Information and Market Power*

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Abstract

We analyze demand function competition with a finite number of agents and private information. We show that the nature of the private information determines the market power of the agents and thus price and volume of equilibrium trade.

We establish our results by providing a characterization of the set of all joint distributions over demands and payoff states that can arise in equilibrium under any information structure, extending results of Bergemann and Morris (2013b).

In demand function competion, the agents condition their demand on the endogenous information contained in the price. We compare the set of feasible outcomes under demand function to the feasible outcomes under Cournot competition. We find that the first and second moments of the equilibrium distribution respond very different to the private information of the agents.

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1 Introduction

Consider a market where consumers submit demand functions, specifying their demand at any given price, and the price is then chosen to clear the market. A consumer exercises *market power* if, by withholding reducing demand at any given price, he can strategically influence the market price. The standard measure of market power is *price impact*: how much do changes in the consumer's demand shift market prices? Market power leads to inefficiency, as consumers' marginal value exceeds the market price. Market power is linked to the number of firms in the market. As the number of firms increases, market power decreases, and we have a competitive economy in the limit.

This standard understanding of market power is based on symmetric information. But if there is asymmetric information, so that the level of market clearing prices conveys payoff relevant information to consumers, then there is an additional channel of price impact. The slope of the demand curves that consumers submit will then reflect not only market power - i.e., their ability to strategically control prices - but also their desire to reflect the information contained in market prices in their demand. These two aspects of the market - strategic manipulation of prices and learning from prices - are intrinsically linked and interact in subtle ways.

The key variable in the interaction of strategic manipulation and learning is what consumers learn from prices. Consider a setting where consumers valuations have both common and idiosyncratic components. Suppose that consumers are uncertain of their own valuations and their own confounds information about the common and idiosyncratic components. The market price will end up reflecting the common component. A higher expected common component reflected in a high market price will lead him to increase his expectation of the common component. But since common and idiosyncratic components are confounded in his information, it is as if he is observing his idiosyncratic component with noise that is correlated with the common component. A higher expected common component then corresponds to him knowing that the noise realization in his information about the common component is higher than expected, and this will lead him to have a lower expectation of the idiosyncratic component. How consumers want to condition on the market price will then depend on which effect predominates, which in turn will depend on the way in which information gets confounded.

If consumers' information overweights the common component, then the impact of information about the common component on the expected idiosyncratic component will be small. Thus a higher price will signal a higher valuation to consumers. Thus the slope, or price sensitivity, of the demand curve that he submits will increase. But anticipating the increased price sensitivity of the equilibrium demand of (other) consumers, any individual consumer will have a higher price impact. We show that however large the number of consumers, and however valuations are distributed between common and idiosyncratic components, consumer's price impact can be arbitrarily large if there is asymmetric information which confounds common and idiosyncratic consumers' valuations. Arbitrarily high price impact of consumers implies arbitrarily low gains from trade and thus large inefficiency.

Conversely, if consumers' information overweights the common component, the impact of information about the common component on the expected idiosyncratic component will be large and will predominate over the direct impact. Thus a higher price will signal a lower idiosyncratic valuation to consumers. Thus the price sensitivity of demand decrease and, anticipating this equilibrium effect, any individual consumer will have a lower price impact. We show that however small the number of consumers (as long as there are at least two), and however valuations are distributed between common and idiosyncratic components, consumers' price impact can be arbitrarily small if there is confounding asymmetric information and thus we can be arbitrarily close to a competitive outcome.

Thus we provide a sharp account of how price impact depends on the information structure.

The market setting is that analyzed in the seminal work of Klemperer and Meyer (1989) on *supply function competition*, where market participants submitted supply curves when there is uncertainty about demand and symmetric information. Because of our interest in strategic uncertainty in financial markets, we focus on a model of *demand function competition*. With appropriate relabelling and sign changes, the model is identical to supply function competition. We follow an important paper of Vives (2011a) in introducing asymmetric information into this setting, and we also follow Vives (2011a) in assuming symmetric consumers with normally distributed common and idiosyncratic components of their values, which allows a tractable and transparent analysis of closed form linear equilibria.

Vives (2011a) also highlighted the interaction of market power with asymmetric information. We depart from Vives (2011a) in studying what can happen in his model, in all (linear) equilibria, for all possible (symmetric and normal) information structures. While he assumes that consumers observe conditionally independent normal signals of their true valuations (which reflect common and idiosyncratic components), we break the link between the degree of confounding in the information structure (the degree to which common and idiosyncratic signals can be distinguished) and the accuracy of signals. This allows us to offer a sharper account of the how market power and asymmetric information interact.

Our characterization of all equilibria for all information structures is of independent interest. Rather than fixing a parameterized class of information structures and solving for equilibria, we show that we can characterize the set of outcomes that can arise without explicit reference to the underlying information structure. We have pursued this strategy in other recent work (most closely related is Bergemann, Heumann, and Morris (2015)) in purely game theoretic settings. In this paper, we pursue this strategy in a game which provides strategic foundations for competitive equilibria. Thus we view the results in this paper as providing a benchmark model for studying the role of information in markets, and present a number of novel questions that can be asked and answered using this approach.

First, in studying a substantive economic question, such as the interaction of market power and information, we can identify which features of the information structure drive results, rather than solving within a low dimensional parameterized class of information structures. If we look at the joint distribution of quantities and prices that can arise in equilibrium, it turns out that extremal distributions, arise when agents observe "noise-free" information structures, where each agent observes perfectly a linear combination of the common component and his idiosyncratic component, but where the weight on the two components is not equal to the weights in that agent's valuation. Any other outcome that could arise in any other (perhaps multidimensional) information structure could also arise in an information structure where agents observed one dimensional confounding signals with noise. The impact of the noise is merely to reduce variation in the outcome. Thus our transparent analysis under noise free information structures frames and bounds what could happen all information structures.

Second, in understanding possible outcomes in a market setting, we can abstract from the consumers' information structures and strategies (i.e., the demand curves that they submit) and identify equilibrium conditions on the joint distribution of quantities (and thus prices) condition on the distribution of valuations. We show that - in our environment - there is a three dimensional array of possible distributions, which can be parameterized by three variables. The price impact is a strictly positive real number. A second parameter is the non-negative correlation between the idiosyncratic component of a consumer's action with idiosyncratic component of his valuation. A third parameter is the non-negative correlation between the total quantity demanded and the common component of consumers' valuations. Any valuations of these three variables are consisting with any underlying distribution of valuations. However, once these variables are fixed, everything else about the equilibrium outcome is pinned down.

Third, by distinguishing between noisy information and confounding information, we offer a classification as to how private information affects the bidding behavior, both in the specific uniform price auction that we analyze here, but more generally in bidding environments with interdependent values.

