iii) All workers draw their potential skill in the other sector according to the transition matrix $D_i$.

(iv) Search: The opportunity to search arrives to a worker with probability $\lambda_{ei}$ for a worker provide the optimal tradeoff. In the equilibrium, the collection of the vector $x$ can have the following form: In the submarket that attracts skill-$h_n$ workers, $x_n \in [x_{n'}, x_n]$ and $x_{n'} = x \equiv \min_n(x_{n'})$ for all $n' \neq n$.

10We assume separation occurs before the random draws of the potential sector in the other sector in order to simplify the separation decision as a function of the skill level determined by the transition matrix $A$. If separation is assumed to occur after the draw according to $D$, then the separation decision will be a function of both the skill in the incumbent sector and the potential skill in the other sector.
employed in sector $i$ and with probability $\lambda_{ui}$ for an unemployed worker in sector $i$. If a worker can search, he receives the shock that determines whether he can search in his incumbent sector or in the other sector. Then, the labor market opens. Firms choose where to create vacancies. In the sector where a worker has the opportunity to search, the worker chooses which submarket to search. If a worker gets a new match, he accepts the new job and quits the current one.

(v) The lottery and the realization of $z$: After a worker makes a transition to a new job, the lottery is played to determine the prize $x_{t}$ and the corresponding wage share $\omega_{t}$. Then, match-specific productivity $z$ of the new match is realized. The period ends.

### 4. Individuals’ Decisions and Equilibrium

#### 4.1. Workers’ search decisions and value functions

Let us represent a match by $(\omega, z, h)$, where $\omega$ is the wage share, $z$ is match-specific productivity and $h$ is the worker’s current skill level. An unemployed worker’s status is denoted as a match $(b, 0, h)$. The skill level $h$ changes over time, but $\omega$ and $z$ remain fixed throughout the match. We will often suppress the dependence of $\omega$ on the skill that the worker had initially at the time of the match. Denote $V_{i,t}(\omega, z, h)$ as the value function of a worker in match $(\omega, z, h)$ in sector $i$ in period $t$, and $V_{ui,t}(h)$ as the value function of an unemployed worker of skill $h$ in sector $i$ in period $t$.\footnote{The phrase “unemployed in sector $i$” means that the worker’s most recent employment was in sector $i$. It is possible that the worker searched in the other sector but did not find a match. It is necessary to associate unemployed workers with particular sectors because skills are sector-specific.} For convenience, we measure value functions at the end of a period.

Consider a worker whose value will be $\nu$ at the end of the period if he does not change his position in the period. Suppose that he has an opportunity to search in sector $i$ in period $t$ and that the worker’s skill in sector $i$ is $h$ at the time of search. The worker chooses a submarket $(x, h)$ in sector $i$ to search. The offer $x$ is the expectation of the function $V$ over the lottery outcome and match-specific productivity. The tightness of the submarket is $\theta_{i,t}(x, h)$ and the matching probability for the worker is $p(\theta_{i,t}(x, h))$. Thus, the return to the worker’s search is:

$$R_{i,t}(\nu, h) = \max_{x} p(\theta_{i,t}(x, h))(x - \nu).$$

\[4.1\]
The worker’s optimal search decision solves the maximization problem above. Denote the policy function for optimal search as \( x = s_{i,t}(v, h) \) and denote the worker’s matching probability induced by optimal search as

\[
\tilde{\rho}_{i,t}(v, h) = p(\theta_{i,t}(s_{i,t}(v, h), h)).
\] (4.2)

If \( \theta_{i,t}(x, h) \) is a decreasing function of \( x \), then \( s_{i,t}(v, h) \) is an increasing function in \( v \) and, for all \( v \) such that \( p(\theta_{i,t}(s_{i,t}(v, h), h)) > 0 \), \( s_{i,t}(v, h) \) is strictly increasing in \( v \).

The formulation of the optimal search problem in (4.1) is valid for both employed workers and unemployed workers, with the appropriate value of the worker’s status quo, \( v \). The value \( v \) is equal to \( V_{i,t}(\omega, z, h) \) for a worker employed in match \((\omega, z, h)\) in sector \( i \), to \( V_{i',t}(\omega', z', h') \) for a worker employed in match \((\omega', z', h')\) in sector \( i' \neq i \) who searches in sector \( i \), to \( V_{u,t}(h) \) for an unemployed worker in sector \( i \), and to \( V_{u',t}(h') \) for an unemployed worker in sector \( i' \neq i \) who searches in sector \( i \). Here, \( h \) and \( h' \) are the worker’s skill at the time of search if the worker stays at the current position.

Optimal search generates transitions from unemployment to employment and between jobs. Denote \( \Phi_{u,t}(v, h) \) as the job-finding probability for an unemployed worker in sector \( i \) whose skill immediately before search is \( h \) and whose value at the end of the period will be \( v \) if he remains unemployed. This probability is calculated before the worker knows which sector he can search. With probability \( \lambda_{ui}\alpha_i \) the worker can search in sector \( i \), in which case he will find a job with probability \( \tilde{\rho}_{i,t}(v, h) \). With probability \( \lambda_{ui}(1 - \alpha_i) \) the worker can search in sector \( i' \neq i \), in which case he will find a job with probability \( \tilde{\rho}_{i',t}(v, h') \) where \( h' \) is the worker’s potential skill in sector \( i' \). Since this potential skill is drawn before search according to \( D_i \), then

\[
\Phi_{u,t}(v, h) = \lambda_{ui}\alpha_i\tilde{\rho}_{i,t}(v, h) + \lambda_{ui}(1 - \alpha_i) \sum_{h'} D_i(h, h')\tilde{\rho}_{i',t}(v, h'), \quad i' \neq i.
\] (4.3)

Although this formulation is valid for any value \( v \) of an unemployed worker, in the equilibrium \( v = V_{u,t}(h) \). Similarly, denote \( \Phi_{i,t}(v, h) \) as the job-to-job transition probability for an employed worker in sector \( i \) whose skill immediately before search is \( h \) and whose value at the end of the period will be \( v \) if he stays with the current job. In contrast to an unemployed worker, this employed worker has a search opportunity with probability \( \lambda_{ei} \).
Thus,
\[ \Phi_{i,t}(v, \hat{h}) = \lambda_{ei}\alpha_{i}\tilde{p}_{i,t}(v, \hat{h}) + \lambda_{ei}(1 - \alpha_{i}) \sum_{h'} D_{i}(\hat{h}, h')\tilde{p}_{i',t}(v, h'), \quad i' \neq i. \] (4.4)

In the equilibrium, \( v = V_{i,t}(\omega, z, \hat{h}) \) in (4.4).

