Fiscal Reform and Government Debt in Japan: A Neoclassical Perspective∗

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Abstract

Past government spending in Japan is imposing a significant fiscal burden that is reflected in a net debt to output ratio near 150 percent. In addition, an aging Japanese society implies that public expenditures and transfers payments relative to output are projected to continue to rise until at least 2050. In this paper we use a standard growth model to measure the size of this burden in the form of additional taxes required to finance these projected expenditures and to stabilize government debt. The fiscal adjustment needed is very large, in the range of 30-40% of total consumption expenditures. Using a distorting tax such as the consumption tax or the labor income tax requires either tax to rise to unprecedented highs, although the former is much less distorting than the latter. The extremely high tax rates we find highlight the importance of considering alternatives that attenuate the projected increases in public spending and/or enlarge the tax base.

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1 Introduction

Large government expenditures in response to low economic growth in the 1990’s and 2000’s has caused Japan to accumulate the highest debt to output ratio among developed economies. In addition, this ratio is expected to continue to rise due to projected increases in public pensions and health expenditures. Figure 1 shows that net debt to Gross National Product (GNP) has risen from around 15% of GNP in the early 1990s to about 110% in 2010.¹

![Figure 1: Net Debt to GNP Ratio](image)

The projected increases in public expenditures are due to the fact that Japan is facing a significant societal aging problem. Figure 2 shows projected increases in two dependency ratios. The first is the number of 65 and older individuals to the number of 20 to 64 year old individuals and the second is the ratio of 70 and older individuals to the number of people between 20 and 69.² The first dependency ratio implies that about 3 workers are currently supporting 1 retiree. In about 60 years, this ratio is projected to increase to just over 1 worker paying taxes to support 1 retiree. If individuals continue to work until age 70, the dependency ratio is projected to increase to 3 workers supporting 2 retirees.

¹Net debt is defined as the difference between financial liabilities and assets of the general government in Japanese national accounts. Our measure of GNP reflects adjustments to the national accounts to be described in Section 3.

Clearly much additional revenue will be required if pension and health benefits are to remain at current levels. Figure 3 shows projections for the expenditure to output ratios by Fukawa and Sato (2009). Without additional revenue, these large increases in government expenditures would lead to further increases in government debt relative to output. While the debt to output ratio has already reached unprecedented levels, it is reasonable to expect that this cannot go on forever. At some point, it will be necessary for the Japanese government to take action to stabilize the debt.
In this paper, we develop a neoclassical growth model that builds on Hayashi and Prescott (2002) and Chen, İmrohoğlu, and İmrohoğlu (2006) by incorporating a strong domestic demand for government bonds and explore alternative ways of financing the projected increases in government expenditures. The basic question we address is “What are the revenue requirements and new taxes needed to finance future government expenditures and at the same time reduce the level of indebtedness to 60% of output in the long run?”

The model is a one sector deterministic growth model in which the private sector has perfect foresight about population growth rates, government policy and factor prices. Both the quantity and the price of bonds are endogenously determined in our model; government purchases of goods and services and transfer payments are exogenous. The government raises revenue by taxing factor incomes, interest income and consumption. The representative household values consumption, leisure, and government bonds and markets are complete. By including bonds in the utility function, the model is made consistent with the very strong domestic demand for government bonds in Japan. A stand-in firm hires labor and rents capital from households. We incorporate forecasts of government purchases and transfer payments from Fukawa and Sato (2009) and projections of future population growth rates produced by the Japanese government.

After calibrating the model to the Japanese economy, we compute a transition path from observed initial conditions in Japan in 1981 to a steady-state in the distant future. In our benchmark exercise, we compute the additional revenue in the form of reductions in transfer payments (or, equivalently, lump sum taxes) that must be raised in order to

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Sakuragawa and Hosono (2010) employ an alternative approach that incorporates intermediation costs to obtain low equilibrium interest rates on government debt.
accomplish this transition. Next, we compute alternative transition paths in which the
government increases distorting tax rates by raising some combination of the consumption
tax or labor income tax.

Of course, the tax increase required to stabilize Japanese debt will depend on how
high the debt to output ratio is allowed to grow before taking action. In our neoclassical
growth model, any debt to output ratio is sustainable in steady state; there is no endogenous
point at which debt can climb no further.\textsuperscript{4} Hence, we experiment with different arbitrary
threshold debt to output ratios (we assume 250 percent as our base case) that, once reached,
triggers tax increases that eventually bring this ratio down to a long run value of 60 percent.

Our main finding is that the revenue required to finance the projected increases
in government expenditures and to stabilize Japanese government debt once the debt to
output ratio reaches 250% is on the order of 30-40% of aggregate consumption per year.
Furthermore, if the government uses distorting taxes such as the consumption tax, this tax
rate must be increased to an unprecedented level of over 60%. If the tax system is reformed by
broadening the tax base, the consumption tax would still have to increase to approximately
40%.\textsuperscript{5} The labor tax, however, is sufficiently distorting that it cannot be used by itself
to accomplish the transition, even when tax base broadening is implemented. Instead, the
consumption tax must be increased along with the labor income tax rate.\textsuperscript{6} If the government
takes fiscal action sooner (at a debt to output ratio below 250%), the required increase in
tax rates is less dramatic but still substantial.

Although the neoclassical growth model does not impose a maximum debt to output
ratio, when compared with the experiences of other countries, it is perhaps surprising that
Japan can continue to borrow at quite low interest rates. The majority of Japanese govern-
ment debt, above 90%, is held domestically. According to Hoshi and Ito (2014), this home
bias is driven by Japanese institutional investors (Japanese banks, insurance companies, pen-
sion funds, and the Bank of Japan) who hold about 85% of the Japanese Government Bonds
(JGB’s).\textsuperscript{7} Consequently, we model Japan as a closed economy where we have calibrated the

\textsuperscript{4}Hoshi and Ito (2014) endogenize the maximum debt to output ratio by assuming that government debt
cannot exceed the total financial assets held by the private sector. This leads them to conclude that the
maximum debt to output ratio between 250% and 350%. We assume a maximum ratio of 250% in our
benchmark case.

\textsuperscript{5}Note that we have abstracted from deductions, exclusions, and progressive tax rates that characterize
the Japanese tax code and have set the labor income tax rate equal to the average marginal tax rate in Japan.
Hence, the amount of revenue raised in our model exceeds that actually raised by the Japanese tax system
by 8% of output. In our basic calibration, we assume this excess revenue is added to transfer payments as
a lump sum tax rebate to households. In some experiments, we eliminate this tax rebate and allow this
revenue to be used to lower the level of debt each period. We call this reform “tax base broadening.”

\textsuperscript{6}Keen, Pradhan, Kang, and de Mooij (2011) also argue for increasing the consumption tax in Japan on
the grounds that it is a less distorting way to raise the necessary revenue than the labor income tax.

\textsuperscript{7}As for the reasons for this home bias, Hoshi and Ito (2014) mention several factors. First, holding
JGB’s involves zero currency risk, which has been historically high for foreign assets. To the extent that
Japanese banks are averse to currency risk the JGB’s would appear very attractive. Second, for balance
sheet reasons, pension funds and insurance companies seem to be prefer holding long term JGBs since
their liabilities in Japanese yen. Third, holding JGB’s makes it easier for Japanese banks to satisfy capital
adequacy requirements. JGB’s do not contribute to the riskiness of bank assets and hence do not raise the
minimum amount of capital banks must hold. Fourth, the Japanese economy has not grown much over the
last two decades and there may not have been sufficiently profitable loans that the banks would have wanted
domestic demand for Japanese government bonds so that the equilibrium interest rate is close to that observed during our sample period. An alternative would be to view Japan as a small open economy. However, given the realized interest rates in our experiments, it would be difficult to envision Japan borrowing at a lower interest rate if international borrowing were an option.

We are contributing to a large literature studying the additional revenue required to stabilize the debt to output ratio in Japan. Early examples include Broda and Weinstein (2005) and Doi (2008). A recent example is Doi, Hoshi, and Okimoto (2011) who estimate the tax revenue as a fraction of GDP required to sustain the debt at the 2010 level. They find that revenue has to go to 40-47% of GDP (relative to 33% of GDP in 2010). They do not, however, consider what would be required if one were to use distorting taxes to raise this additional revenue. Hoshi and Ito (2014) find similar results.

Work that follows an approach similar to ours include İmrohoroğlu and Sudo (2011a) who also use the methodology of Hayashi and Prescott (2002). They measure the impact of raising the consumption tax rate in Japan from 5% to 15% and find that this leads to primary surpluses for several years, but eventually primary deficits re-emerge and the fiscal situation worsens. İmrohoroğlu and Sudo (2011b) examine whether a decade long growth miracle may increase tax revenue sufficiently to allow for fiscal balance and find that the sort of high growth episodes experienced in the past are not going to provide a solution to Japan’s fiscal problems.\(^8\)

The paper is organized as follows. Section 2 describes the model economy and calibration is discussed in Section 3. Section 4 presents our quantitative results. Section 5 concludes.