The remained of the paper is organized as follows. Section 2 describes the model and the payoff environment. Section 3 describes the Bayes Nash equilibrium in a small class of information structures that we refer to as noise free information structures. Section 4 introduces a second solution concept, Bayes correlated equilibrium, and with it, we can describe the equilibrium behavior for all possible information structures. Section 5 analyzes the origin of market power in environment with private information in more detail. Section 6 compares the feasible equilibrium outcomes under demand competition with the ones arising in quantity competition. Section 7 concludes.

2 Model

Payoffs We consider an economy with finite number of agents (buyers), indexed by $i \in N = \{1, ..., N\}$. There is a divisible good which is purchased by the agents. The realized utility of a trader who buys an amount a_i of the asset at price p is given by:

$$u_i(\theta_i, a_i, p) \triangleq \theta_i a_i - \frac{1}{2} a_i^2 - a_i p,$$

where θ_i is the (marginal) willingness to pay, the payoff state, of trader i. The aggregate demand of the buyers is denoted by A with

$$A = \sum_{i=1}^{N} a_i.$$

The asset is supplied by a competitive market of producers represented by an aggregate supply function:

$$p = r_0 + rA.$$

Thus, the aggregate supply is equivalently represented by a convex cost function

$$c(A) \triangleq r_0 A + \frac{1}{2} r A^2.$$

We assume that the willingness to pay, θ_i, is symmetrically and normally distributed across agents. Thus, for any pair of agents $i, j \in N$ their willingness to pay is distributed according to:

$$\begin{pmatrix} \theta_i \\ \theta_j \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \rho_{\theta\theta}\sigma_\theta^2 \\ \rho_{\theta\theta}\sigma_\theta^2 & \sigma_\theta^2 \end{pmatrix} \right). \tag{1}$$

The expected willingness to pay is given by the mean $\mu_\theta \in \mathbb{R}_+$ and the variance is denoted by σ_θ^2. The correlation across agents is given by the correlation coefficient $\rho_{\theta\theta}$. By symmetry, and for notational convenience we omit the subscripts in description of the moments (thus μ_θ instead of μ_{θ_i}, σ_θ instead of σ_{θ_i}, and $\rho_{\theta\theta}$ instead of $\rho_{\theta_i\theta_j}$). For the symmetrically distributed random variables $\{\theta_i\}_{i=1}^N$ to form a feasible multivariate normal distribution it has to be that $\rho_{\theta\theta} \in [-\frac{1}{N-1}, 1]$.

With the symmetry of the payoff states across agents, a useful and alternative representation of the environment is obtained by decomposing the random variable into a common and an idiosyncratic component. Thus for a given profile of realized payoff states $(\theta_1, ..., \theta_N)$, we define the average payoff state:

$$\bar{\theta} \triangleq \frac{1}{N} \sum_{i \in N} \theta_i, \tag{2}$$

and, correspondingly, we define the idiosyncratic component of agent i payoff state:

$$\Delta\theta_i \triangleq \theta_i - \bar{\theta}. \tag{3}$$

Now, we can describe the payoff environment in terms of the common and idiosyncratic component $\theta_i = \bar{\theta} + \Delta\theta_i$ with

$$\begin{pmatrix} \bar{\theta} \\ \Delta\theta_i \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{\bar{\theta}}^2 & 0 \\ 0 & \sigma_{\Delta\theta}^2 \end{pmatrix} \right), \tag{4}$$

where the variance of common component is given by

$$\sigma_{\bar{\theta}}^2 = \rho_{\theta\theta}\sigma_\theta^2$$

and the variance of the idiosyncratic component is the residual

$$\sigma_{\Delta\theta}^2 = (1 - \rho_{\theta\theta})\,\sigma_\theta^2.$$

We shall use the decomposition of the individual variable into a common and an (orthogonal) idiosyncratic component later also for the action and the signal variables. Henceforth we use a bar above a variable to denote the average over all agents, as in $\bar{\theta}$ and a Δ to denote the idiosyncratic component relative to the average, as in $\Delta\theta_i$.

Signals Each agent receives a signal, possibly noisy, possibly multi-dimensional about his payoff state and the payoff state of all the other agents. We shall restrict attention throughout the paper to symmetric and normally distributed signals. The signal that agent i receives, $s_i = \left(s_i^1, ..., s_i^K\right)$, is a K-dimensional vector for some finite K. The joint distribution of the signals (types) and the payoff states of the agents can therefore be described for any pair of agents i and j as a multivariate normal distribution:

$$\begin{pmatrix} \theta_i \\ \theta_j \\ s_i \\ s_j \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_\theta \\ \mu_\theta \\ \mu_s \\ \mu_s \end{pmatrix}, \begin{pmatrix} \Sigma_{\theta\theta} & \Sigma_{\theta s} \\ \Sigma_{\theta s} & \Sigma_{ss} \end{pmatrix} \right), \tag{5}$$

where μ_s is the mean realization of the signal vector s_i and the submatrices $\Sigma_{\theta\theta}$, $\Sigma_{\theta s}$ and Σ_{ss} together form the variance-covariance matrix of the joint distribution. The only restriction that we impose on the joint distribution is that the submatrix $\Sigma_{\theta\theta}$ coincides with the common prior distribution of the payoff states given earlier by (1), and that jointly the submatrices form a valid variance-covariance matrix, that is the entire matrix is positive semi-definite. With minor abuse of language, we refer to joint distribution of states and signals as an information structure $\mathcal{I}$.

Strategies The agents simultaneously submit demand functions $x_i(s_i, p)$. The demand function $x_i(s_i, p)$ represents the demand of agent i at price p given the private information conveyed by the signal s_i:

$$x_i : \mathbb{R}^K \times \mathbb{R} \to \mathbb{R}. \tag{6}$$

The equilibrium price p^* is determined by the submitted demand functions and the market clearing condition:

$$p^* = r_0 + r \sum_{i \in N} x_i(s_i, p^*).$$

We analyze the symmetric Bayes Nash equilibrium in demand function competition. Given the market clearing condition, the equilibrium demand function $x^*(s_i, p)$ solves for each agent i and each signal realization s_i the following maximization problem:

$$x(s_i, p) \in \underset{x_i(\cdot) \in C(\mathbb{R})}{\arg\max}\, \mathbb{E}[u_i(\theta_i, x_i(p^*), p^*)|s_i]. \tag{7}$$

subject to the market clearing condition:

$$p^* = r_0 + r \sum_{j \neq i} x(s_j, p^*) + x_i(p^*). \tag{8}$$

For the moment, we shall merely require that for every signal s_i the submitted demand function $x_i(p)$ is a continuous function defined the real line $\mathbb{R}$, thus $x_i(\cdot) \in C(\mathbb{R})$. In the subsequent equilibrium analysis, we find that given the linear quadratic payoff environment, and the normality of the signal and payoff environment, the resulting equilibrium demand function is a linear function in the signal vector s_i and the price p.