We now compute workers' value functions. Consider first a skill-\( h \) worker in sector \( i \) who is unemployed at the end of period \( t - 1 \). The worker receives the unemployment benefit \( b_{i}(h) \) in period \( t \). If the worker survives (with probability \( \rho \)), he has the opportunity to search for a job. Note that if the worker remains unemployed at the end of period \( t \), the worker's value will be \( V_{ui,t}(h) \). This is the worker's reservation value in search. Let \( V_{ui,t}^{e}(V_{ui,t}(h), h) \) denote the worker's expected value of search in period \( t \). The worker's value function at the end of period \( t - 1 \) is:
\[ V_{ai,t-1}(h) = \beta[b_{i}(h) + \rho V_{ui,t}^{e}(V_{ui,t}(h), h)]. \] (4.5)

The value \( V_{ui}^{e} \) is generated by the search opportunity. The unemployed worker in sector \( i \) is able to search in sector \( i \) with probability \( \lambda_{ai}\alpha_{i} \), in which case the return to search is \( R_{i,t}(V_{ui,t}(h), h) \). The worker is able to search in the other sector \( i' \neq i \) with probability \( \lambda_{ai}(1 - \alpha_{i}) \), in which case the worker's potential skill is \( h' \) in sector \( i' \) which was drawn before search according to the distribution \( D_{i}(h, h') \). Given \( h' \), the return to search in sector \( i' \) is \( R_{i',t}(V_{ui,t}(h), h') \). Thus,
\[ V_{ui,t}^{e}(v, h) = v + \lambda_{ai}\alpha_{i}R_{i,t}(v, h) + \lambda_{ai}(1 - \alpha_{i}) \sum_{h'} D_{i}(h, h')R_{i',t}(v, h'), \] (4.6)
where \( v = V_{ui,t}(h) \) in the equilibrium. In (4.6), an unemployed worker's probability of getting an offer and the optimal search choice are embedded in \( R_{i} \) and \( R_{i'} \).

Next, consider a skill-\( h \) worker employed in a match \((\omega, z, h)\) in sector \( i \) at the end of period \( t - 1 \). The worker receives wage \( \omega Y_{i,t}(z, h) \) in period \( t \). If the worker survives (with probability \( \rho \)), his skill is augmented to \( \hat{h} \) according to the probability distribution \( A_{i}(h, \hat{h}) \). Given the realization \( \hat{h} \), the worker chooses whether or not to separate into unemployment. If the worker chooses to stay at the job, the value is \( V_{i,t}^{e}(v, \hat{h}) \), where \( v = V_{i,t}(\omega, z, \hat{h}) \). If the worker chooses to separate into unemployment, the skill level will be \( \tilde{h} = h_{1} \) with probability \( ch_{1}/\hat{h} \), and \( \tilde{h} = \hat{h} \) with probability \( 1 - ch_{1}/\hat{h} \), where \( c \in (0, 1) \) is a constant.
Given $\tilde{h}$, the worker value will be $V_{ui,t}(V_{ui,t}(\tilde{h}), \tilde{h})$, and so the worker’s expected value over $\tilde{h}$ is

$$E_\tilde{h} V_{ui,t}(V_{ui,t}(\tilde{h}), \tilde{h}) = \frac{c_{h1}}{h} V_{ui,t}(V_{ui,t}(h1), h1) + (1 - \frac{c_{h1}}{h}) V_{ui,t}(V_{ui,t}(\tilde{h}), \tilde{h}). \quad (4.7)$$

We assume that the worker must incur an adjustment cost in terms of disutility to separate into unemployment, which is expressed as $\xi(d - \delta_i)V_{ui,t}$. The function $\xi$ is assumed to have the following properties: $\xi(0) = \xi'(0) = 0$, $\xi > 0$ for all $d \neq \delta_i$, $\xi' > 0$ for all $d > \delta_i$, and $\xi'' > 0$. Then, the worker’s value function at the end of period $t - 1$ is

$$V_{i,t-1}(w, z, h) = \beta \left\{ \omega Y_{i,t}(z, h) + \rho \sum_h A_i(h, \tilde{h}) \times \max_{d_{i,t} \in [\delta_i, 1]} [(d_{i,t} - \xi(d_{i,t} - \delta_i))] E_\tilde{h} V_{ui,t}(V_{ui,t}(\tilde{h}), \tilde{h}) + (1 - d_{i,t}) V_{i,t}^e(v, \tilde{h}) \right\}, \quad (4.8)$$

where $v = V_{i,t}(w, z, h)$.

To compute $V_{i,t}^e(v, \tilde{h})$, consider the worker with skill $\tilde{h}$ in the search stage. If the worker remains at the current job at the end of the period, the value will be $v = V_{i,t}(w, z, \tilde{h})$, which is the worker’s reservation value in search in period $t$. The worker has an opportunity to search with probability $\lambda_{ai}$. With this opportunity, the worker is able to search in sector $i$ with probability $\alpha_i$, in which case the return to search is $R_{i,t}(v, \tilde{h})$. With the opportunity to search, the worker is able to search in the other sector $i' \neq i$ with probability $(1 - \alpha_i)$, in which case the worker’s potential skill is $h'$ in sector $i'$ which is drawn before search according to the probability distribution $D_i(\tilde{h}, h')$. Given $h'$, the return to search in sector $i'$ is $R_{i',t}(v, h')$. Thus,

$$V_{i,t}^e(v, \tilde{h}) = v + \lambda_{ai} \alpha_i R_{i,t}(v, \tilde{h}) + \lambda_{ai} (1 - \alpha_i) \sum_{h'} D_i(\tilde{h}, h') R_{i',t}(v, h'), \quad (4.9)$$

where $v = V_{i,t}(w, z, \tilde{h})$ in the equilibrium and $i' \neq i$. In (4.9), the probability of getting an offer and the optimal search choice are embedded in $R_i$ and $R_{i'}$. 