2 Model

In this section we describe the details of our model. Upper case variables are per capita values that grow along a balanced growth path. Lower case variables are stationary along a balanced growth path. The time period of the model is one year.

The economy is populated by a representative household with \(N_t\) members at time \(t\). The size of the household is assumed to grow at a time-varying growth factor \(\eta_t\) so that \(N_{t+1} = \eta_t N_t\). There is no uncertainty in our economy; households are assumed to have perfect foresight.

The fiscal analysis in this paper takes as given time series on tax rates, government spending (\(G_t\)), transfer payments (\(TR_t\), the working age population (\(N_t\)), and total factor productivity (\(A_t\)), where actual time series are used from 1981-2010. Forecasts and assumptions are used to extend these series to 2050 and beyond. In addition, we assume that the tax rates, the ratios of government purchases and transfer payments to output, and the growth rates of \(N_t\) and \(A_t\) are all eventually constant and the economy converges to a balanced growth path. A one sector neoclassical growth model is used to endogenously determine hours worked (\(h_t\)), consumption (\(C_t\)), output (\(Y_t\)), the stock of capital (\(K_{t+1}\)), tax revenues, to make. Finally, the persistent deflation in Japan over the last two decades has raised the return on JGB’s.

\(^8\)We consider a similar exercise and arrive at the same conclusion in section 4.4.3.
government debt \((B_t)\), and the price of government bonds \((q_t)\), from 1981 into the infinite future.

# 2.1 Government

We begin by describing the government’s budget constraint. The government is assumed to collect revenue from taxing household consumption at the rate \(\tau_{c,t}\), labor income at the rate \(\tau_{h,t}\), capital income at the rate \(\tau_{k,t}\), and interest on government bonds at the rate \(\tau_{b,t}\). Given time series for \(G_t\) and \(TR_t\), the quantity of one-period discount bonds \((B_{t+1})\) that are issued by the government is determined by the following budget constraint (where all quantities are in per capita terms):

\[
G_t + TR^*_t + B_t = \eta_t q_t B_{t+1} + \tau_{c,t} C_t + \tau_{h,t} W_t h_t
+ \tau_{k,t} (r_t - \delta) K_t + \tau_{b,t} (1 - q_t - 1) B_t.
\]  

Here, in addition to variables already defined, \(W_t\) and \(r_t\) denote the wage rate and the return to capital, and \(\delta\) is the depreciation rate of capital.

The government is also assumed to be subject to a “debt sustainability” rule that forces the government to retire debt when the debt to output ratio reaches some arbitrary value \(b_{\text{max}}\) that we specify. We include this feature for two reasons. First, the solution procedure we use for computing equilibrium paths requires that the economy ultimately converge to a steady state with a constant bond to output ratio. Without some additional constraint, this convergence would not be guaranteed. Second, while there is no natural limit to how much debt the government in our model can issue, such a limit almost certainly exists in actual economies. Although we will deviate from this assumption later in the paper, we initially assume that debt reductions required to stabilize the debt to output ratio are financed by reducing transfer payments, which is equivalent to a lump-sum tax in our model. Hence, if the government is required by our debt sustainability rule to spend \(D_t\) on debt reduction, \(TR^*_t\) in the government budget constraint (1) is given as follows:

\[
TR^*_t = TR_t - D_t.
\]  

The debt sustainability rule works in the following way. The value of \(D_t\) is initially zero and remains so until the debt to output ratio reaches the value \(b_{\text{max}}\). At this point, the government is forced to begin retiring a sufficient amount of debt by reducing transfers (setting \(D_t > 0\)) so that the debt to output ratio falls and ultimately converges to some value \(\bar{b} = \overline{B_t}/\overline{Y_t}\). More formally, \(D_t\) is set according to the following formula:

\[
D_t = \begin{cases} 
\kappa(B_t - \overline{B_t}) & \text{if } B_s/Y_s \geq b_{\text{max}} \text{ for some } s \leq t, \\
0 & \text{otherwise.} 
\end{cases}
\]  

In practice, we choose the parameter \(\kappa > 0\) to be as small as possible so that \(B(t)/Y(t)\) converges to \(\bar{b}\) and does not continue to increase without limit. That is, we experiment with alternative values of \(\bar{b}\) and \(b_{\text{max}}\) and, in each case, \(\kappa\) is assigned the smallest value such that convergence to \(\bar{b}\) is attained.
Later, we will consider rules that replace \( \tau_{c,t} \) and/or \( \tau_{h,t} \) with larger tax rates that are sufficient to bring \( B_t/Y_t \) to its steady state level \( \bar{b} \).

### 2.2 Household’s Problem

The household at time 0 is endowed with initial holdings of per capita physical capital \( K_0 > 0 \), and real, one-period, zero-coupon, discount bonds \( B_0 > 0 \). In addition, each member of the household is endowed with one unit of time each period that can be used for market activities \( h_t \) or leisure \( 1 - h_t \). Given a sequence of wages, rental rates for capital, government bond prices \( \{W_t, r_t, q_t\}_{t=0}^{\infty} \), tax rates on consumption, and labor, capital and bond income, and per-capita transfer payments \( \{\tau_{c,t}, \tau_{h,t}, \tau_{k,t}, \tau_{b,t}, TR_t\}_{t=0}^{\infty} \), the household chooses a sequence of per member consumption, hours worked, capital, and real bond holdings \( \{C_t, h_t, K_{t+1}, B_{t+1}\}_{t=0}^{\infty} \) to solve the following problem:

\[
\max_{t=0}^{\infty} \sum_{t=0}^{\infty} \beta^t N_t [\log C_t - \alpha \frac{h_t^{1+1/\psi}}{1 + 1/\psi} + \phi \log(\mu_t + B_{t+1})]
\]

subject to

\[
(1 + \tau_{c,t})C_t + \eta_t K_{t+1} + q_t \eta_t B_{t+1} = (1 - \tau_{h,t})W_t h_t + [(1 + (1 - \tau_{k,t})(r_t - \delta)] K_t + [1 - (1 - \eta_{t-1}) \tau_{h,t}] B_t + TR_t^*,
\]

where \( K_0 > 0 \) and \( B_0 > 0 \) are given initial conditions. Here \( K_{t+1} \) is per member holdings of capital at time \( t + 1 \). \( \eta_t K_{t+1} \) expresses the same quantity of capital per member at time \( t \). The household’s maximization is subject to a budget constraint where after-tax consumption expenditures and resources allocated to wealth accumulation in the form of capital and bond holdings are financed by after-tax labor income, after-tax capital income and holdings of capital, after-tax proceeds of bond holdings chosen in the previous period, and transfer payments from the government. The parameter \( \beta \) denotes the household’s subjective discount factor. The disutility of work is described by \(-\alpha < 0 \) and \( \phi > 0 \) denotes the household’s preferences for government bonds. We use \( \psi \) to denote the intertemporal elasticity of substitution (IES) of labor.

Since about 95% of the Japanese government bonds are held domestically, we assume that Japan is a closed economy where all debt is held by Japanese citizens, i.e. the Japanese household in our model. In addition, Japanese government bonds historically have had yields less than the return to physical capital. As a result, we introduce government debt in the utility function, with \( \phi > 0 \).

Finally, \( \mu_t \) is a parameter that limits the curvature of the period utility function over bonds. Essentially, it represents assets that might be perfect substitutes to Japanese government issued bonds in generating utility to households. We allow this parameter to

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\[\text{Footnote:} \text{For example, consider a simplified version of the model in which the representative household solves } \max_{t=0}^{\infty} \beta^t \{\log c_t + \phi \log b_{t+1}\} \text{ subject to } c_t + k_{t+1} + q_t b_{t+1} = w_t + r_k k_t + (1 - \delta) k_t. \text{ The first order conditions are given by } \frac{1}{\beta^t} = \beta \frac{r_k}{r_{k+1}} \frac{\phi}{\phi - 1} - \frac{w_t}{\eta_t} + \frac{\phi}{\phi - 1} = 0, \text{ and } R_t = r_t + 1 - \delta. \text{ Steady-state implies } q - \frac{1}{R} = \frac{\phi c}{\beta} > 0, \text{ which means that the return on } k, \text{ denoted by } R, \text{ dominates that on } b \text{ which is equal to } 1/q. \]

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\[\text{Footnote:} \text{This parameter helps us to match the volatility of the bond prices.}\]
move at the same rate of balanced growth as the rest of the economy so that the detrended version is a constant. In particular, \( \mu_t = \mu A_t^{1/(1-\theta)} \).

### 2.3 Firm’s Problem

A stand-in firm operates a constant returns to scale Cobb-Douglas production technology

\[
N_t Y_t = A_t (N_t K_t)^\theta (N_t h_t)^{1-\theta}
\]

\[
N_{t+1} K_{t+1} = (1 - \delta) N_t K_t + N_t X_t.
\]

Capital depreciates at the rate \( \delta \). The income share of capital is given by \( \theta \). \( A_t \) is total factor productivity which grows exogenously at the rate \( \gamma_t \), so we have \( A_{t+1} = \gamma_t A_t \). Per capita gross investment is denoted by \( X_t \).