Definition 1 (Symmetric Bayes Nash Equilibrium)
The demand functions $\{x(s_i, p)\}_{i=1}^{N}$ constitute a symmetric Bayes Nash equilibrium if for every agent i and every signal realization s_i, the best response condition (7) and the market clearing condition (8) are satisfied.

3 Noise Free Information Structures

We begin our analysis with the conventional approach. Namely, we fix the information structure of the agents and then determine the structure of the equilibrium demand function. The novel aspect in the analysis is the nature of the private information that we refer to as noise free information structure. Namely, the signal of each agent is a linear combination of the idiosyncratic and the common component of the payoff state. In turn, the resulting structure of the equilibrium demand functions will suggest a different and novel approach that describes the equilibrium directly in terms of the outcomes of the game, namely the quantities, price, and payoff states in a way that is independent of the specific information structure.

3.1 Noise Free Information Structure

We begin the equilibrium analysis with a class of one dimensional signals. For now, we shall consider the class of information structures in which the one dimensional signal of each agent i is given by:

$$s_i = \Delta\theta_i + \lambda \cdot \bar{\theta}. \tag{9}$$

where $\lambda \in \mathbb{R}$ is the weight that the common component of the payoff state receives in the signal that the agent i receives. As the idiosyncratic and the common component, $\Delta\theta_i$ and $\bar{\theta}$ respectively of the signal are normal distributed, the signal s_i is also normally distributed. We call the signal s_i noise-free as it is generated by the component of the payoff state, and no extraneous noise enters the signal. However to the extent that the composition of the idiosyncratic and common component in the signal differ from its composition in the payoff state, that is as long as $\lambda \neq 1$, agent i faces residual uncertainty about his willingness to pay since the signal s_i confounds the idiosyncratic and the common component. Given the multivariate normal distribution, the conditional expectation of agent i about his payoff state θ_i given by

$$\mathbb{E}\left[\theta_i \left| s_i \right.\right] = \mu_\theta + \frac{\lambda \rho_{\theta\theta} \sigma_\theta^2 + \left(1 - \rho_{\theta\theta}\right) \sigma_\theta^2}{\lambda^2 \rho_{\theta\theta} \sigma_\theta^2 + \left(1 - \rho_{\theta\theta}\right) \sigma_\theta^2} \left(s_i - \lambda \mu_\theta\right),$$

and the conditional expectation about the common component, or for that matter the payoff state θ_j of any other agent j is given by

$$\mathbb{E}\left[\overline{\theta} \left| s_i \right.\right] = \mathbb{E}\left[\theta_j \left| s_i \right.\right] = \mu_\theta + \frac{\lambda \rho_{\theta\theta} \sigma_\theta^2}{\lambda^2 \rho_{\theta\theta} \sigma_\theta^2 + \left(1 - \rho_{\theta\theta}\right) \sigma_\theta^2} \left(s_i - \lambda \mu_\theta\right),$$

where the later conditional expectation is less responsive to the signal s_i as the idiosyncratic component of agent i payoff state drops out in the updating rule. In the above expressions of the conditional expectations we can cancel the total variance σ_θ^2 of the payoff state, and what matters is only the relative contribution of each component of the shock, 1 and λ, respectively.

In the important special case of $\lambda = 1$, we can verify that $\mathbb{E}\left[\theta_i \left| s_i \right.\right] = \theta_i$. By contrast, if $\lambda > 1$, then the signal overweighs the common component relative to its payoff relevance, and if $\lambda < 1$, then the signal underweights the common component relative to its payoff relevance. If $\lambda = 0$, then the signal s_i only conveys information about the idiosyncratic component, if $\lambda \to \pm\infty$, then the signal s_i only conveys information about the common component.

3.2 Price Impact

Each agent submits a demand function that describes his demand for the asset at a given market price. Thus the demand of each agent is conditioned on the private signal s_i and the equilibrium price p. Heuristically, each agent i therefore solves a pointwise maximization problem given the conditioning information (p, s_i):

$$\max_{a_i} \mathbb{E}\left[\theta_i a_i - \frac{1}{2} a_i^2 - p a_i \middle| p, s_i\right]. \tag{10}$$

The resulting first order condition is

$$\mathbb{E}[\theta_i | s_i, p] - p - a_i - \frac{\partial p}{\partial a_i} a_i = 0$$

and thus determines the demand of agent i:

$$a_i = \frac{\mathbb{E}[\theta_i | s_i, p] - p}{1 + \frac{\partial p}{\partial a_i}}. \tag{11}$$

Importantly, with a finite number of agents, an increase in the demand by agent i affects the equilibrium price for all the traders, which is, still heuristically, captured by the partial derivative $\partial p / \partial a_i$. We shall refer to this derivative, provided that it exists, as the *price impact* of agent i. The price impact represents the "market power" of agent i in the sense that it indicates how strongly the trader can influence the equilibrium price by changing his own demanded quantity. In the absence of market power, we have $\partial p / \partial a_i = 0$, and the resulting demanded quantity by agent i always equals the difference between his marginal willingness to pay and the price of the asset:

$$a_i = \mathbb{E}[\theta_i | s_i, p] - p.$$

In the presence of market power, $\partial p / \partial a_i > 0$, each agent i lowers his demand, and hence we will observe a strategic demand reduction.

3.3 Demand Function Equilibrium

Next, we derive the equilibrium demand functions of the traders. Given the linear quadratic payoff environment, and the normality of the state and signal environment we will find that in equilibrium the traders submit symmetric linear demands:

$$x_i(s_i, p) \triangleq \beta_0 + \beta_s s_i + \beta_p p.$$

The demand function of each trader i consists of a stochastic intercept of his demand function, $\beta_0 + \beta_s s_i$, and the price sensitivity $\beta_p p$, where the slope of the individual demand function β_p is typically negative.[1]

[1] The demand function competition, plotted as $(q(p), p)$ therefore can be viewed as intermediate structure between Cournot competition and Bertrand competition. Cournot competition generates a vertical demand function in which the buyer fixed demand q and the price responds whereas Betrand competition generates a horizontal demand function in which the buyer fixes the price p and realized demand q adjusts. With symmetric information, there exists a continuum of symmetric price equilibria in which the supply is shared among the buyers (where the continuum can be maintained due to the discontinuity in the supply as a function of the price). With asymmetric information, the bidding game is a generalization of the first price auction with variable supply which appears to be an open problem in the auction theory. The resulting equilibrium is likely to allocate all units to the winning bidder, and zero

Given the candidate equilibrium demand functions, the market clearing condition can be written as

$$p = r_0 + r \sum_{i=1}^{N} \left(\beta_0 + \beta_s s_i + \beta_p p\right). \tag{12}$$

For a moment let us consider the market clearing condition from the point of view of a specific trader k. Given the candidate equilibrium strategies of the other agent, trader $k$can anticipate that a change in his demanded quantity a_k will impact the equilibrium price:

$$p = r_0 + r \sum_{i \neq k} \left(\beta_0 + \beta_s s_i + \beta_p p\right) + r a_k.$$