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Note: The adjustment cost is needed to make the separation policy function a continuous function of the worker’s value $v$. Although a continuous policy function is not necessary to make a worker’s value function continuous, it is necessary to ensure a firm’s value function to be continuous. Because the firm’s value function will be used to determine the market tightness function through the free entry condition, existence of an equilibrium cannot be guaranteed in general if a firm’s value function is not continuous. We will make the adjustment cost sufficiently small by adjusting the parameters in the cost function $\xi$. 

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12 The adjustment cost is needed to make the separation policy function a continuous function of the worker’s value $v$. Although a continuous policy function is not necessary to make a worker’s value function continuous, it is necessary to ensure a firm’s value function to be continuous. Because the firm’s value function will be used to determine the market tightness function through the free entry condition, existence of an equilibrium cannot be guaranteed in general if a firm’s value function is not continuous. We will make the adjustment cost sufficiently small by adjusting the parameters in the cost function $\xi$. 

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21
Let us characterize more explicitly the optimal decision in (4.8) on separation into unemployment. A worker chooses to separate into unemployment with probability one if the expected value of searching off the job is higher than that of searching on the job. Since the lower bound on this separation probability is determined by Nature as $\delta_i$ in sector $i$, the optimal separation decision $d_{i,t}$ is given as

$$d_{i,t}(v, \hat{h}) = \begin{cases} 
\delta_i, & \text{if } V^e_{i,t}(v, \hat{h}) > E_h V^e_{ui,t} \\
1, & \text{if } V^e_{i,t}(v, \hat{h}) < [1 - \xi'(1 - \delta_i)]E_h V^e_{ui,t} \\
\delta_i + \xi'^{-1}(1 - \frac{V^e_{i,t}(v, \hat{h})}{E_h V^e_{ui,t}}), & \text{otherwise},
\end{cases}$$

(4.10)

where $v = V_{i,t}(\omega, z, \hat{h})$, $E_h V^e_{ui,t} = E_h V^e_{ui,t}(V_{ui,t}(\hat{h}), \tilde{h})$, and $\xi'^{-1}$ is the inverse function of $\xi'$.

### 4.2. Firms’ decisions and value functions

Consider a firm in match $(\omega, z, h)$ in sector $i$ at the end of period $t - 1$. The firm is employing a skill-$h$ worker whose wage share is $\omega$ and whose match-specific productivity is $z$. Denote the firm’s value function at the end of period $t - 1$ as $J_{i,t-1}(\omega, z, h)$. Production in period $t$ yields output $Y_{i,t}(z, h)$. The firm pays wage to the worker and retains profit $(1 - \omega)Y_{i,t}(z, h)$. If the worker survives (with probability $\rho$), his skill is augmented to $\hat{h}$ according to the probability distribution $A_i(h, \hat{h})$. Given the realization $\hat{h}$, the worker chooses to separate into unemployment with probability $d_{i,t}(v, \hat{h})$, where $v = V_{i,t}(\omega, z, \hat{h})$. If the worker does not separate into unemployment, the worker leaves the job for another job with probability $\Phi_{i,t}(v, \hat{h})$ which is given by (4.4). If the worker stays with the firm at the end of period $t$, the firm’s value will be $J_{i,t}(\omega, z, \hat{h})$. Thus,

$$J_{i,t-1}(\omega, z, h) = \beta \left\{ (1 - \omega)Y_{i,t}(z, h) + \rho \sum_{\hat{h}} A_i(h, \hat{h}) \left[ 1 - d_{i,t}(v, \hat{h}) \right] \left[ 1 - \Phi_{i,t}(v, \hat{h}) \right] J_{i,t}(\omega, z, \hat{h}) \right\}$$

(4.11)

where $v = V_{i,t}(\omega, z, \hat{h})$. Recall that the job-to-job transition probability $\Phi_{i,t}$ has incorporated the probability distribution $D_t$ according to which the worker draws the potential skill in the other sector. Because both $d_{i,t}(v, \hat{h})$ and $\Phi_{i,t}(v, \hat{h})$ are continuous functions of $v$, the right-hand side of (4.11) is a continuous function of $v$, and so is $J$. The value $J_{i,t-1}$ depends on $V_{i,t-1}$ through the wage share $\omega$.

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13 Notice that $\omega$ and $z$ affect the separation decision $d_{i,t}$ only through the value function $V_{i,t}(\omega, z, \hat{h})$. Given $V_{i,t}$, the optimal separation decision is independent of $\omega$ and $z$. Part of the reason for this independence is that the market tightness $\theta_{i,t}(x, \hat{h})$ is independent of $z$ given $(x, \hat{h})$ in period $t$.
The function \( J_i(\omega, z, h) \) may not be monotonic in \( \omega \), in general, because the worker unilaterally chooses whether to separate into unemployment. To see this, consider two wage shares \( \omega_1 \) and \( \omega_2 \) that induce the values \( v_1 = V_{i,t}(\omega_1, z, \hat{h}) \) and \( v_2 = V_{i,t}(\omega_2, z, \hat{h}) \) to satisfy \( V_{i,t}^\epsilon(v_1, \hat{h}) = [1 - \xi'(1-\delta_i)]V_{ui,t}^\epsilon \) and \( V_{i,t}^\epsilon(v_2, \hat{h}) = V_{ui,t}^\epsilon \). Let \( \varepsilon > 0 \) be a sufficiently small number and, for the sake of argument, suppose that \( V_{i,t}(\omega, z, \hat{h}) \) is strictly increasing in \( \omega \) in the neighborhoods of \( \omega_1 \) and \( \omega_2 \). By (4.10), a worker at \( \omega_1 - \varepsilon \) chooses to separate into unemployment with probability one, which destroys the firm’s future value \( J_{i,t}(\omega_1 - \varepsilon, z, \hat{h}) \). In contrast, a worker at \( \omega = \omega_2 + \varepsilon \) separates into unemployment with probability \( \delta_i < 1 \). When the adjustment cost of separation is sufficiently small, \( \omega_2 \) is only slightly higher than \( \omega_1 \). In this case, the firm’s value at \( \omega_2 + \varepsilon \) is higher than the value at \( \omega_1 - \varepsilon \). That is, \( J_i(\omega, z, h) \) is increasing near \( \omega \) where the worker’s separation choice changes from 1 to \( \delta_i \).\(^{14}\)

Away from these values of \( \omega \), \( J_i(\omega, z, h) \) is likely to be a decreasing function. Thus, \( J_i(\omega, z, h) \) is likely to be non-monotonic. This non-monotonicity makes the set of payoffs induced by \( J_i \) non-convex and, hence, gives rise to a role of the lottery formulated below.