### 2.4 Equilibrium

Given a government fiscal policy \( \{G_t, TR_t, D_t, B_t, \tau_{h,t}, \tau_{k,t}, \tau_{c,t}, \tau_{b,t}\}_{t=0}^{\infty} \), a debt sustainability rule \( \{\kappa, \bar{b}, b_{\text{max}}\} \), and the paths of working age population \( \{N_t\}_{t=0}^{\infty} \) and technology \( \{A_t\}_{t=0}^{\infty} \), a competitive equilibrium consists of an allocation \( \{C_t, h_t, K_{t+1}, B_{t+1}\}_{t=0}^{\infty} \), factor prices \( \{W_t, r_t\}_{t=0}^{\infty} \) and the bond price \( \{q_t\}_{t=0}^{\infty} \) such that

- the allocation solves the household’s problem,
- the allocation solves the firm’s profit maximization problem with factor prices given by: \( W_t = (1 - \theta) A_t K_t^\theta h_t^{\theta - \theta} \) and \( r_t = \theta A_t K_t^{\theta - 1} h_t^{1 - \theta} \),
- the government budget is satisfied,
- given a value for \( b_{\text{max}} \), the value of \( \kappa \) in the fiscal sustainability rule is sufficiently large to guarantee convergence of \( B_t/Y_t \) to \( \bar{b} \),
- the market for bonds clears,
- and the goods market clears: \( C_t + [\eta_t K_{t+1} - (1 - \delta) K_t] + G_t = Y_t \).

### 2.5 Detrended Equilibrium Conditions

In this subsection we derive the detrended equilibrium conditions to use in solving the model numerically. Given a trending per capita variable \( Z_t \) we obtain its detrended per capita counterpart by

\[
z_t = \frac{Z_t}{A_t^{1/(1-\theta)}}.
\]
The first set of detrended equilibrium conditions is given below.

\[ \frac{(1 + \tau_{c,t+1})^{1/(1-\theta)} c_{t+1}}{(1 + \tau_{c,t}) c_t} = \beta [1 + (1 - \tau_{k,t+1})(r_{t+1} - \delta)], \quad (5) \]

\[ \frac{\phi}{\mu + b_{t+1}} + \frac{\beta \eta_t [1 - (1 - q_t)\tau_{b,t+1}]}{(1 + \tau_{c,t+1})c_{t+1}} = \frac{q_t \eta_t \gamma_t^{1/(1-\theta)}}{(1 + \tau_{c,t}) c_t}, \quad (6) \]

\[ \alpha h_t^{1/\psi} = \frac{(1 - \tau_{h,t}) w_t}{(1 + \tau_{c,t}) c_t}, \quad (7) \]

\[ \eta_t \gamma_t^{1/(1-\theta)} k_{t+1} = (1 - \delta) k_t + x_t. \quad (9) \]

Equation (5) is the typical Euler equation arising from the choice of capital stock at time \( t \). The bond Euler equation is given by (6). The first order condition for hours worked is shown in equation (7). The production function and the law of motion for capital are given in equations (8) and (9), respectively. The budget constraint for the household is given below in equation (10)

\[ (1 + \tau_{c,t}) c_t + \eta_t \gamma_t^{1/(1-\theta)} k_{t+1} + q_t \eta_t \gamma_t^{1/(1-\theta)} b_{t+1} = (1 - \tau_{h,t}) w_t h_t + [1 - (1 - q_{t-1})\tau_{b,t}] b_t + tr_t - dt + [1 + (1 - \tau_{k,t})(r_t - \delta)] k_t. \quad (10) \]

The government budget equation is given by equation (11)

\[ g_t + tr_t + b_t = q_t \eta_t \gamma_t^{1/(1-\theta)} b_{t+1} + \tau_{c,t} c_t + \tau_{h,t} w_t h_t + \tau_{k,t} (r_t - \delta) k_t + \tau_{b,t} (1 - q_{t-1}) b_t + d_t. \quad (11) \]

Equation (12) is the detrended debt sustainability rule

\[ d_t = \begin{cases} \kappa (b_t - \overline{b}_t \overline{y}) & \text{if } b_s/y_s \geq b_{\text{max}} \text{ for some } s \leq t, \\ 0 & \text{otherwise}. \end{cases} \quad (12) \]

where \( \overline{y} \) is the value of \( y_t \) along the balanced growth path and \( \overline{b} \) is the targeted debt to output ratio along the balanced growth path.

Finally, the market clearing conditions are given below in equations (13), (14) and (15)

\[ r_t = \theta k_t^{\theta-1} h_t^{1-\theta}, \quad (13) \]

\[ w_t = (1 - \theta) k_t^{\theta} h_t^{-\theta}, \quad (14) \]

\[ c_t + x_t + g_t = y_t. \quad (15) \]

Hence we have 10 equations, (5) through (14), in 10 unknowns \( \{c_t, x_t, h_t, y_t, k_{t+1}, b_{t+1}, d_t, q_t, w_t, r_t\} \) at each time period \( t \).
2.6 Solution Procedure

We take as given a value for $k_{1981}$ and a sequence \{τ_{c,t}, τ_{h,t}, τ_{b,t}, η_{t}, γ_{t}, g_t, tr_t\}_{t=1981}^{\infty}$, where the elements of this sequence are constant beyond some date. These constant values determine the steady state to which the economy ultimately converges. We use a shooting algorithm, similar to that in Hayashi and Prescott (2002) and Chen, İmrohoroglu, and İmrohoroglu (2006), to determine the value of $c_{1981}$ (or, equivalently, $k_{1982}$) such that the sequence of endogenous variables \{c_t, x_t, h_t, y_t, k_{t+1}, b_{t+1}, d_t, q_t, w_t, r_t\} determined by equations (5) through (14) converges to the steady state. That is, the shooting algorithm guarantees that the capital stock sequence satisfies the transversality condition. Note that our fiscal sustainability rule guarantees that the bond to output ratio is equal to $\bar{\delta}$ in the steady state achieved in the limit.

3 Calibration

The structural parameters of our model are calibrated based on information from the sample period, which consists of annual data from 1981 to 2010.\(^{11}\) We take the capital-output and bond-output ratios in 1981 as initial conditions and use the sample paths for total factor productivity (TFP), population growth rates, tax rates, government purchases and transfer payments as exogenous inputs to the model. In addition we make assumptions about the values for these exogenous variables beyond the sample period in order to calculate equilibrium transition paths from 1981 toward the eventual steady state.

**Population:** Our measure of population, $N_t$, is working age population between the ages of 20 and 69. What matters for the equilibrium path computed, however, is not the level but the sequence of population growth factors (see equations 5 to 15). We use the actual values between 1981 and 2010 and rely on official projections for 2011-2050. We assume that the population stabilizes after 2050; that is $\eta_t = 1$, $t \geq 2050$. The projections for 2011-2050 are the medium-fertility and medium-mortality variants of population forecasts calculated by the National Institute of Population and Social Security Research.\(^{12}\)

**National Accounts:** Our measure of output is Gross National Product adjusted to include income from foreign capital, following Hayashi and Prescott (2002). In particular, we define the model’s capital stock, $K_t$, as consisting of private fixed capital, held domestically and in foreign countries. We add net exports and net factor payments from abroad to measured private investment.

Government investment, including net land purchases, is assumed to be expensed. Therefore we treat it as part of government consumption and subtract depreciation of government capital from government consumption. We summarize these choices in Table 1 below:

\(^{11}\)The main reason for taking the year 2010 as the last year for our sample is to abstract from the huge public expenditures in 2011 following the Great Tohoku Earthquake, tsunami and the nuclear disaster.

\(^{12}\)http://www.ipss.go.jp/index-e.asp.
Table 1: Adjustments to National Account Measurements

\[
C = \text{Private Consumption Expenditures} \\
I = \text{Private Gross Investment} + \text{Change in Inventories} + \text{Net Exports} + \text{Net Factor Payments from Abroad} \\
G = \text{Government Final Consumption Expenditures} + \text{General Government Gross Capital Formation} + \text{Government Net Land Purchases} - \text{Book Value Depreciation of Government Capital} \\
Y = C + I + G
\]

**Labor input:** For \( h_t \) we take the product of employment per working age population and average weekly hours worked, normalized by dividing by 98, which is our assumption on discretionary hours available per week.

**Government Accounts:** Our measure of government purchases of goods and services, \( G_t \) in Table 1, also includes Japanese public health expenditures. Transfer payments, \( TR_t \), includes social benefits (other than those in kind, which are included in \( G_t \)) that are mostly public pensions, plus other current net transfers minus net indirect taxes. We also add 8% of output to our measure of transfers since our modeling of flat tax rates leads to higher tax revenue than in the data as we abstract from all deductions and exemptions that are present in the complicated Japanese tax code. That is, the tax revenue collected minus this 8% of output is close to the actual tax revenue collected by the government.