In other words, from the point of view of trader k, the market clearing condition is represented by a residual supply function for trader k :

$$p = r_0 + r \sum_{i \neq k} \left(\beta_0 + \beta_s s_i\right) + r\left(N-1\right)\beta_p p + r a_k,$$

and after collecting the terms involving the market clearing price p on the lhs:

$$p = \frac{r}{1-\left(N-1\right) r \beta_p} \left(\frac{r_0}{r} + \sum_{i \neq k} \left(\beta_0 + \beta_s s_i\right) + a_k \right).$$

The residual supply function that trader k is facing thus has a random intercept determined by the signal realizations $s_{-k} = (s_1, ..., s_{k-1}, s_{k+1}, ...s_N)$ of the other traders and a constant slope that is given by the responsiveness of the other traders to the price. Thus, we observe that under the hypothesis of a symmetric linear demand function equilibrium, the price impact of trader k, $\partial p / \partial a_k$ is going to be a constant as well, and we denote it by m :

$$m \triangleq \frac{\partial p}{\partial a_k} = \frac{r}{1-\left(N-1\right) r \beta_p}. \tag{13}$$

Thus, the price impact of trader k is determined in equilibrium by the price sensitivity β_p of all the other traders. Typically, the demand function of the buyers is downward sloping, or $\beta_p < 0$. Thus, an increase in the (absolute) price sensitivity $\left|\beta_p\right|$ of the other traders decreases the price impact m, the market power, of trader k.

units to the losing bidders, and hence lead to a very inefficient allocation relative to a discriminatory price auction in which bidders submit bids for every marginal unit. Ausubel, Cramton, Pycia, Rostek, and Weretka (2014) analyze a corresponding model of *fixed* supply under uniform price *and* pay-as-you-bid auction rules.

We now return to the market clearing condition (12) and express it in terms of the signals received. Thus, after rearranging the market clearing condition, we find that the equilibrium price is informative about the average signal $\overline{s}$ received by the agents:

$$\frac{1}{N}\sum_{i=1}^{N} s_i = \frac{\left(1 - Nr\beta_p\right) p - r_0 - Nr\beta_0}{Nr\beta_s}. \tag{14}$$

The average signal $\overline{s}$ (following the same notation as for the payoff state in (2)) is perfectly informative of the average state $\overline{\theta}$

$$\overline{s} = \frac{1}{N}\sum_i s_i = \frac{1}{N}\sum_i \left(\Delta\theta_i + \lambda\overline{\theta}\right) = \lambda\overline{\theta}.$$

Thus, in equilibrium, the conditional expectation of each agent regarding his expected payoff state $\mathbb{E}[\theta_i|s_i, p]$ equals $\mathbb{E}[\theta_i|s_i, \overline{s}]$:

$$\mathbb{E}[\theta_i|s_i, p] = \mathbb{E}[\theta_i|s_i, \overline{s}] = s_i + \left(\frac{1-\lambda}{\lambda}\right)\overline{s} = \theta_i, \tag{15}$$

which in turn allows each agent to infer his payoff state θ_i perfectly.

We can now find the equilibrium demand function by requiring that the quantity demanded satisfies the best response condition (11) for every signal realization s_i and every price p, or equivalently every average signal realization $\overline{s}$:

$$\beta_0 + \beta_s s_i + \beta_p p = \frac{\mathbb{E}[\theta_i|s_i, p] - p}{1+m} = \frac{\mathbb{E}[\theta_i|s_i, \overline{s}] - p}{1+m},$$

using the fact the price impact $\partial p/\partial a_i$ equals a constant m introduced earlier in (13).

Thus, using the resulting conditional expectation in terms of $(s_i, \overline{s})$, or more explicitly the market clearing condition expressed in terms of the average signal $\overline{s}$, we can find the equilibrium demand function by matching the three coefficients $\left(\beta_0, \beta_s, \beta_p\right)$ and the price impact m:

$$\begin{aligned}\beta_0 + \beta_s s_i + \beta_p p &= \frac{s_i + \frac{1-\lambda}{\lambda}\overline{s} - p}{1 + \frac{\partial p}{\partial a_i}} \\ &= \frac{s_i + \frac{1-\lambda}{\lambda}\left(\frac{(1-Nrc_p)p - P_0 - Nrc_0}{Nrc_s}\right) - p}{1+m}.\end{aligned}$$

The results are summarized in the following proposition that describes the linear and symmetric equilibrium demand function.

Proposition 1 (Demand Function Equilibrium with Noise Free Signals)
For every noise free information structure λ, there exists a unique symmetric linear Bayes Nash equilibrium. The coefficients of the linear demand function are given by

$$\beta_0 = -\frac{(1-\lambda)\, r_0}{Nr}, \quad \beta_s = \frac{1}{1+m}, \quad \beta_p = \frac{1-\lambda}{Nr} - \frac{\lambda}{1+m}, \tag{16}$$

and

$$m = \frac{1}{2}\left(-Nr\frac{(N-1)\lambda - 1}{(N-1)\lambda + 1} + \sqrt{(Nr\frac{(N-1)\lambda - 1}{(N-1)\lambda + 1})^2 + 2Nr + 1} - 1\right). \tag{17}$$

The coefficients of the demand function, $\beta_0, \beta_s, \beta_p$ are described in terms of the primitives of the model and the price impact m, which in turn is a function of the primitives r, N, and the information structure λ. Importantly, even when we hold the payoff environment as represented by r and N (and the distribution of payoff states fixed), the price impact varies greatly with the information structure and we develop the impact that the information structure has on the equilibrium strategy of the agents in more detail in the next subsection. For now we observe that from the equilibrium strategies we can immediately infer the realized demands in equilibrium which have a very transparent structure.

Corollary 1 (Realized Demand)
In the unique symmetric linear Bayes Nash equilibrium, the realized demand a_i of each agent is given by: $a_i = \Delta a_i + \bar{a}$ with:

$$\Delta a_i = \frac{\Delta\theta_i}{1+m} \text{ and } \bar{a} = \frac{\bar{\theta}}{1+r+m}. \tag{18}$$

The description of the equilibrium trades in (18) indicate that the individual trades are measurable with respect to idiosyncratic as well as the common component of the payoff shock of agent i, $\Delta\theta_i$ and $\bar{\theta}$ respectively. Thus the demand functions remain equilibrium strategies even under complete information, and hence form an ex post equilibrium, and not merely a Bayes Nash equilibrium. Moreover, as the equilibrium strategy does not depend on the conditional expectation, the above characterization of the equilibrium is valid beyond the multivariate normal distribution. As long as we maintain the quadratic payoff environment, any symmetric and continuous joint distribution of the payoff state $(\theta_1, ..., \theta_N)$ would lead to the above characterization of the ex post equilibrium in demand functions.