A firm chooses which sector and which submarket in the sector to create vacancies. Once inside a submarket \((x, h)\) in sector \(i\), a firm chooses a lottery \((x_{\ell,i}, \pi_{\ell,i})_{\ell=1,2}\) to deliver the value \(x\) to a worker. The prize \(x_{\ell}\) of the lottery and the wage share \(\omega_{\ell}\) are related to each other as follows:

\[
x_{\ell} = \mathbb{E}_z V_{i,t}(\omega_{\ell}, z, h),
\]

where \(\mathbb{E}_z\) denotes the expectation over \(z\). Let us denote the inverse of (4.12) as \(\omega_{\ell} = \Omega_{i,t}(x_{\ell}, h)\). Then, a firm’s expected value of a match in submarket \((x, h)\) in sector \(i\) is

\[
J_{i,t}^{\epsilon}(x, h) = \max_{(x_{\ell,i}, \pi_{\ell,i})} \sum_{\ell=1,2} [\pi_{\ell,i} \mathbb{E}_z J_{i,t}(\Omega_{i,t}(x_{\ell}, h), z, h)]
\]

\[
\text{s.t. } \sum_{\ell=1,2} \pi_{\ell,i}x_{\ell,i} \geq x, \quad \pi_{1,i} + \pi_{2,i} = 1, \quad \pi_{1,i}, \pi_{2,i} \in [0,1],
\]

and \(\omega_{\ell} = \Omega_{i,t}(x_{\ell}, h) \in [0,1] \text{ for } \ell = 1, 2\).

Denote the optimal choice of the lottery by the policy functions \((x_{\ell,i}(x, h), \pi_{\ell,i}(x, h))_{\ell=1,2}\), and the corresponding wage share by the policy function \((\omega_{\ell,i}(x, h))_{\ell=1,2}.^{15}\)

\(^{14}\)If the adjustment cost of separation is zero, then \(\omega_1 = \omega_2\), and \(J_i(\omega, z, \hat{h})\) has a discontinuous jump at this common value of \(\omega_1\) and \(\omega_2\).

\(^{15}\)Recall that the firm’s value function \(J_i(\omega, z, h)\) can have increasing sections with respect to \(\omega\) around 23.
4.3. Market tightness

The function $\theta_{i,t}(x, h)$ describes the tightness of the submarket $(x, h)$ in sector $i$ in period $t$, which is determined by free-entry of vacancies into the submarkets. A firm can enter any submarket in either sector to post vacancies. In sector $i$, the cost of posting a vacancy for one period is $k_i$. In submarket $(x, h)$, a vacancy is filled with probability $q(\theta_{i,t}(x, h))$, in which case the firm’s expected value over match-specific productivity is $J^c_{i,t}(x, h)$. If the vacancy is not filled, the firm’s value is zero. Thus, the expected profit of posting a vacancy in submarket $(x, h)$ in sector $i$ is $q(\theta_{i,t}(x, h))J^c_{i,t}(x, h) - k_i$. This expected profit cannot be strictly positive in the equilibrium: If it were strictly positive, then a firm would want to create infinitely many vacancies per applicant in submarket $(x, h)$, which would induce $q(\theta_{i,t}(x, h)) = 0$ that contradicts strictly positive expected profit of a vacancy. If a firm creates a positive number of vacancies in the submarket, the expected profit of a vacancy must be zero. On the other hand, if the expected profit of a vacancy is strictly negative, a firm will create no vacancy in the submarket. Thus, the tightness must satisfy:

$$q(\theta_{i,t}(x, h))J^c_{i,t}(x, h) \leq k_i \quad \text{and} \quad \theta_{i,t}(x, h) \geq 0, \quad \text{all } (x, h), \quad (4.14)$$

where the two inequalities hold with complementary slackness.\(^{16}\)

4.4. Definition of equilibrium

The distribution of workers at the end of period $t$ can be described by $(u_{i,t}(h), g_{i,t}(\omega, z, h))$, where $i \in \{1, 2\}$, $h \in H$, $\omega \in [0, 1]$ and $z \in Z$. The number $u_{i,t}(h)$ denotes the measure of skill-$h$ workers who are unemployed in sector $i$ at the end of period $t$. The number $g_{i,t}(\omega, z, h)$ denotes the measure of workers in match $(\omega, z, h)$ in sector $i$ at the end of period $t$, i.e., the measure of skill-$h$ workers who are employed in sector $i$ at a wage share the levels where the separation probability increases from $\delta_i$ to 1, because increasing $\omega$ even slightly at such values has a large benefit to the firm by reducing the separation probability. If this increasing section occurs in $[0, \omega_0]$ for some $\omega_0 > 0$, then the objective function in (4.13) is increasing in $x$ for $x \in [\underline{x}, x_0]$, where $\underline{x}$ is the lower bound on $x$ and $x_0 > \underline{x}$. In this case, it is never optimal to offer any value $x \in [\underline{x}, x_0)$, because such an offer is dominated by the offer $x_0$. To generate the function $J^c_i(x, h)$ for such values of $x$, we allow the constraint $\sum_{\ell=1,2} \pi_{\ell,i}x_{\ell,i} \geq x$ to be slack, so that $J^c_i(x, h) = J^c_i(x_0, h)$ for $x \in [\underline{x}, x_0]$. This flat section of $J^c_i(x, h)$ over $x$ is inconsequential in the equilibrium because no offer with $x < x_0$ is made in the equilibrium.

\(^{16}\)We impose (4.14) for all submarkets $(x, h)$, not just for those that are active in the equilibrium. A similar restriction has been imposed on beliefs out of the equilibrium in the literature (e.g., Acemoglu and Shimer, 1999, Shi, 2009, and Menzio and Shi, 2010a, 2011).
and match-specific productivity \( z \). The distribution of workers over values is denoted \( g_{v_i,t}(v, z, h) \), which can be calculated from \( g_{i,t}(\omega, z, h) \) for any worker value \( v = V_{i,t}(\omega, z, h) \).