As we mentioned in the Introduction (see Figure 3), Japan’s already high debt to output ratio is projected to rise even further due to the aging of the population. Fukawa and Sato (2009) estimate an increase of 3 percentage points in the ratio of government purchases to output and a 4 percentage point rise in transfer payments to output from 2010 to 2050. According to Fukawa and Sato (2009), the projected increase in government purchases is nearly entirely due to the expected increase in public long term care expenditures, driven by the increased longevity of the population. Similarly, the projected increase in transfer payments are driven by expected increases in public pension expenditures.\(^{13}\) These estimates are very similar to those calculated independently by İmrohoroğlu, Kitao, and Yamada (2013).\(^{14}\)

\(^{13}\)The projections in Fukawa and Sato (2009) are based on a system of about 40 regression equations (in addition to definitional relations and equations describing the evolution of the population in different age groups) which is estimated from Japanese data sources over the sample period 1980-2003. The population projections used are the same as those used in this paper. In addition, they assume a rate of growth of real GDP of about 2%.

\(^{14}\)İmrohoroğlu, Kitao, and Yamada (2013) build a micro-data based large-scale overlapping generations model for Japan and incorporate the Japanese pension rules in detail. Using existing pension law and fiscal parameters and the medium variants of fertility and survival probability projections, they produce future time paths for government purchases and transfer payments.
Tax Rates: Our measure of labor income tax rates, \( \tau_{h,t} \), comes from the estimates of average marginal labor income tax rates by Gunji and Miyazaki (2011). The last value is 0.3324 for 2007 and we assume that this same value holds for 2008 and beyond in the benchmark calibration.

The capital income tax rate, \( \tau_{k,t} \), is constructed following the methodology in Hayashi and Prescott (2002). The value of this tax rate for 2010 is 0.3557. We assume that this value remains unchanged for 2011 and beyond.

A consumption tax rate of \( \tau_{c,t} = 3\% \) was introduced in Japan in 1989, and it was raised to 5% in 1997 and to 8% in 2014. It is scheduled to rise to 10% in 2015. In our benchmark calibration, we assume that the consumption tax rate stays constant at 10% beyond 2015.

The tax rate on interest from government bonds, \( \tau_{b,t} \), is equal to 20% for all time periods. This tax is imposed on the semianual interest income from coupon-bearing bonds and is withheld (15% income tax plus 5% local tax) at the time the interest is paid.

Figure 4 below shows the tax rates used except for the tax on bond interest income, which is constant throughout at 20%.

![Figure 4: Tax Rates](image)

Technology parameters: Given the data described above, the Cobb-Douglas production function allows us to calculate total factor productivity:

\[
A_t = Y_t / (K_t^{\theta} L_t^{1-\theta}).
\]

The capital income share, \( \theta \), is set equal to 0.3783, which is the sample (1981-2010) average of the annual ratio of capital income to our adjusted measure of GNP. Given this, we can compute the growth factor of TFP, \( \gamma_t = A_{t+1} / A_t \), from the actual data between 1981
and 2010. For 2011 and beyond, we assume that $\gamma_t = 1.015^{1-\theta}$. This implies a growth rate of 1.5% for per capita output along the balanced growth path. Finally, we compute a time series for the depreciation rate of capital following the methodology of Hayashi and Prescott (2002) and set $\delta = 0.0842$, which is the sample average.

Table 2 summarizes our choices of TFP and population growth factors.

<table>
<thead>
<tr>
<th></th>
<th>1981−2010</th>
<th>2011−2050</th>
<th>2051−∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_t$</td>
<td>Actual Values</td>
<td>1.015$(1-\theta)$</td>
<td>1.015$(1-\theta)$</td>
</tr>
<tr>
<td>$\eta_t$</td>
<td>Actual Values</td>
<td>Government Projections</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Preference parameters:** There are five preference parameters, $\beta, \alpha, \psi, \phi,$ and $\mu$, in the utility function given by equation (4), where $\mu = \mu_t/A_t^{1/(1-\theta)}$. These are held constant throughout our analysis. The parameter $\psi$ is the Frisch elasticity of labor supply, taken as 0.5, following Chetty, Guren, Manoli, and Broda (2012).

For the three preference parameters $\beta, \alpha,$ and $\phi$, we use the equilibrium conditions given in equations (16), (17), and (18) for the sample period to obtain values for each year and, from that, averages over the sample.

\[
\beta_t = \frac{(1 + \tau_{c,t+1})\gamma_t^{1/(1-\theta)} c_{t+1}}{(1 + \tau_{c,t})c_t \left[1 + (1 - \tau_{h,t+1}) \left(\frac{\theta_{k,t+1}}{k_{t+1}} - \delta\right)\right]} \tag{16}
\]

\[
\alpha_t = \frac{h_t^{-1/\psi} (1 - \tau_{h,t})(1 - \theta) y_t}{(1 + \tau_{c,t})c_t h_t} \tag{17}
\]

\[
\phi_t = \eta_t (\mu + b_{t+1}) \left[\frac{q_t \gamma_t^{1/(1-\theta)}}{(1 + \tau_{c,t}) c_t} - \frac{\beta_t [1 - (1 - q_t) \tau_{b,t+1}]}{(1 + \tau_{c,t+1}) c_{t+1}}\right]. \tag{18}
\]

Note, however, that the equilibrium condition in equation (18) contains the equilibrium price of government bonds, $q_t$. The empirical counterpart to $q_t$ that we compute reflects the fact that government debt in actual economies is comprised of bond holdings of varying maturities while our model economy includes only one period discount bonds. In particular, let $B_t$ be beginning of period debt and $P_t$ be interest payments made in period $t$, both measured in current Yen. In addition, let $F_t$ be the GNP deflator. We compute the price of bonds in period $t$ as follows:

\[
q_t = \frac{B_{t+1}/F_t}{(B_{t+1} + P_{t+1})/F_{t+1}}. \tag{19}
\]

Using data on $B_{t+1}, F_t,$ and $P_{t+1}$ over the sample period, we compute $q_t$ and feed the values into the equilibrium conditions above to calculate the sample values of the preference parameters.
Figure 5 shows the sequence of \( \{q_t\}_{t=1981}^{2009} \) calculated from the data using equation 19 and the sequence implied by our benchmark calibration. In addition, we show the same sequence when \( \phi = 0 \) and bonds earn the same rate of return as capital. With \( \phi > 0 \), households are willing to hold government debt at a higher bond price and lower return than in the \( \phi = 0 \) case.

![Figure 5: Bond Prices](image)

In Figure 6, we compare the rates of return on capital and bonds in our model, both before and after tax. The rate of return dominance of capital over bonds is apparent in this figure.
The remaining preference parameter $\mu$, which is the detrended value of $\mu_t$, is chosen to minimize the sum of squared differences between the bond price implied by our model and its data counterpart.

Table 3 reports the values for the structural parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>0.3783 Sample Average, 1981-2010</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0842 Sample Average, 1981-2010</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9677 Equation (16), Sample Average</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>22.6331 Equation (17), Sample Average</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.5 Chetty et al (2012)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.063 Equation (18), Sample Average</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.1 Fit $q_t$ for 1981-2010</td>
</tr>
</tbody>
</table>

4 Quantitative Experiments

4.1 Benchmark Experiment

In our benchmark exercise, the government reduces transfers by $D_t$ (or, equivalently, levies a lump sum tax of this amount) when the bond to output ratio reaches some critical value $b_{\text{max}}$. Later, we will consider alternative fiscal policies that impose increases in distorting
taxes and/or broadening the tax base to retire debt. In what follows, we describe how we implement the fiscal sustainability rule introduced earlier in equation (12).

4.1.1 Revenue Required to Stabilize Debt

For \( b_{\text{max}} \), the maximum net debt to output ratio beyond which fiscal austerity kicks in, we consider three values, 200\%, 250\%, and 300\%. For most countries, these values may seem too high. For Japan, however, these may be more reasonable. Indeed, the (net) debt to output ratio for 2013 is around 150\%. In addition, setting \( b_{\text{max}} \) equal to 250\% is consistent with the maximum sustainable debt to output ratio estimated by Hoshi and Ito (2014).

We assume that the debt to output ratio along the balanced growth path, \( \overline{b} \), is equal to 60\%. This is loosely motivated by the debt to output ratio that was once viewed as a requirement to be part of the European Monetary Union. We will consider alternative values for this parameter in section 4.4.2.

As mentioned earlier, \( \kappa \) is set equal to the smallest value such that convergence to \( \overline{b} \) is attained. Figure 7 illustrates how we choose \( \kappa \) for the benchmark value of \( b_{\text{max}} = 250\% \). The upper panel of this figures shows the paths implied by our model for the debt to output ratios under three possible values for \( \kappa \). The endogenous date at which our sustainability rule is triggered is 2021, at which time the government begins to retire a fraction \( \kappa \) of the debt in excess of the steady-state value, \( \overline{b} \). A value of \( \kappa \), 0.05, as can be seen in Figure 7 is insufficient to rein in the debt to output ratio which continues to grow beyond 2021. The two larger values shown, 0.1 and 0.15, do succeed in bringing the debt to output ratio under control. The value 0.1 is the smallest value among these three that guarantees convergence to \( \overline{b} \).