We observe that the level of realized demand depends on the payoff state and the equilibrium price impact. Thus we find that an increase in the market power leads to an uniform reduction of

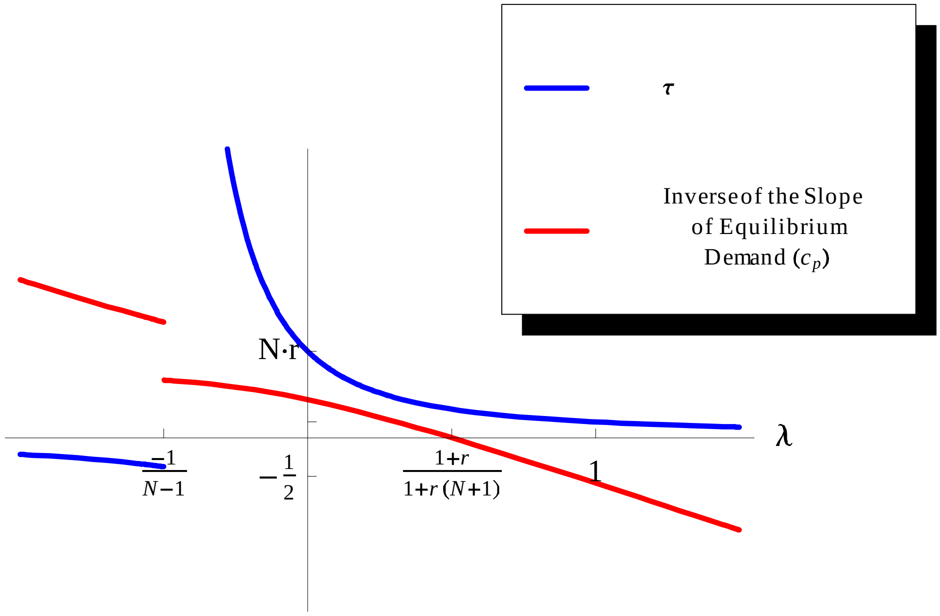

Figure 1: Equilibrium Demand and Price Impact with Noise Free Information

the realized demand for all realizations of the payoff state. Notably, the socially efficient level of trade would be realized at $m = 0$, and an increase in market power leads to lower volume of trade. The idiosyncratic component Δa_i in the realized demand is not affected by the supply condition r, as the sum of the idiosyncratic terms sum to zero, by definition of the idiosyncratic trade. However, even the level of idiosyncratic demand is affected by the price impact as it influences the response of all the other traders.

3.4 Information Structure and Market Power

We derived the equilibrium demand for a given information structure λ in Proposition 1. But the above analysis also lays the groundwork to understand how the information structure impacts the demand function and consequently the price impact and the market power of each each agent.

Figure 1 graphically summarizes the price impact m and the price sensitivity of the equilibrium demand function β_p as a function of the information structure λ as derived earlier in Proposition 1.

Earlier we noticed that the price impact of a given trader is supported by the price sensitivity of the other traders, see (13). Thus the central aspect to understand is how each trader responds to the new information contained in the price relative to his private signal s_i. From (15), we can write the conditional expectation of the agent about his payoff state given his signal s_i and the signals of all the other agents s_{-i} as:

$$\mathbb{E}[\theta_i | s_i, s_{-i}] = \left(\frac{N-1}{N} + \frac{1}{\lambda N} \right) s_i + \left(\frac{1-\lambda}{\lambda} \right) \frac{1}{N} \sum_{j \neq i} s_j, \tag{19}$$

where the conditional expectation follows from the fact that the signals are noise free and that the equilibrium is informative. Noticeably, the expectation does not refer to either the nature of the supply function or the moments of the common prior. Thus the conditional expectation of agent i is a linear combination of his own signal and the signal of the other agents. Importantly, as the weight λ on the common component turns negative there is a critical value

$$\lambda = -\frac{1}{N-1}, \tag{20}$$

at which the conditional expectation of agent i is independent of his own signal s_i, and equal to the sum of the signals of the other agents:

$$\mathbb{E}[\theta_i | s_i, s_{-i}] = -\sum_{j \neq i} s_j.$$

We can thus expect the demand behavior to change dramatically around the critical information structure λ in which each agent learns nothing from his own signal in the presence of all the other signals. Proposition 2 and 3 therefore describe separately the qualitative behavior of the equilibrium behavior for $\lambda \geq -1/(N-1)$ and $\lambda < -1/(N-1)$.

Indeed for the moment, let us suppose that the signal overweighs the common shock relative to its payoff importance, that is $\lambda > 1$. Each trader submits a demand function given his private information, and has to decide how to respond to the price, that is how to set β_p, negative or positive, small or large ? We saw that the equilibrium price p will be perfectly informative about the average signal $\overline{s}$ and thus about the average state $\overline{\theta}$. In particular, a higher price p reflects a higher average state $\overline{\theta}$. So how should the demand of agent i respond to a market clearing price p that is higher (or lower) than expected on the basis of the private signal ? Thus suppose the equilibrium price p reflects a higher common state $\overline{\theta}$ than agent i would have expected on the basis of his conditional expectation $\mathbb{E}\left[\overline{\theta}\,|s_i\right]$, or $\overline{\theta} > \mathbb{E}\left[\overline{\theta}\,|s_i\right]$. As the signal is noise free, and given by the weighted linear combination $s_i = \Delta\theta_i + \lambda\overline{\theta}$, agent i will have to revise his expectation $\mathbb{E}\left[\Delta\theta_i\,\middle|\overline{\theta}, s_i\right]$ about the idiosyncratic component downwards, after all the weighted sum of the realization $\Delta\theta_i$

and $\overline{\theta}$ still have to add up to s_i. Also, given s_i and the realization of the common component $\overline{\theta}$,, the conditional expectation equals the realized idiosyncratic component, $\mathbb{E}\left[\Delta\theta_i\left|\overline{\theta}, s_i\right.\right] = \Delta\theta_i$, and thus

$$\mathbb{E}\left[\Delta\theta_i\left|\overline{\theta}, s_i\right.\right] - \mathbb{E}\left[\Delta\theta_i\left|s_i\right.\right] = \Delta\theta_i - \mathbb{E}\left[\Delta\theta_i\left|s_i\right.\right] = -\lambda\left(\overline{\theta} - \mathbb{E}\left[\overline{\theta}\left|s_i\right.\right]\right), \tag{21}$$

to balance the upward revision of the common component. Moreover, since the signal overweighs the common component, the downward revision of the idiosyncratic component occurs at the multiple λ of the upwards revision, thus leading to a decrease in the expectation of the payoff state θ_i :

$$\mathbb{E}\left[\theta_i\left|\overline{\theta}, s_i\right.\right] - \mathbb{E}\left[\theta_i\left|s_i\right.\right] = \theta_i - \mathbb{E}\left[\theta_i\left|s_i\right.\right] = -\left(\lambda - 1\right)\left(\overline{\theta} - \mathbb{E}\left[\overline{\theta}\left|s_i\right.\right]\right) < 0. \tag{22}$$

We can therefore conclude that in equilibrium the realized price p and the realized willingness to pay θ_i move in the opposite direction, thus suggesting that $\beta_p < 0$. Moreover as the weight λ on the common component in the signal increases, the downward revision of the expectation becomes more pronounced, and hence induces each agent to lower his demand more aggressively in response to higher prices, thus depressing β_p further. And we observed earlier, see (13), that from the point of view of agent i, the sensitivity to price by all other agents implies that the price impact of agent i is decreasing. Together this leads to the comparative static result of price sensitivity and price impact with respect to the information structure parametrized by the weight λ, as stated below in Proposition 2.1 and 3.1. As λ is increasing, the price sensitivity decreases and the resulting price impact m converges to 0 as $\lambda \to \infty$.