An equilibrium in the economy consists of workers’ policy function for optimal search, \( s_{i,t}(v, h) \), policy function for optimal separation into unemployment, \( d_{i,t}(v, h) \), policy function for the lottery, \( (x_{i,t}(x, h), \pi_{i,t}(x, h))_{t=1,2} \), the implied policy function for the wage share, \( (\omega_{i,t}(x, h))_{t=1,2} \), workers’ value functions \( V_{ui,t}(h) \) and \( V_{i,t}(\omega, z, h) \), firms’ value functions \( J_{i,t}(\omega, z, h) \) and \( J_{i,e}^e(x, h) \), the market tightness function \( \theta_{i,t}(x, h) \), and the distribution of workers \( (u_{i,t}(h), g_{i,t}(\omega, z, h)) \) that meet the following requirements:

(i) \( s_{i,t}(v, h) \) solves (4.1), and \( d_{i,t}(v, h) \) solves the maximization problem in (4.8);  
(ii) \( V_{ui,t}(h) \) satisfies (4.5), \( V_{i,t}(\omega, z, h) \) satisfies (4.8), and \( J_{i,t}(\omega, z, h) \) satisfies (4.11);  
(iii) \( (x_{i,t}(x, h), \pi_{i,t}(x, h))_{t=1,2} \) solve (4.13), yield \( J_{i,e}^e(x, h) \), and induce \( \omega_{i,t}(x, h) \) as in (4.12);  
(iv) \( \theta_{i,t}(x, h) \) satisfies (4.14);  
(v) Given the initial distribution \( (u_{i,0}(h), g_{i,0}(\omega, z, h)) \), the distribution in any period \( t \), \( (u_{i,t}(h), g_{i,t}(\omega, z, h)) \), is consistent with the flows of workers generated by optimal search \( s \), optimal separation \( d \), exogenous exit and entry of workers from the labor market, skill accumulation \( A_i \) and skill depreciation \( D_i \).

We have explained requirements (i) - (iv). Requirement (v) governs equilibrium dynamics of the distribution of workers. Optimal search by unemployed workers generates a flow of workers from unemployment into employment. Optimal search by employed workers generates job-to-job transitions between different matches \( (\omega, z, h) \). Optimal separation generates a flow from employment into unemployment. Death reduces the measure of workers in the support of the distribution, while birth introduces new workers into the market through unemployment at the lowest skill level \( h_1 \). The skill accumulation process \( A_i \) moves workers up the skill level in sector \( i \), while the skill depreciation process \( D_i \) moves workers down the skill level when workers change sectors. Although we can write down the law of motion of the distribution to reflect these flows explicitly, the task is cumbersome and it requires a large amount of new notation. Moreover, this task is not necessary for the numerical computation, as we will explain below.

An important property of the equilibrium is block recursivity; that is, individuals’ policy functions, value functions and the market tightness function are all independent of the distribution of workers.\(^{17}\) This property has been analyzed in previous models of directed

\(^{17}\) The time subscript \( t \) on these functions does not indicate the dependence of these functions on the
search, such as Shi (2009), Menzio and Shi (2010a,b, 2011), and Gonzalez and Shi (2010).

Block recursivity is an of directed search and free entry of vacancies. To explain, let us start with workers’ search decisions. With directed search, a worker optimally chooses to search in the submarket that offers the best tradeoff between the expected value and the matching probability. For this decision, the worker only needs the information on the tightness in each submarket, which is provided by the tightness function. If the tightness function is independent of the distribution of workers, so is a worker’s optimal search decision. This feature of optimal search implies that the gain from search is independent of the distribution of workers. Thus, a worker’s value function depends only on the value of the worker’s current status, the particular offer he chooses to search for, and the matching probability associated with that particular offer. Similarly, the tightness function provides all the information needed for a firm to optimally choose where to create vacancies, how many vacancies to create, and the lottery needed to deliver an offer. So, the decision on vacancy creation, the choice of the lottery, and a firm’s value function are independent of the distribution of workers. Finally, with free entry of vacancies, the expected profit of a vacancy must be zero in every submarket where the expected value of a vacancy over match-specific productivity is at least as large as the cost of a vacancy. This condition pins down the tightness in each submarket, which is indeed independent of the distribution of workers.

Block recursivity does not imply that the distribution of workers is unimportant for the aggregate economy or the sectoral reallocation. On the contrary, the dynamics of the distribution are critical for assessing the dynamics of a sector’s employment share, the levels of wage and output in each sector, and wage inequality. What block recursivity enables us to do is to simplify the characterization and computation of the dynamic equilibrium substantially by eliminating the distribution as a state variable in individuals’ optimal decisions. Specifically, we can first compute the policy functions, the value functions, and the market tightness function without any reference to the distribution of workers. Then, we can compute the dynamics of the distribution by simply counting the flows of workers. In the actual procedure of computation, we will not even need to resort to the law of motion of the distribution of workers. Instead, we will use the policy functions to simulate

\[ Y_{i,t}(z, h), \text{ unemployment benefit, } b_i(h), \text{ and new workers’ participation, } \zeta_t. \]
the equilibrium to obtain the dynamics of the distribution.

The equilibrium has two main sources of residual wage inequality. One is match-specific productivity, which is exogenously assumed. Because two matches that are identical ex ante can have different realizations of match-specific production, the workers in the two matches can have different wage levels even if they have the same wage share. The second source is search frictions which induce workers’ wage shares and values to diverge endogenously. For example, two workers with the same skill \( h \) in sector \( i \) can have different search outcomes. The one who gets a new match will have a different value from the one who fails to get a new match. This difference in the value will induce the two workers to search for different values next period. More generally, a worker’s policy function for optimal search, \( x = s_{i,t}(v, h) \), is an increasing function of the worker value of staying in the current position, \( v \). Thus, the higher a worker’s value of staying in the current position, the higher the value the worker will search for in the future. This feature of optimal search induces dispersion in search values and wage shares even if match-specific productivity, skill accumulation and skill depreciation are absent.\(^{18}\)

5. Quantitative Analysis

5.1. Functional forms

The revenue function in sector 1 is

\[
Y_{1,t}(z, h) = [y_0 h + r(h)(y_t - y_0)] z,
\]

and the revenue function in sector 2 is \( Y_{2,t}(z, h) = y_0 z h \). The process of \( y_t \) is the driving force of the sectoral shift that we explore in this paper. The initial value of \( y \) is \( y_0 = 1 \) and the time path of \( y \) is

\[
y_t = \begin{cases} 
  y_0 + \frac{t}{T}(y^* - y_0), & \text{if } 1 \leq t \leq T - 1 \\
  y^*, & \text{if } t \geq T,
\end{cases}
\]

where \( y^* > y_0 \) and \( T > 1 \). That is, the relative marginal revenue in sector 1, \( y_t \), will grow linearly between periods 1 and \( T \), and will reach the new stationary level \( y^* \) in period \( T \).