The lower panel of Figure 7 provides a plot of \( d_t/c_t \), which is the revenue required to retire debt as a fraction of consumption expenditures, for different values of \( \kappa \). We refer to this as the “consumption tax equivalent revenue requirement.”

A value of \( \kappa = 0.15 \) would allow the debt to output ratio to fall more quickly, but, as can be seen from the bottom panel of Figure 7, this value would involve collecting more revenue than necessary in the initial periods after the trigger is activated.
Figure 7: $B_t/Y_t$ and $D_t/C_t$ in the Benchmark Economy

Note that as long as debt exceeds the steady-state value, $d_t$ remains positive. As the difference between actual debt and its steady-state value becomes small, the revenue requirement gets smaller and eventually, when the government purchases to output and transfer payments to output become stationary in 2050, the value of $d_t$ converges to zero.

Figures (8) and (9) illustrate the paths of the bond to output ratios and the revenue requirements for the two alternative values for $b_{max}$. For $b_{max} = 200\%$ and $b_{max} = 300\%$, we choose $\kappa = 0.12$ and $\kappa = 0.085$, respectively. As one would expect, the higher is $b_{max}$, the later is the date at which the fiscal sustainability rule is activated. In all of the cases considered, however, that date occurs before 2025. In addition, the revenue requirement increases with the value of $b_{max}$. 
Figure 8: Alternative Times Paths for Debt to Output Ratio

Figure 9: Revenue Requirement as Fraction of Aggregate Consumption
4.1.2 Comparison with Japanese Data

Although our primary interest is in projecting the path for endogenous variables beyond our sample period, we first report the time paths over the sample period 1981-2010 generated by our calibrated model and their counterparts from Japanese data. This allows us to evaluate similarities and differences between actual data and those generated by the model.

Figure 10 shows data and model comparisons for hours worked, capital stock and output. The last two variables are normalized so that 1981 values equal 100.

![Figure 10: Labor, Capital, and Output](image)

A striking aspect of Figure 10 is that our model does not quite match the observed time path for hours worked. During the 1990’s, labor supply fell significantly in Japan. Hayashi and Prescott (2002) attribute some of this decline to the legislated reduction in the length of the work week in Japan, a feature that is absent in our model. As a result, the model predicts a flatter hours path than observed in the data.

Figure 11 illustrates data and model consumption and investment, normalized to equal 100 in 1981, and the capital-output ratio. Here, one can see that the model predicts higher investment during the 1990’s and early 2000’s than actually observed. This may be due to the substitution toward capital goods with lower depreciation rates during this period, something that is not featured in our model. Toward the end of the 1981-2010 period, there is more agreement between the model and the data.

15Using the methodology for computing the depreciation rate in Hayashi and Prescott (2002), we find that the rate of depreciation fell from 9.1% in 1989 to 7.5% in 2010.
Figure 11: Consumption, Investment, and Capital-Output Ratio

Figure 12 shows the debt to output ratio from the Japanese data and that generated by our model. Recall that we add 8% of output to our measure of transfers to account for the fact that our model abstracts from exemptions and deductions in the Japanese tax code. With this assumption, our endogenous debt to output ratio is very much in line with that in the data.
4.2 Steady State Analysis with Distorting Taxes

Our benchmark experiment relies on reducing transfers to finance the fiscal burden Japan is facing. In practice, however, reducing transfers significantly may not be politically feasible and the government may have to raise distorting taxes such as the consumption tax and/or the labor income tax.

Before we consider the short run effects, it will be useful to describe the long run consequences of using distorting taxes. As equation (7) indicates, both tax rates distort the static first order condition governing the labor/leisure decision. This allows us to define an effective tax rate as a function of \( \tau_c \) and \( \tau_h \) using equation (7):

\[
(1 - \tau) = (1 - \tau_h)/(1 + \tau_c), \quad \text{which implies that} \quad \tau = (\tau_c + \tau_h)/(1 + \tau_c).
\]

(20)

Note that there are different combinations of \( \tau_h \) and \( \tau_c \) that produce the same \( \tau \). However, the amount of revenue raised by these taxes is not a function of \( \tau \), but instead a function of \( \tau_c \) and \( \tau_h \) separately. In other words, although the distortions created by these combinations are the same, the revenue raised is not. In particular, the total tax revenue of the government in steady state is given by

\[
\bar{\tau}_c \bar{c} + \bar{\tau}_h \bar{w} \bar{h} + \bar{\tau}_k (\bar{r} - \delta) \bar{k} + \bar{\tau}_b (1 - \bar{q}) \bar{b},
\]

where a bar over a variable indicates its steady state value. Note that steady state quantities such as consumption, hours worked, capital stock, bond holdings, as well as the wage rate, interest rate and the bond price depend on the particular configuration of \( \bar{\tau}_h \) and \( \bar{\tau}_c \). Throughout this section, we set \( \bar{\tau}_k = 35.57\% \) and \( \bar{\tau}_b = 20\% \).
In Figure 13 we set $\tau_c = 10\%$, which is the steady state value for this tax rate in our benchmark calibration, and plot a normalized measure of total tax revenues as a function of the steady state labor income tax rate, $\tau_h$. In particular, we take steady state revenue, multiply by 100 and divide by steady state government purchases plus transfers. That is, we plot revenues multiplied by $100/(G + TR)$.

![Figure 13: Labor Income Tax Rate and Steady State Tax Revenue](image)

Using our calibrated value for the Frisch elasticity of labor supply, $\psi$ equal to 0.5, one can see from Figure 13 that there exists no labor income tax rate in steady state that can raise the needed steady state revenues. We show, however, that it is possible to raise sufficient revenue if we assume a very low value for the Frisch elasticity and are willing to consider the resulting very high value for $\tau_h$. We regard these low elasticity/high labor tax rate cases to be unreasonable parameterizations and therefore we do not pursue these cases further. Instead, we consider increases in the consumption tax rate, perhaps in combination with increases in the labor income tax rate, that will raise the required revenue in the steady state.
Figure 14: Consumption Tax Rate and Steady State Tax Revenue

Figure 14 depicts the steady state relationship between the consumption tax rate and revenue setting $\tau_h = 0.3324$, which is the steady state value of this tax rate in our benchmark calibration. Note that the value of the consumption tax rate needed to raise the required revenue is about 40% and not sensitive to the Frisch elasticity of labor supply. Still, a consumption tax of this magnitude would give pause to policymakers.
In Figure 15 we show combinations of $\tau_h$ and $\tau_c$ that raise the required steady state revenue. By increasing the labor income tax rate one can reduce the corresponding consumption tax rate until the steady state is on the wrong side of the “Laffer” curve. In addition, this figure shows that the tax distortion as defined above is minimized (i.e. the effective tax is minimized) by setting the labor income tax rate equal to zero and raising all of the revenue by a consumption tax equal to about 80%. In other words, although consumption and labor income tax rates both raise the effective tax rate and distort labor supply, for a given amount of revenue, it is better to only use the consumption tax and set the labor income tax rate equal to zero, although we recognize that a zero labor income tax and an 80 percent consumption tax is not a politically feasible option.

4.3 Fiscal Policy Experiments

Our benchmark exercise computes the additional revenue that must be raised to achieve fiscal stability in the steady state but does not incorporate the distortions that would be caused by higher tax rates. In this subsection, we compare the results from our benchmark experiment in which transfers are reduced to achieve fiscal stability with ones that use the revenues from increases in the consumption tax and/or the labor income tax to achieve this goal. We focus on consumption and labor income tax rates because they do not involve the overwhelming distortions associated with increasing the capital income tax in this environment. We also consider the possibility of broadening the tax base. Recall that in our calibration, due to the fact that we assume flat tax rates and do not incorporate the deductions and exclusions that are part of the Japanese tax code, we added 8% of output to transfer payments (a lump sum tax rebate). That is, the tax revenue collected by the tax system in our model
minus 8% of output is roughly equal to the actual revenue collected by the Japanese tax system in the sample period. When we implement what we call “tax broadening” this lump sum tax rebate is eliminated and the government is able to keep additional revenue equal to 8% of output. This we presume is something the Japanese government could achieve by introducing a flat tax and eliminating deductions and exclusions from their tax code.

In our experiments we assume that tax rates and transfers are equal to their benchmark values until the debt to output ratio hits the threshold level, \( b_{\text{max}} \). We denote these values by \( \tau_{c,t}^B, \tau_{h,t}^B \), and \( TR_t^B \). Once the threshold is hit, say at \( t = T_1 \), some or all of these fiscal parameters will change from their benchmark values. For example, if tax broadening is assumed, then \( TR_t \) will equal \( TR_t^B - 0.08Y_t \) for \( t \geq T_1 \).