Proposition 2 (Information Structure and Market Power I)
For $\lambda \geq -1/\left(N-1\right)$:

1. *the price impact $m \in (0, \infty)$ and the price sensitivity $\beta_p \in \left(-\infty, \frac{1}{r(N-1)}\right)$ are decreasing in λ;*
2. *as $\lambda \to -1/\left(N-1\right)$, we have:*

$$\lim_{\lambda \downarrow -1/(N-1)} m = +\infty, \quad \lim_{\lambda \downarrow -1/(N-1)} \beta_p = \frac{1}{r\left(N-1\right)};$$

3. *as $\lambda \to \infty$, we have:*

$$\lim_{\lambda\to\infty} m = 0, \quad \lim_{\lambda\to\infty} \beta_p = -\infty.$$

Thus we find that the information structure has a profound effect on the responsiveness of the agents to the price p, and through the equilibrium price sensitivity of the agents, it affects the market power of each agent as captured by the price impact. The ensuing comparative static result holds locally for all $\lambda \in \mathbb{R}$ except for the critical value of $\lambda = -1/(N-1)$, where we observe a discontinuity in the price sensitivity and the price impact of the agents.

Even before we reach the critical value $\lambda = -1/(N-1)$, the interaction between the equilibrium price and the equilibrium update on the willingness to pay becomes more subtle if the common shock $\overline{\theta}$ receives a smaller weight in the signal than it receives in the payoff state of agent i. A higher than expected price still implies a higher than expected common shock, but because $\lambda < 1$, the resulting downward revision of the idiosyncratic component $\Delta\theta_i$ is smaller, and in consequence the resulting revision on the payoff state θ_i balances in favor of the payoff state. Thus a higher price now indicates a higher expected payoff state θ_i, and if λ falls sufficiently below 1, then the price sensitivity β_p even turns positive, in fact it hits zero at $\lambda = (1+r)/(1+r(N+1)) < 1$.

Proposition 3 (Information Structure and Market Power II)
For $\lambda < -1/(N-1)$:

1. *the price impact $m \in \left(-\frac{1}{2}, 0\right)$ and the price sensitivity $\beta_p \in \left(\frac{1+2r}{r(N-1)}, \infty\right)$ are decreasing in λ;*
2. *as $\lambda \to -1/(N-1)$, we have:*
$$\lim_{\lambda \uparrow -1/(N-1)} m = -\frac{1}{2}, \quad \lim_{\lambda \uparrow -1/(N-1)} \beta_p = \frac{1+2r}{r(N-1)};$$
3. *as $\lambda \to -\infty$, we have:*
$$\lim_{\lambda \to \infty} m = 0, \quad \lim_{\lambda \to \infty} \beta_p = +\infty.$$

It remains to understand the source of the discontinuity in the price sensitivity and the price impact at the critical value of $\lambda = -1/(N-1)$. We observed that as the information structure λ approaches the critical value, the impact that the agent i's signal has on his expectation of θ_i converges to 0 as

$$\mathbb{E}[\theta_i | s_i, s_{-i}] = \left(\frac{N-1}{N} + \frac{1}{\lambda N}\right) s_i + \left(\frac{1-\lambda}{\lambda}\right) \frac{1}{N} \sum_{j \neq i} s_j. \tag{23}$$

This implies that the total weight that an agent puts on his own signal goes to 0 as he is only learning from the stochastic intercept of the residual supply that he faces.[2] Importantly, as λ approaches

[2]Note that the weight an agent puts on his own signal is not only the coefficient β_s but also depends on the coefficient β_p

$-1/(N-1)$ from the right, a high signal implies a low payoff type θ_i for agent i who receives the signal, but also for all other agents as $\lambda \in (-1/(N-1), 0)$ implies that

$$\frac{\partial \bar{\theta}}{\partial s_i}, \frac{\partial \theta_i}{\partial s_i} \leq 0. \tag{24}$$

Now, as λ is close to $-1/(N-1)$ a small upward shift in the quantity bought by any agent i is interpreted as a large upward shift in the equilibrium expectation of agent i as he is only putting a small weight on his own signal. But importantly, while a smaller signal realization s_i leads agent i to revise his expectation about θ_i moderately upwards, by all other agents $j \neq i$, it will correctly by interpreted as a much larger increase in their expectation of θ_j and thus lead to a huge upward shift in the amount bought by all other agents. In the limit as $\lambda \to -1/(N-1)$, an arbitrarily small increase in the amount bought by agent i leads to an arbitrarily large increase in the equilibrium expectation of θ_j for all other agents which implies an arbitrarily high increase in the quantity bought by other agents, which implies an arbitrarily large increase in price. Thus, a small change in the quantity bought by agent i implies an arbitrarily high positive impact on prices, which leads to arbitrarily large market power as stated in Proposition 2.1.

Now, the discontinuity at $\lambda = -1/(N-1)$ arises as when λ approaches $-1/(N-1)$ from the left, the sign in the updating rule for the payoff state of agent i and all the other agents differ, namely evaluating (23) at $\lambda \leq -1/(N-1)$ gives us:

$$\frac{\partial \theta_i}{\partial s_i} \geq 0 \text{ and } \frac{\partial \bar{\theta}}{\partial s_i} \leq 0. \tag{25}$$

In turn, an small increase in the payoff state of agent i caused by the signal s_i implies a large decrease in the common component $\bar{\theta}$, and hence a large decrease in the payoff state θ_j of the all other agents, $j \neq i$. When we translate this into demand behavior, we find that a small increase in quantity demand by agent i implies a large decrease in the demand by all the other agents. Thus, an increase in the quantity bought by agent i reduces the price at which he buys, as other agents are decreasing the quantity they buy. In fact, this means that the price impact m turns from positive to negative when $\lambda < -1/(N-1)$. In fact if we consider the objective function of agent i :

$$\mathbb{E}[\theta_i|s_i]a_i - \frac{1}{2}a_i^2 - pa_i, \tag{26}$$

and notice that $\partial p/\partial a_i = m$, so that

$$\mathbb{E}[\theta_i|s_i]a_i - \left(\frac{1}{2} + m\right) a_i^2,$$

then with < 0, it is as if the objective function of agent is becoming less concave as m decreases. Thus, as λ approaches $-1/(N-1)$ from the left, even though an agent i's signal s_i is almost non-informative of his type, as the agent is almost risk neutral, his demand response to his own signal remains bounded away from 0, which also bounds the response of other agents to any shift in the amount bought by an individual agent, explaining the finite limit from the left.