\(^{18}\)Lotteries increase this heterogeneity further by creating the possibility that two workers with the same skill \( h \) who are matched with the same offer \( x \) may draw different prizes in a lottery.
The function $r(h)$ captures how changes in $y_t$ in sector 1 interact with the skill level. It is specified as

$$r(h) = \left\{ \begin{array}{ll} \frac{h - \psi h_1}{h_{N(h)} - \psi h_1} h_{N(h)}, & \psi \geq 0. \\ \end{array} \right.$$  

This is a parsimonious specification for a wide range of interactions between $y_t$ and $h$, captured by a single parameter $\psi$. If $\psi < 1$, then $r(h)$ is strictly increasing in $h$ for all $h \in H$. In this case, matches in sector 1 with high worker skills benefit more from the increase in $y_t$ than matches with low worker skills. If $1 \leq \psi < h_{N(h)}/h_1$, then $r(h)$ is strictly decreasing in $h$ for $h < \psi h_1$ and strictly increasing in $h$ for $h > \psi h_1$. In this case, matches in sector 1 with worker skills at the two ends benefit more from the increase in $y_t$ than matches with worker skills in the middle. This non-monotonic effect may be present in the data. If $\psi > h_{N(h)}/h_1$, then $r(h)$ is strictly decreasing in $h$ for all $h \in H$. In this case, matches in sector 1 with low worker skills benefit more from the increase in $y_t$ than matches with high worker skills.

The matching function is such that a worker’s matching probability in a submarket with tightness $\theta$ is

$$p(\theta) = \frac{\theta}{(1 + \theta^\gamma)^{1/\gamma}}, \quad \gamma > 0.$$  

Then, the matching probability function for a vacancy in the submarket is $q(\theta) = \frac{p(\theta)}{\theta} = (1 + \theta^\gamma)^{-1/\gamma}$. Output of home production is assumed to have the form

$$b_t(h) = b_0[Y_{1,t}(\mathbb{E}z, h) + Y_{2,t}(\mathbb{E}z, h)]/2,$$  

where $b_0 > 0$. This form allows $b_t$ to increase with the increase in the revenue in sector 1. If $b_t(h)$ is set to be constant over time, instead, the increase in the revenue in sector 1 may reduce unemployment sharply during the transition.

For skill accumulation on the job, the skill level of a worker employed in sector $i$ changes from $h_n$ in the current period to $h_{n+n'}$ in the next period according to the probability:

$$A_i(h_n, h_{n+n'}) = \left\{ \begin{array}{ll} 1, & \text{if } n' = 0 \text{ and } n = N \\ a_i \in (0, 1), & \text{if } n' = 0 \text{ and } n < N \\ (1 - a_i) \left[ \sum_{j=1}^{N-n} \left( \frac{n'}{N} \right)^n \right]^{-1}, & \text{if } 0 < n' \leq N - n, \end{array} \right.$$  

where $\eta_i \geq 0$ is a constant. Let us explain this specification. If the worker’s skill is at the highest level, it remains the same. If the worker’s skill is below the highest level, then
his skill stays at the current level with probability \( a_i \) and increases to a higher level \( h_{n+n'} \) with probability \((1-a_i)\left[\sum_{j=1}^{N-n} \left(\frac{n'}{j}\right)^{n+1}\right]^{-1}\). The parameter \( \eta_1 \) controls how the new skill is distributed. If \( \eta_1 = 0 \), the worker makes a transit to each of the higher skill levels with the same probability \((1-a_i)/(N-n)\). If \( \eta_1 > 0 \), then the farther the new skill level is above the worker’s current level, the lower the probability that the worker makes a transit to that new level. In the limit \( \eta_1 \to \infty \), the worker either stays at the current skill level or moves up by one level.

Similarly, for the skill loss when switching sectors, the probability with which a worker in sector \( i \) with skill \( h_n \) draws a potential skill level \( h_{n-n'} \) in the other sector is:

\[
D_i(h_n, h_{n-n'}) = \begin{cases} 
1, & \text{if } n' = 0 \text{ and } n = 1 \\
\sigma_i \in (0, 1), & \text{if } n' = 0 \text{ and } n > 1 \\
(1 - \sigma_i) \left[\sum_{j=1}^{n-1} \left(\frac{n'}{j}\right)^{n+1}\right]^{-1}, & \text{if } 0 < n' \leq n - 1,
\end{cases}
\tag{5.2}
\]

where \( \eta_2 > 0 \). That is, if the worker’s current skill is at the lowest level, the worker’s potential skill in the other sector remains at the lowest level; if the worker’s skill is above the lowest level, then his potential skill in the other sector remains at the current level in sector \( i \) with probability \( \sigma_i \) and is equal to a lower level \( h_{n-n'} \) with probability \((1-a_i)/(N-n)\). If \( \eta_2 = 0 \), then the worker’s potential skill in the other sector will be equal to each of the lower levels with probability \((1-a_i)/(n-1)\). If \( \eta_2 > 0 \), then the farther a skill level is below the worker’s current level in sector \( i \), the lower the probability that the worker’s potential skill in the other sector will be equal to that level. In the limit \( \eta_2 \to \infty \), the worker’s potential skill in the other sector will be equal to either the current skill in sector \( i \) or one level below the current skill.