In each case we consider, we compute a new value for one tax rate, either for the consumption tax or the labor income tax, so that the government’s steady state budget constraint is satisfied given the value of \( b \) that we assume. This steady state tax rate is denoted by \( \overline{\tau}_c \) or \( \overline{\tau}_h \). In addition, after the threshold bond to output ratio is hit (when \( t = T_1 \)), this tax rate is raised by an amount \( \pi \) over and above its steady state level to pay for the projected increase in expenditures and to pay off the accumulated debt so that the debt to output ratio ultimately converges to \( \overline{b} \). In particular, \( \pi \) is the smallest such value that facilitates convergence to the steady state. Once \( \overline{b} \) has been reached, (at date \( t = T_2 \)) this additional increase is eliminated and the tax rate is set equal to its steady state value.\(^\text{16}\)

This fiscal policy can be summarized as follows (where \( x = c \) or \( h \) and \( t > 2010 \)):

\[
\tau_{x,t} = \begin{cases} 
\tau_{x,t}^B & \text{if } t < T_1 \text{ (i.e. } B_s/Y_s \leq b_{\text{max}} \text{ for all } s \leq t) \\
\tau_x + \pi & \text{if } T_1 \leq t < T_2 \text{ (i.e. } B_s/Y_s > b_{\text{max}} \text{ for some } s \leq t \text{ and } B_t/Y_t > \overline{b}) \\
\overline{\tau}_x & \text{if } t \geq T_2 \text{ (i.e. } B_t/Y_t \leq \overline{b})
\end{cases}
\]

Table 4 below summarizes the five experiments we consider in this section. In the first two experiments, the tax instrument used to achieve fiscal balance in the steady state is the consumption tax, while experiments 3-5 use the labor income tax. In the last set of experiments, the consumption tax is also increased at date \( T_1 \) since, as discussed in the previous section, it is impossible to achieve fiscal balance in the steady state by increasing only the labor income tax rate.

\(^\text{16}\)Given that the economy is close to its steady state but has not necessarily converged, we also activate the debt sustainability rule, equation (12), at this date. By allowing \( d_t \) to adjust according to this rule, we guarantee convergence to the steady state.
Table 4: Characterization of Fiscal Experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\tau_{c,t}^1$</th>
<th>$\tau_{h,t}^1$</th>
<th>$TR_t^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\begin{cases} \tau_{c,t}^B &amp; t &lt; T_1, \ \tau_{c,t}^1 + \pi_1 &amp; T_1 \leq t &lt; T_2, \ \tau_{c,t}^1 &amp; t \geq T_2. \end{cases}$</td>
<td>$\tau_{h,t}^B$ for all $t$,</td>
<td>$TR_t^1 = TR_t^B$ for all $t$,</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\tau_{c,t}^2$</th>
<th>$\tau_{h,t}^2$</th>
<th>$TR_t^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$\begin{cases} \tau_{c,t}^B &amp; t &lt; T_1, \ \tau_{c,t}^2 + \pi_2 &amp; T_1 \leq t &lt; T_2, \ \tau_{c,t}^2 &amp; t \geq T_2. \end{cases}$</td>
<td>$\tau_{h,t}^B$ for all $t$,</td>
<td>$TR_t^2 = \begin{cases} TR_t^B &amp; t &lt; T_1, \ TR_t - 0.08Y_t &amp; t \geq T_1. \end{cases}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\tau_{c,t}^3$</th>
<th>$\tau_{h,t}^3$</th>
<th>$TR_t^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$\begin{cases} \tau_{c,t}^B &amp; t &lt; T_1, \ \tau_{c,t}^3 + 0.3 &amp; t \geq T_1, \end{cases}$</td>
<td>$\tau_{h,t}^B$ for all $t$,</td>
<td>$TR_t^3 = TR_t^B$ for all $t$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\tau_{c,t}^4$</th>
<th>$\tau_{h,t}^4$</th>
<th>$TR_t^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$\begin{cases} \tau_{c,t}^B &amp; t &lt; T_1, \ \tau_{c,t}^4 + 0.3 &amp; t \geq T_1, \end{cases}$</td>
<td>$\tau_{h,t}^B$ for all $t$,</td>
<td>$TR_t^4 = \begin{cases} TR_t^B &amp; t &lt; T_1, \ TR_t^B - 0.08Y_t &amp; t \geq T_1. \end{cases}$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\tau_{c,t}^5$</th>
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<th>$TR_t^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>$\begin{cases} \tau_{c,t}^B &amp; t &lt; T_1, \ \tau_{c,t}^5 + 0.05 &amp; t \geq T_1, \end{cases}$</td>
<td>$\tau_{h,t}^B$ for all $t$,</td>
<td>$TR_t^5 = \begin{cases} TR_t^B &amp; t &lt; T_1, \ TR_t^B - 0.08Y_t &amp; t \geq T_1. \end{cases}$</td>
</tr>
</tbody>
</table>
4.3.1 Experiments 1 and 2 (Consumption Tax)

Figure 16 shows the projected time path of $\tau_{c,t}$ in experiments 1 and 2 when this tax rate is used to finance the projected increases in expenditures and to reduce the debt to output ratio (eventually) to 60%. The labor income tax rate in these experiments, which is also shown in this figure, is assumed to follow the same path as in the benchmark case.

The difference between experiments 1 and 2 is the presence of tax broadening in experiment 2 and its absence in experiment 1. The solid line depicts the time path of the consumption tax in experiment 1 when it is the only fiscal policy instrument used to achieve fiscal balance. In this case, there is a very sharp increase in the consumption tax in $T_1 = 2018$ from 10% to 61.4%. Eventually, at $T_2 = 2092$, the consumption tax rate reaches its steady state value of $\tau_{c}^1 = 47.4\%$.

When we assume in experiment 2 that the Japanese government broadens the tax base, which we capture by reducing transfers by 8% of output, the consumption tax rises from 10% to 40.0% at $T_1 = 2019$. It then converges to its steady state value of $\tau_{c}^2 = 27.0\%$ in 2087. The significant reduction in transfers allows the government to contain the increase in the consumption tax needed to pay for the projected increases in public expenditures and at the same time bring the debt to output ratio down to the prescribed value of 60%.

4.3.2 Experiments 3–5 (Labor Income Tax)

As Figure 13 shows, it is impossible to raise sufficient revenue to cover the projected expenditures at the steady state when the government relies exclusively on increasing the labor income tax. In Figure 17 we consider policies that adjust additional fiscal instruments to complement the increase in the labor income tax so that sufficient revenues are raised.
In our first two labor income tax experiments, denoted experiments 3 and 4, when the debt to output ratio reaches the first trigger value of 250% in year $T_1$, the consumption tax rate is increased exogenously from 10% to 40%.\footnote{We were unable to compute an equilibrium in experiment 3 unless the consumption tax rate were increased to this level.}

In experiment 3, the labor income tax rate rises to 56.7% in year $T_1 = 2020$, eventually falling to its steady state value of $\tau^3_h = 41.7%$. In experiment 4, in addition to the increase in the consumption tax to 40%, we allow for tax base broadening as in experiment 2. Now, the trigger is activated in year $T_1 = 2019$ at which point the labor income tax rate actually drops slightly from 33.2% to 33.1%, eventually falling to its steady state level of $\tau^4_h = 20.1%$.

While a very large permanent increase in the consumption tax is required when there is no tax base broadening, clearly no such increase is needed in experiment 4. Hence, in experiment 5, we increase the consumption tax to just 15% (five percentage points above its benchmark value) at date $T_1$ while also allowing for tax base broadening.\footnote{Again, the five percentage point increase in the consumption tax rate was required to compute an equilibrium.} This turns out to happen in year $T_1 = 2023$, at which time the labor income tax rate is raised to 63.2%, falling eventually to its steady state level of $\tau^5_h = 46.2%$.

These experiments show that even with tax base broadening, very large increases in either the consumption tax or the labor income tax (or both) are required. Such fiscal policies are not likely to be adopted—at best Japan would have to live with a 40% consumption tax for a number of decades. Unless one is willing to accept these extremely high tax rates,
Japan needs to find ways to significantly reduce spending and/or find additional sources of revenue.

4.3.3 Comparing the Benchmark with Alternative Policies

In this section we compare the equilibrium transition paths of key macroeconomic variables under the benchmark fiscal policy and two alternative fiscal policies associated with experiments 2 and 5 in which transfers are reduced by 8% of output. Recall that in experiment 2, the first trigger of 250% debt to output ratio is hit in 2019 and the consumption tax is increased from 10% to 40%, eventually declining to its steady state level of 27%. In experiment 5, the trigger is activated in 2023 when the consumption tax rate is raised to 15% and the labor income tax rate is increased to 63.2%, eventually falling to its steady state level of 46.2%.

Before we begin describing how endogenous variables respond in these various experiments, it is useful to describe how increases in the consumption and labor income tax rates affect household decisions. In our environment, changes over time in the consumption tax distort saving decisions. In particular, when the consumption tax rate is expected to increase at some future date, consumption today is expected to be relatively cheap compared to consumption in the future. As a result, saving is reduced and current consumption is increased. When the consumption tax increase is implemented, there is a drop in consumption and a rise in investment.