It has been prominently noted that demand function competition under complete information typically has many, often a continuum, of equilibrium outcomes. In a seminal contribution, Klemperer and Meyer (1989) showed that only one of these outcomes survives if the game is perturbed with a small amount of imperfect information. The equilibrium market power under this perturbation is the same outcome as when each agent receives a noise free signal with $\lambda = 1$.

The set of outcomes described by noise free signals coincide with the outcomes described by slope takers equilibrium, as described by Weretka (2011). We established in Proposition 1 that any feasible market power can be decentralized as an ex-post complete information equilibrium.

It might be helpful to complete the discussion with the limit case in which there is no exogenous supply of the asset, that is as $r \to \infty$, and we require that the average net supply $\overline{a} = 0$ for all realization of payoff states and signals.

Proposition 4 (Equilibrium with Zero Net Supply)

1. *If $\lambda \in [-1/(N-1), 1/(N-1)]$, then there does not exist an equilibrium in linear symmetric demand functions;*

2. *If $\lambda \notin [-1/(N-1), 1/(N-1)]$, the coefficients of the linear equilibrium demand function are:*

$$\beta_s = \frac{1}{1+m}, \ \beta_p = \frac{-\lambda}{1+m},$$

and the market power is given by:

$$m = \frac{1}{\lambda(N-1)-1}.$$

We could alternatively consider the case with elastic supply and then let $r \to \infty$. We then find that the expression for m would show that as $r \to \infty$, we have $m \to \infty$ for all $\lambda \in [-1/(N-1), 1/(N-1)]$. By contrast, for $\lambda \notin [-1/(N-1), 1/(N-1)]$, if $r \to \infty$, then $m \to 1/\left(\lambda(N-1)-1\right)$.

The non-existence of equilibrium for the case of zero net supply is known in the literature. It is usually attributed to the presence of only two agent. The above result shows that for every finite number of agents there exist information structures for which there are equilibria and information structure for which there are no equilibria. The range of value of λ for which there is no equilibrium decreases with the number of agents. Finally, we note that as the literature focused exclusively on the case of noisy, but non-confounding information structure, and hence $\lambda = 1$, the non-existence of equilibrium arises only for $N = 2$.

4 Equilibrium Behavior for All Information Structures

Until now we have analyzed the demand function competition for a small and special class of information structures. We now extend the analysis to the equilibrium behavior under all possible (symmetric and normally distributed) information structures. To do so, we introduce a solution concept that we refer to as Bayes correlated equilibrium. This solution concept will describe the equilibrium behavior in terms of a distribution of outcomes, namely action and states, and the price impact.

4.1 Definition of Bayes Correlated Equilibrium

The notion of Bayes correlated equilibria will be defined independently of any specific information structure of the agents. We shall merely require that the joint distribution of outcomes, namely prices, quantities, and payoff states form a joint distribution such that there exists a price impact m under which the best response conditions of the agents is satisfied and the market clears.

The equilibrium object is therefore a joint distribution, denoted by μ, over prices, individual and average quantity, and individual and average payoff state, $(p, a_i, \bar{a}, \theta_i, \bar{\theta})$. As we continue to restrict the analysis to symmetric normal distribution, such a joint distribution is a multivariate distribution given by:

$$\begin{pmatrix} p \\ a_i \\ \bar{a} \\ \theta_i \\ \bar{\theta} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_p \\ \mu_a \\ \mu_a \\ \mu_\theta \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \sigma_p^2 & \rho_{ap}\sigma_a\sigma_p & \rho_{p\bar{a}}\sigma_{\bar{a}}\sigma_p & \rho_{\theta p}\sigma_\theta\sigma_p & \rho_{p\bar{\theta}}\sigma_{\bar{\theta}}\sigma_p \\ \rho_{ap}\sigma_a\sigma_p & \sigma_a^2 & \rho_{a\bar{a}}\sigma_{\bar{a}}\sigma_a & \rho_{a\theta}\sigma_\theta\sigma_a & \rho_{a\bar{\theta}}\sigma_{\bar{\theta}}\sigma_a \\ \rho_{\bar{a}p}\sigma_{\bar{a}}\sigma_P & \rho_{a\bar{a}}\sigma_{\bar{a}}\sigma_a\sigma_{\bar{a}} & \sigma_{\bar{a}}^2 & \rho_{\theta\bar{a}}\sigma_\theta\sigma_{\bar{a}} & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}} \\ \rho_{\theta p}\sigma_\theta\sigma_p & \rho_{a\theta}\sigma_\theta\sigma_a & \rho_{\theta\bar{a}}\sigma_\theta\sigma_{\bar{a}} & \sigma_\theta^2 & \rho_{\theta\bar{\theta}}\sigma_\theta\sigma_{\bar{\theta}} \\ \rho_{p\bar{\theta}}\sigma_{\bar{\theta}}\sigma_p & \rho_{a\bar{\theta}}\sigma_{\bar{\theta}}\sigma_a & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{\theta}}\sigma_{\bar{a}} & \rho_{\theta\bar{\theta}}\sigma_\theta\sigma_{\bar{\theta}} & \sigma_{\bar{\theta}}^2 \end{pmatrix} \right). \tag{27}$$

With a symmetric distribution across agents, the correlation between any two agents, say in the demand a_i and a_j is identical to the correlation between the demand a_i and the average demand $\bar{a}$.

Thus it suffices to track the outcome of an individual agent i and the average outcome, where we use the notation introduced earlier in (2) and (3) to denote the common and idiosyncratic component of a_i, namely $\overline{a}$ and Δa_i.

Definition 2 (Bayes Correlated Equilibrium)
A Bayes correlated equilibrium is a joint (normal) distribution of $(p, a_i, \bar{a}, \theta_i, \bar{\theta})$ (as given by (27)) and a price impact $m \in (-1/2, \infty)$ such that best response condition holds for all i, a_i, p:

$$\mathbb{E}[\theta_i | a_i, p] - p - a_i - m a_i = 0; \tag{28}$$

and the market clears:

$$p = r_0 + rN\bar{a}. \tag{29}$$

The best response condition (28) reflects the fact that in the demand function competition each trader can condition his demand, and hence his expectation about the payoff state on the equilibrium price. Given the price p and the price impact m, the equilibrium quantity a_i must then be optimal for agent i. In addition the market clearing condition determines the price as a function of the realized demand, $N\bar{a}$, which by definition is equal to $\Sigma_i a_i$. Since the price p is perfectly collinear with the aggregate demand $N\bar{a}$, we will frequently refer to a Bayes correlated equilibrium in terms of the variables $(a_i, \bar{a}, \theta_i, \bar{\theta})'$, without making an explicit reference to the price.