When a worker exits the labor force exogenously, the worker is replaced with a new worker who enters the labor force through unemployment. The new worker enters sector 1 with probability \( \zeta_i \) and sector 2 with probability \( 1-\zeta_i \). We assume \( \zeta_i = 0.5 + (y_t - y_0)f_d \), where \( f_d \) is a parameter. This specification focuses on the variations in the participation over time that are induced by the changes in the relative marginal revenue between the two sectors.
5.2. Parameter values

We consider the terminal stationary state and use data for the years 2004/2005, assuming that all fundamental parameters are constant over time. Furthermore, since the economy looks symmetric in 1960 we conjecture that all sector-specific parameters are actually equal across sectors:

\[
\begin{align*}
\delta_1 &= \delta_2 = \delta, \\
\lambda_{u1} &= \lambda_{u2} = \lambda_u, \\
\lambda_{e1} &= \lambda_{e2} = \lambda_e, \\
k_1 &= k_2 = k, \\
a_1 &= a_2 = a, \\
\sigma_1 &= \sigma_2 = \sigma, \\
\eta_1 &= \eta_2 = \eta.
\end{align*}
\]

To simplify estimation, we estimate a subset of the parameters outside of the model. These parameters are listed in Table 1. The values of \( a \) and \( \eta \) and the support of the skill distribution are marked “experimental” because they will be estimated (outside the model) but such estimation has not been performed. We will take the observations of workers who stayed in the same firm in consecutive periods and use their wage changes over time to estimate the support of the distribution, the persistence of the skill and the speed of skill accumulation on the job.

<table>
<thead>
<tr>
<th>parameter</th>
<th>meaning</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta = 0.987 )</td>
<td>discount factor</td>
<td>real interest rate</td>
</tr>
<tr>
<td>( b_0 = 0.6 )</td>
<td>coefficient in home production</td>
<td>macro estimate</td>
</tr>
<tr>
<td>( \rho = 0.99 )</td>
<td>exit probability</td>
<td>sample average</td>
</tr>
<tr>
<td>( \zeta = 0.5 )</td>
<td>prob. that a new worker enters sector 1</td>
<td>baseline</td>
</tr>
<tr>
<td>( \delta = 0.015 )</td>
<td>exogenous separation in sector ( i )</td>
<td>macro estimate</td>
</tr>
<tr>
<td>( \lambda_u = 1 )</td>
<td>search prob. of an unemployed worker</td>
<td>normalization</td>
</tr>
<tr>
<td>( c = 0.5 )</td>
<td>skill loss parameter in unemployment</td>
<td>micro estimate</td>
</tr>
<tr>
<td>( \xi = 0.01 )</td>
<td>parameter in the cost of separation</td>
<td>sufficiently small</td>
</tr>
<tr>
<td>( a = 0.95 )</td>
<td>skill persistence when staying in the same sector ( i = 1, 2 )</td>
<td>experimental value</td>
</tr>
<tr>
<td>( \eta = 3 )</td>
<td>parameter controlling distribution of skill changes</td>
<td>experimental value</td>
</tr>
<tr>
<td>( h \in {0.5, 1.5} )</td>
<td>evenly distributed support of skills</td>
<td>experimental values</td>
</tr>
</tbody>
</table>

We compute the final steady state of the model and use it to estimate the remaining parameters. These parameters describe the final level of the relative revenue in services, \( y^* \), the non-uniform benefit of \( y \) across skills, \( \psi \), the skill distribution, \( f_h \), the distribution from which match-specific productivity is drawn, \( f_z \), the cost of vacancy creation, \( k \), on-the-job search frequency, \( \lambda_e \), inter-sectoral search probability, \( 1 - \alpha \), the parameter in the
matching function, $\gamma$, and the skill loss when changing sectors, $1 - \sigma$. The estimation uses the following moments:

(i) Density of changes of log wages, i.e. its 10 percentiles, at the following transitions: (a) within sector, across firms; (b) across sectors.

(ii) Transition rates: (a) sector $i$ to sector $j$ (two moments); (b) sector to unemployment (two moments); (c) unemployment in sector to sector (four moments).

(iii) Stocks: Unemployment rate; service-sector unemployment share; service-sector employment share.

(iv) Log wage income: 10th Percentiles.

(v) Service-sector employment and unemployment shares;

(vi) Log relative wage income in the service sector.

5.3. Preliminary results

The full estimation of the model will be time consuming. To get some indications of how the model behaves quantitatively, we compute the model with the following values of the parameters:

\[ y^* = 1.20, \; \psi = 0, \; k = 0.1, \; \gamma = 0.2, \; \lambda^e = 1, \; \alpha = 0.8, \; \sigma = 0.5, \]
\[ h \text{ is uniformly distributed on 5 levels in } [0.5, 1.5], \]
\[ z \text{ is the normal distribution truncated on } [0.8, 1.2] \]
and then allocated to 11 levels in the interval.

Note that because $\psi = 0$ in this case, the increase in $y$ benefits all skills equally in the sense that $Y_{2,t}(z, h) = y_t z h$. Also, because $\sigma = 0.5$, there is a large loss in skill when a worker changes sectors.

Suppose that the economy is in the steady state in period 0 with $y_0 = 1$. At the beginning of period 1, the path of the relative marginal revenue per effective labor in the service sector to the non-service sector is realized to be changing over time according to the above process $\{y_t\}_{t \geq 1}$, which reaches the new stationary level at $T = 140$ (i.e., in thirty five years). We compute the initial steady, the final steady state, and the transitional dynamics between the two steady state. The procedure of computing the stationary equilibrium is described in Appendix A. Note that at time $T$, all value and policy functions become stationary, but the economy has not reached the steady state yet because the distribution of workers at $T$ is still away from the steady state. We compute the model for additional
100 periods and take the result at $T + 100$ as an approximation for the final steady state, which is depicted in the following figures as a pseudo period $T + 1$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image.png}
\caption{Shares of services in employment and unemployment}
\end{figure}

With the above parameter values, Figure 5.1 depicts the shares of services in employment and unemployment. Both shares increase steadily after a relative short length of time of non-monotonic adjustments. The model’s prediction of the employment share of services in the final steady state is close to the observed value in the data. This result can be taken as an indication that the final value of the relative marginal revenue, $y_T = 1.20$, is an appropriate one. It is important to recognize that the steady increase in the employment share of services is not automatically implied by the assumed process of $y_t$. Although $y_t$ increases steadily, the entire time path of $y$ is assumed to be realized at $t = 1$. If there were no frictions, this would imply that the present value of the gain to switching from non-services to services is the largest at $t = 1$: the earlier a worker makes this switch, the earlier the worker can benefit from the growing marginal revenue in the service sector. That is, a frictionless model would predict a large jump, and possibly overshooting, in the share of workers in services at $t = 1$. This large initial jump is prevented in this model by sector-specific skills and search frictions. With sector-specific skills, the skill loss when changing sectors reduces the magnitude of the transition of workers from non-services to services. With search frictions, workers who are employed in non-services and have a significant room for wage growth in non-services are unlikely to switch to services until they
get either close to the top of the wage distribution in non-services or unemployed.