As discussed in section 4.2, both tax rates appear in the first order condition for labor and hence distort labor supply. If a tax rate increase is designed to raise a particular amount of revenue, the required consumption tax rate increase leads to a much smaller decrease in labor supply than does the required increase in the labor income tax rate.
In Figure 18, we show the effective tax rate $\tau$ (defined in equation 20) across the three selected experiments all of which assume the same path for government expenditures. The figure shows that the resulting increase in the effective tax rate is much higher when the labor tax is the primary tool used to achieve fiscal sustainability than if the consumption tax is used instead. Because the labor income tax is more distorting in this environment, hours worked and output are more depressed when the labor income tax is the primary fiscal instrument employed.
Figure 19: Consumption and Investment

Figure 19 shows the transition paths of consumption and investment. As the top frame in this figure shows, there is a consumption boom in experiment 2 which starts even before 2010 in anticipation of the huge increase in the consumption tax in 2019. When the consumption tax does increase in 2019, there is a very sharp decline in consumption. The mirror image of this consumption path can be seen in the bottom frame of Figure 19 as investment drops significantly for several years until the consumption tax is raised in 2019 at which time there is a large jump in investment.

This figure also shows that in experiment 5 there is a large increase in investment prior to 2023 when the labor income tax rate jumps. At this date, there is a very sharp decline in investment as well as consumption.
Figure 20: Labor, Capital, and Output

Figure 20 depicts the responses of labor, capital and output in the selected experiments. In the top frame of Figure 20, there is a sharp decline in hours worked, and as a result, in output, in response to the large increase in the labor income tax rate of experiment 5. In experiment 2, however, the distortion on labor supply is much smaller and spread over time. In the second and third frames of Figure 20 the effects of the changes in hours worked and investment are reflected in changes in the capital stock and output.
Figure 21: Debt to Output Ratio and Interest Rate on Government Bonds

Figure 21 shows the transition paths for bond to output ratios and interest rates on government bonds under the same three scenarios. The interest rates predicted by our model for the three experiments follow different paths along the transition due to our closed economy assumption. Presumably, the interest rates would be identical across experiments and constant over time in a small open economy environment. However, the future interest rates only fluctuate between 3 to 6%, so total borrowing costs should be similar to what one would obtain using an open economy model.

4.4 Sensitivity Analysis

Here we examine the sensitivity of our results to some alternative calibration and policy options. In particular, we consider how our results change with different values for the parameter $\psi$, which is the intertemporal elasticity of substitution of labor supply and, in this model, also the Frisch elasticity of labor supply. In addition, we also explore alternative assumptions regarding the trigger bond to output ratio ($b_{max}$) and/or the steady state bond to output ratio, $\bar{b}$. Finally, we also study how our findings change under different assumptions regarding future total factor productivity growth.
4.4.1 Labor Supply Elasticity

The goal of this subsection is to explore how our results change if labor supply is even more inelastic than in our base calibration where $\psi = 0.5$. Hence, we consider a very low value for the Frisch elasticity, $\psi = 0.25$, and perform Experiments 2 and 5 using this value. We compare the results obtained with what has been previously reported when $\psi = 0.5$. Table 5 describes our findings.

<table>
<thead>
<tr>
<th></th>
<th>Experiment 2</th>
<th>Experiment 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi = 0.5$</td>
<td>$\psi = 0.25$</td>
<td>$\psi = 0.5$</td>
</tr>
<tr>
<td>$T_1$</td>
<td>2019</td>
<td>2019</td>
</tr>
<tr>
<td>$\tau_{C,T_1}$</td>
<td>40.0%</td>
<td>38.7%</td>
</tr>
<tr>
<td>$\tau_{H,T_1}$</td>
<td>(*)</td>
<td>(*)</td>
</tr>
<tr>
<td>$\tau_T_1$</td>
<td>52.3%</td>
<td>51.9%</td>
</tr>
<tr>
<td>$T_2$</td>
<td>2087</td>
<td>2067</td>
</tr>
<tr>
<td>$\tau_{C,T_2}$</td>
<td>27.0%</td>
<td>25.7%</td>
</tr>
<tr>
<td>$\tau_{H,T_2}$</td>
<td>(*)</td>
<td>(*)</td>
</tr>
<tr>
<td>$\tau_T_2$</td>
<td>47.4%</td>
<td>46.9%</td>
</tr>
</tbody>
</table>

$T_1$: Date when $B/Y$ reaches 250%
$T_2$: Date when $B/Y$ is less than or equal to 60%.
$\tau_{c,i}$: Consumption tax rate at date $i$
$\tau_{h,i}$: Labor income tax rate at date $i$
$\bar{\tau}_i = (\tau_{c,i} + \tau_{h,i})/(1 + \tau_{c,i})$: Effective tax rate at date $i$
(*): Indicates no change from benchmark calibration.

In Table 5 we show the first year that our fiscal sustainability rule is triggered, which occurs when the debt to output ratio exceeds 250%, and the new values for the relevant tax rates. Columns labeled $\psi = 0.5$ report previously reported findings shown in Figures 16 and 17.

In experiment 2, assuming a smaller Frisch elasticity results in a relatively minor changes in our findings. With a smaller distortion in labor supply, a slightly smaller consumption tax during the transition (38.7% versus 40%) can now provide fiscal sustainability. The only notable difference is the periods required before transitioning to the steady state tax rate, which is 25.7% versus 27% in our original calibration. In particular, transition now takes 20 periods less than when $\psi = 0.5$.

In experiment 5, the length of transition is about the same, but the labor income tax rate during the transition is now lower at 54.5% relative to 63.2%. The smaller Frisch elasticity results in a smaller distortion in labor supply and therefore a tax rate not as high as that required when $\psi = 0.5$.  

34
4.4.2 The Debt to Output Ratio

Our previous analysis postulated that the Japanese government would raise taxes when the debt to output ratio reaches its ‘trigger’ value of 250% sufficient to bring this ratio down to 60% in the steady state. In this subsection, we conduct some variations on Experiment 2 in which the fiscal authority is assumed to target different debt to output ratios, either the trigger value at which point taxes are raised (\(b_{max}\)) or its steady state value \(\bar{b}\).

First, we present the findings from a computation in which the government acts at the same trigger value of 250% for the debt to output ratio, but keeps it at this level in the steady state instead of reducing it to 60%. This would necessitate a higher consumption tax in the steady state to service the higher debt compared to the baseline case. During the transition, however, the consumption tax rate needed to deal with the aging-related fiscal expenditures would not be as high since there is no additional requirement to reduce debt to a lower steady state value.

Indeed, according to Table 6, the equilibrium consumption tax is lower than in the baseline case along the transition, but higher in the final steady state. In fact, the transition tax rate and steady state tax rates are identical and equal to 38.8%.

In the last column of Table 6, we report the values of the consumption tax when the government is assumed to act much sooner with a trigger value for debt to output at 150% and to stabilize debt at this level forever. In this case, the consumption tax during the transition period is significantly lower at 32.2% compared to the baseline value of 40.0%. By acting sooner, the government spreads the tax increases needed to deal with the aging population over a longer period of time and therefore doesn’t have to raise taxes as much. However, as in the previous case, given that the government does not bring the debt to output ratio down to a lower steady state value, this higher tax rate must be maintained forever.

<table>
<thead>
<tr>
<th>Table 6: Experiment 2 with Higher Debt to Output Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_{max} = 250%)</td>
</tr>
<tr>
<td>(\bar{b} = 60%)</td>
</tr>
<tr>
<td>(baseline case)</td>
</tr>
<tr>
<td>(T_1)</td>
</tr>
<tr>
<td>(\tau_{c,T_1})</td>
</tr>
<tr>
<td>(T_2)</td>
</tr>
<tr>
<td>(\tau_{c,T_2})</td>
</tr>
</tbody>
</table>

| \(T_1\) : Date when \(B/Y\) reaches \(b_{max}\) |
| \(T_2\) : Date when \(B/Y\) is less than or equal to \(\bar{b}\) |
| \(\tau_{c,i}\) : Consumption tax rate at date \(i\) |

4.4.3 Higher Productivity Growth

So far, our analysis has assumed a TFP growth rate of 0.93% which delivers a balanced growth rate of 1.5% in the steady state. Forecasting growth rates is difficult and therefore
it will be useful to provide alternative projections based on other growth rates. Faster growth in economic activity would increase the tax base and as a result allow for a lower consumption tax rate to pay for the projected increase in government expenditures. In Table 7, we show how the equilibrium consumption tax in Experiment 2 responds to alternative assumptions regarding economic growth.

**Table 7: Experiment 2 with Different Growth Rates**

<table>
<thead>
<tr>
<th>({\gamma_t}_{t=2011}^{\infty})</th>
<th>(T_1)</th>
<th>(\tau_{c,T_1})</th>
<th>(T_2)</th>
<th>(\tau_{c,T_2})</th>
<th>(g_{GDP})</th>
</tr>
</thead>
<tbody>
<tr>
<td>baseline case</td>
<td>2019</td>
<td>40.0%</td>
<td>2087</td>
<td>27.0%</td>
<td>0.9885</td>
</tr>
<tr>
<td>({\gamma_t}_{t=2015}^{2024})</td>
<td>2019</td>
<td>41.0%</td>
<td>2098</td>
<td>27.0%</td>
<td>1.0006</td>
</tr>
<tr>
<td>({\gamma_t}_{t=2015}^{2024})</td>
<td>2019</td>
<td>38.0%</td>
<td>2091</td>
<td>27.0%</td>
<td>1.0147</td>
</tr>
<tr>
<td>({\gamma_t}_{t=2015}^{2024})</td>
<td>2019</td>
<td>35.0%</td>
<td>2094</td>
<td>27.0%</td>
<td>1.0415</td>
</tr>
<tr>
<td>({\gamma_t}_{t=2011}^{\infty})</td>
<td>2020</td>
<td>27.2%</td>
<td>2091</td>
<td>7.2%</td>
<td>1.0088</td>
</tr>
</tbody>
</table>

\(T_1\) : Date when \(B/Y\) reaches \(b_{max}\)

\(T_2\) : Date when \(B/Y\) is less than or equal to \(\bar{b}\)

\(\tau_{c,i}\) : Consumption tax rate at date \(i\)

\(g_{GDP}\) : Gross growth rate of real GDP between 2015-2024

The first row of findings in Table 7 displays the baseline results. The next three rows show the impact on the equilibrium consumption tax during the transition period as well as the steady state, together with the dates when these values are attained. These results are based on TFP growth rates consistent with balanced growth rates of 0%, 2% and 4%, respectively, for ten years from 2015-2024, after which growth returns to the value 1.5% as in our baseline case. The actual rates of growth of output from 2015-2024 also depend on the timing and extent of changes in capital and labor inputs. These rates are reported in the last column of the table.

Since the steady state growth rate under these three alternative temporary growth scenarios is the same, the steady state consumption tax is 27.0% as in the baseline case. If growth is slower than that assumed in the baseline case, then the transitional consumption tax rate is slightly higher at 41%. If growth is higher, then the consumption tax along the transition is lower. However, even if Japan experiences fast growth for ten years consistent with a balanced growth rate of 4%, the consumption tax is still 35% until 2094. In other words, very fast economic growth for 10 years—a ‘miracle’ decade—is insufficient to put a significant dent in Japan’s looming fiscal crisis.

Finally, we explore the effect of ‘permanently’ faster output growth in the last row of Table 7. In particular, when we assume a TFP growth rate consistent with a balanced growth rate of 2.5%, starting from 2015 and continuing forever, the fiscal burden is much
lowered. Now, a transitional consumption tax rate of 27.2% is sufficient to pay for the projected increases in government expenditures. In addition, the steady state tax rate is 7.2%, which is lower than the baseline value of 27.0% and even lower than the 10% tax rate that the Japanese government plans to levy beginning in 2015.

Hence, temporarily faster growth does not help much; permanently faster growth does, but this is unlikely to be achieved. Given reasonable expectations of economic performance in the near future, the takeaway from the findings in Table 7 is that Japan must consider other policies, including spending reductions, in order to achieve fiscal sustainability.

4.5 Welfare Analysis

In this section we compute welfare differences across our various experiments. For each experiment we compute the consumption equivalent variation relative to Experiment 2. This is done by calculating the percent change in consumption that would be required each period in Experiment 2 to make the present discounted utility the same as in the alternative experiment.

Specifically, let the realized discounted 1981 value of utility in Experiment 2 be denoted by $\hat{W}$. This can be computed as follows using the sequence of consumption, hours and bond holdings that are realized in Experiment 2.

$$\hat{W} = \sum_{t=1981}^{\infty} \beta^t N_t \left[ \log \hat{C}_t - \alpha \frac{\hat{h}_t^{1+1/\psi}}{1 + 1/\psi} + \phi \log (\mu_t + \hat{B}_{t+1}) \right].$$

Let $W$ be the corresponding realized utility for one of our alternative experiments. The consumption equivalent variation, $\lambda$, is the percentage change in consumption required in each period in Experiment 2 that would equate $W$ to $\hat{W}$. That is, $\lambda$ solves the following equation:

$$W = \sum_{t=1981}^{\infty} \beta^t N_t \left[ \log [(1 + \lambda)\hat{C}_t] - \alpha \frac{\hat{h}_t^{1+1/\psi}}{1 + 1/\psi} + \phi \log (\mu_t + \hat{B}_{t+1}) \right].$$

Solving for $\lambda$ yields

$$\lambda = \exp \left( \frac{W - \hat{W}}{\sum_{t=1981}^{\infty} \beta^t N_t} \right) - 1.$$
In Table 8 we report the value of $\lambda$ for each experiment in Table 4 as well as the additional experiments reported in Table 6. The welfare results show that, as expected, welfare is higher if the tax base is broadened since this allows the government to contain the increase in distorting tax rates. For example, if 8% of output is returned to households as a lump sum transfer as in Experiment 1, a higher consumption tax, both along the transition path and at the steady state, is needed. This leads to a welfare cost of 1.11% relative to Experiment 2 in which the government is assumed to keep this 8% of output.

Also, the results indicate that a consumption tax should be used to achieve fiscal stability rather than the labor income tax, holding everything else constant. In particular, note that Experiment 3 leads to a larger welfare loss relative to Experiment 1.

In Experiments 4 and 5 we allow for tax broadening. Experiment 4 provides higher welfare than Experiment 5 since it relies more heavily on increasing the consumption tax while Experiment 5 relies on a higher labor income tax. In particular, the combination of tax broadening and heavier reliance on the consumption tax in Experiment 4 means that the labor income tax rate can be is significantly lower in the long run and hence is superior to Experiment 2 in terms of welfare.

In the last two rows of Table 8 we consider the variations on Experiment 2 that are described in section 4.4.2. These calculations explore alternatives where the ‘trigger’ value of the debt to output ratio $b_{\text{max}}$ is equal to the steady state debt to output ratio $\bar{b}$. According to Table 8, there is a slight welfare gain, +0.17%, from keeping the debt to output ratio at 250% forever. This reflects the value that the household places on slightly lower taxes early on despite higher taxes in the steady state relative Experiment 2. In the last row, we show that a lower trigger value of the debt to output ratio (150%) leads to lower welfare relative to Experiment 2. This suggests that the welfare measure is heavily impacted by agents’ discounting given that they prefer a much higher transitional consumption tax rate in Experiment 2 over a smaller increase in taxes activated much sooner when the trigger value is 150% rather than 250%.

<table>
<thead>
<tr>
<th>$\lambda$ Relative to Experiment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp 1</td>
</tr>
<tr>
<td>Exp 3</td>
</tr>
<tr>
<td>Exp 4</td>
</tr>
<tr>
<td>Exp 5</td>
</tr>
<tr>
<td>$b_{\text{max}} = 2.5$</td>
</tr>
<tr>
<td>$\bar{b} = 2.5$</td>
</tr>
<tr>
<td>$b_{\text{max}} = 1.5$</td>
</tr>
<tr>
<td>$\bar{b} = 1.5$</td>
</tr>
</tbody>
</table>
5 Conclusions

Japan is aging rapidly. The ratio of the number of Japanese over the age of 64 to those between 20 and 64 is projected to increase from 39% in 2010 to above 91% in 2070. This dramatic shift in the share of elderly in the society is expected to raise public retirement and health expenditures significantly. Indeed, the ratio of these aging-related expenditures to output is projected to rise an additional 7 percentage points. In addition, past spending decisions have already caused the net debt to output ratio to soar above 110% by 2010.

In this paper we build a neoclassical growth model to measure the size of the fiscal response needed to restore fiscal balance in Japan. In our model, both consumption and labor income taxes distort labor supply. However, for a given amount of revenue, the consumption tax is less distorting than the labor income tax.

Our main result is that fiscal sustainability requires a large adjustment in tax revenues, in the range of 30-40% of aggregate consumption if we abstract from distortions. Adjusting the consumption or labor income tax rate to achieve this, however, requires that taxes be set to unprecedentedly high levels—tax rates of 40-60% if the government waits until the debt to output ratio reaches 250% before taking fiscal action. The lower end of this range is made possible if revenue equal to 8% of output can be raised by flattening tax rates and eliminating deductions.

The dismal nature of these findings is motivation for research that explores policy measures that will allow some of the fiscal adjustment to come from sources other than higher taxes. These may include reducing expenditures via reforms of public pensions and health expenditures, a new approach to immigration, family policies to raise fertility and to increase female labor force participation (see İmrohoroglu, Kitao, and Yamada (2013)), and microeconomic reforms that promote higher rates of innovation and growth. For example, Braun and Joines (2014) argue, using a calibrated life cycle model, that reductions in public health expenditures generated by higher deductibles is particularly promising. Using a similar model, Kitao (2014) finds that a combination of reducing pension benefits and delaying the date at which one can start collecting benefits is also promising.

Another possibility is for the Japanese government to reduce its holdings of publicly owned nonfinancial assets. As of 2013, the value of state-owned nonfinancial assets and firms in Japan amount to 141.6% of Japanese GDP. In contrast, the ratio of state-owned assets to GDP in the U.S. is 62.0%. Hence, it is possible that significant revenue can be raised by selling some of the state-owned assets in Japan to help achieve fiscal sustainability.

\[\text{State-owned Assets: Setting Out the Store}', \text{ The Economist}, \text{ January 11, 2014.}\]
References