Changes in log relative average wage income in services to non-services are depicted in Figure 5.2. Relative wage income increases at \( t = 1 \) because the workers employed in services immediately receive higher wages as the result of \( y_1 > y_0 \). After the initial increase, wage income in services relative to non-services falls for about fifteen periods and then rises. The magnitude of log relative wage income in the final steady state is comparable with the one observed in the data. The dynamic pattern of relative wage income is also remarkable. Although the fall at the beginning of the transition is small in both the magnitude and the duration in comparison with that observed in the data in 1970s and part of 1980s, it does indicate two important features of the model. First, the decision on whether to move is based on the expected present value in the service sector relative to the non-service sector, not on the spot wage. In fact, some workers are willing to take a job in the service sector even if the initial wage is relatively low, because they expect to be compensated later with faster wage growth that will be generated by the increase in the marginal revenue in services relative to non-services. Second, the move from non-services to services is not uniform across different levels of the skill and match-specific productivity. Workers who have relatively low skills and who are in relatively low match-specific productivity in non-services are more likely to move than workers who are in the middle spectrum of skills and match-specific productivity, because quitting from the current matches has a relatively
low opportunity cost for the former workers. Also, workers who are at the very top of the wage distribution in non-services are more likely to move than those in the middle, because staying in non-services does not provide much additional wage increase in the future.

The willingness to take up jobs in services with relatively low initial wages and the movement of workers with relatively low skills into services tend to increase wage inequality in the service sector. In contrast, the movement of workers with relative high skills into services tends to reduce wage inequality in the service sector. Figure 5.2 depicts the standard deviation in log wage income in services relative to non-services. The immediate increase in wage income in services at $t = 1$ compresses wage inequality in the service sector. After this initial fall, the relative standard deviation in log wage income increases in services relative to non-services. This increase in relative inequality indicates that a predominant fraction of the movers from non-services to services get relatively low wages. As they move up the wage ladder with on-the-job search and skill accumulation, their wages grow over time on average, which pushes up the upper tail of the wage distribution and keeps wage inequality relative larger in services than in non-services.

![Figure 5.3a. Kernel densities of log wage income in the two steady states: services](image)

To get a more detailed picture of the relative change in wage dispersion, we plot the kernel densities of log wage income in the initial steady state and the final steady state. These densities are shown in Figure 5.3a for the service sector and in Figure 5.3b for the non-service sector. The growth in the relative marginal revenue shifts the support of the wage distribution to the right in both sectors, and more so in services than in non-services.
In both sectors, the lower tail of the wage distribution is compressed and the upper tail becomes thicker, as more workers move up the wage ladder. In comparison, the upper tail becomes much thicker in the service sector than in the non-service sector, which explains the increased wage inequality in services relative to non-services.

![Kernel densities of log wage income in the two steady states: non-services](image)

**Figure 5.3b.** Kernel densities of log wage income in the two steady states: non-services

### 6. Conclusion

In this paper we document the facts on the shift of workers from the service sector to the non-sector sector, together with changes in the relative wage and relative inequality between the two sectors. A dynamic search model is constructed to allow for ex ante worker heterogeneity, sector specific skills, skill accumulation/loss, and on-the-job search. A numerical example shows that a steady increase in the relative marginal revenue in services to non-services can generate the observed steady growth of the employment share of services, the non-monotonic dynamics of the relative wage income, and widening wage inequality. The model needs to be structurally estimated to help us learn about fundamental parameters in the process of the relative marginal revenue, search, sector specific skills, and skill dynamics.
Appendix

A. Computation of the Steady State and Dynamics

To compute the steady state, we set \( y_t = y \) and set all functions to be time invariant. Exploring block recursivity of the equilibrium, we first compute the policy functions, the value functions, and the market tightness functions according to (i)-(viii) below:

(i) Start with a guess of a firm’s ex ante value function \( J^e_i(x, h) \).

(ii) Solve (4.14) for the tightness function \( \tilde{\psi}_i(x, h) \).

(iii) Compute a worker’s search decision from the stationary counterpart of (4.1), which yields the optimal search policy function \( s_i(v, h) \). Use this search policy to compute the matching probability \( \tilde{\pi}_i(v, h) \) by (4.2), an unemployed worker’s job-finding probability \( \Phi_i(v, h) \) by (4.3), and an employed worker’s job-to-job transition probability \( \Phi_i(v, h) \) by (4.4).

(iv) Compute the function \( V^e_u(v, h) \) from (4.6), and \( V^e_i(v, h) \) from (4.9). Use these functions to compute the optimal decision of separation into unemployment, \( d_i(v, h) \), from (4.10).

(v) Iterate on (4.5) to obtain an unemployed worker’s value function \( V_{ui}(h) \).

(vi) Iterate on (4.8) to obtain an employed worker’s value function \( V_i(\omega, z, h) \) and on (4.11) to obtain a firm’s value function \( J_i(\omega, z, h) \).

(vii) Solve the maximization problem in (4.13) to obtain the policy functions for the lottery, \( (x_{\ell,i}(x, h), \pi_{\ell,i}(x, h))_{\ell=1,2} \), and the updated ex ante value function of a firm, \( J^e_i(x, h) \).

(viii) Repeat steps (ii)-(vii) until the function \( J^e_i(x, h) \) converges. Invert the function \( V_i(\omega, z, h) \) obtained in step (vi) in the last iteration to obtain the wage share function, \( (\omega_{\ell,i}(x, h))_{\ell=1,2} \), corresponding to the prizes of the lottery in step (vii), \( (x_{\ell,i}(x, h))_{\ell=1,2} \).

Next, we compute the steady-state distribution of workers. We do so by Monte Carlo simulation. Start with an arbitrary initial distribution of workers.

Now we compute the dynamic equilibrium. We assume that the economy is in the steady state in period 0 with \( y = y_0 \). At the beginning of period 1, the path \( \{y_t\}_{t \geq 1} \) changed into the one specified in the main text. Because \( y_t = y^* \) for all \( t \geq T \), then for all \( t \geq T \), the policy functions, the value functions and the market tightness function are given by the new steady state with \( y = y^* \). In contrast, the distribution function of workers does not reach the new steady state at \( t = T \), in general.
References