We introduced the notion of a Bayes correlated equilibrium in Bergemann and Morris (2013b) in a linear best response model with normally distributed uncertainty, and subsequently defined it for a canonical finite games, with a finite number of actions, states and players in Bergemann and Morris (2013a). The present definition adapts the notion of Bayes correlated equilibrium to the demand function competition in two ways. First, it accounts for the fact that the agents can condition on the price in the conditional expectation $\mathbb{E}[\theta_i | a_i, p]$ of the best response condition. Second, it allows the price impact m to be an equilibrium object and thus determined in equilibrium.

In Section 6 we shall compare in detail the set of equilibrium outcomes under demand function competition and under quantity (Cournot) competition. Here it might be informative to simply contrast the Bayes correlated equilibrium conditions. Under quantity competition, each agent has to chose a quantity independent of the price, and hence the price impact $\partial p / \partial a_i$ is determined by the market clearing condition, and hence $\partial p / \partial a_i = r$ for all i and a_i. Likewise, the conditional expectation of each agent can only take into account the choice of the quantity a_i, but cannot condition on the price. Thus the best response condition under quantity competition can now be stated as:

$$\mathbb{E}[\theta_i|a_i] - \mathbb{E}[p|a_i] - a_i - ra_i = 0. \tag{30}$$

By contrast, the market clearing condition and the specification of the equilibrium distribution remain unchanged relative to above definition under demand function competition. Thus, already at this point, we observe that the quantity competition looses one degree of freedom as the price impact is fixed to r by the exogenous supply function, but it gains one degree of freedom as the correlation coefficient between demand a_i and price p is less than 1 as there might be noise in the conditional expectation $\mathbb{E}[p|a_i]$ which did not exist when the demand a_i is contingent on price as in the demand function competition.

We will now provide two different characterizations of the set of feasible outcomes as Bayes correlated equilibrium. The first characterization is purely in statistical terms, namely the moments of the equilibrium distribution. We provide a sharp characterization of what are the feasible outcomes under any information structure. This characterization in particular will allow us to understand how the conditioning on the prices, which are a source of endogenous information, restrict the set of outcomes with respect to an economy where demand decision have to be made in expectation of the realized equilibrium price, such as in the quantity competition. We then provide an equivalence result that formally connects the solution concept of Bayes correlated equilibria to the Bayes Nash equilibrium under all possible (normal symmetric) information structure. In turn, the equivalence result suggest a second characterization in terms of a canonical information structure that allow us to decentralize all outcomes as Bayes Nash equilibrium supported by the specific class of canonical information structures. This characterization provides a link between information structure as exogenous data and the equilibrium outcome. As such it is suitable to provide additional intuition behind the driving mechanisms of the equilibrium price impact.

4.2 Statistical Characterization

We provide a statistical characterization of the set of feasible distributions of quantities and prices in any Bayes correlated equilibrium. We begin with an auxiliary lemma that allows us to reduce the number of variables we need to describe the equilibrium outcome. The reduction in moments that we need to track arises purely from the symmetry property (across agents) of the outcome distribution rather than the equilibrium properties. Under symmetry across agents, we can represent the moments of the individual variables, θ_i and a_i, in terms of the moments of the common component of these variables, namely $\overline{\theta}$ and $\overline{a}$, and the idiosyncratic components, $\Delta\theta_i$ and Δa_i.

By construction, the idiosyncratic and the common component are orthogonal to each other, and hence exactly half of the covariance terms are going to be equal to zero. In particular, we only need to follow the covariance between the idiosyncratic components, $\Delta\theta_i$ and Δa_i, and the covariance between the common components, $\overline{\theta}$ and $\overline{a}$, and we define their correlation coefficients:

$$\rho_{\Delta\Delta} \triangleq \operatorname{corr}(\Delta a_i, \Delta\theta_i) \text{ and } \rho_{\bar{a}\bar{\theta}} \triangleq \operatorname{corr}(\bar{a}, \bar{\theta}). \tag{31}$$

With this we can represent the outcome distribution as follows.

Lemma 1 (Symmetric Outcome Distribution)
If the random variables $(a_1, ..., a_N, \theta_1, ..., \theta_N)$ are symmetric normally distributed, then the random variables $(\Delta a_i, \bar{a}, \Delta\theta_i, \bar{\theta},)$ satisfy:

$$\mu_{\bar{a}} = \mu_a,\ \mu_{\bar{\theta}} = \mu_\theta,\ \mu_{\Delta a} = \mu_{\Delta\theta} = 0,$$

and their joint distribution of variables can be expressed as:

$$\begin{pmatrix} \Delta a_i \\ \bar{a} \\ \Delta\theta_i \\ \bar{\theta} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ \mu_a \\ 0 \\ \mu_\theta \end{pmatrix}, \begin{pmatrix} \frac{(N-1)\sigma_a^2(1-\rho_{aa})}{N} & 0 & \rho_{\Delta\Delta}\sigma_{\Delta a}\sigma_{\Delta\theta} & 0 \\ 0 & \frac{\sigma_a^2((N-1)\rho_{aa}+1)}{N} & 0 & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{a}}\sigma_{\bar{\theta}} \\ \rho_{\Delta\Delta}\sigma_{\Delta a}\sigma_{\Delta\theta} & 0 & \frac{(N-1)\sigma_\theta^2(1-\rho_{\theta\theta})}{N} & 0 \\ 0 & \rho_{\bar{a}\bar{\theta}}\sigma_{\bar{a}}\sigma_{\bar{\theta}} & 0 & \frac{\sigma_\theta^2((N-1)\rho_{\theta\theta}+1)}{N} \end{pmatrix}\right), \tag{32}$$

We can now characterize the set of Bayes correlated equilibrium.

Proposition 5 (Statistical Characterization of Bayes Correlated Equilibrium)
The normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ and the price impact $m \in (-1/2, \infty)$ form a symmetric Bayes correlated equilibrium if and only if:

1. *the joint distribution of variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ is given by (32);*

2. *the mean individual action is:*
$$\mu_a = \frac{\mu_\theta - r_0}{1 + rN + m}; \tag{33}$$

3. *the variance of the common and the idiosyncratic components of the individual action are:*
$$\sigma_{\bar{a}} = \frac{\rho_{\bar{a}\bar{\theta}}\sigma_\theta}{1 + m + r},\ \sigma_{\Delta a} = \frac{\rho_{\Delta\Delta}\sigma_\theta}{1 + m}; \tag{34}$$

4. the correlation coefficients are:

$$\rho_{\Delta\Delta}, \rho_{\bar{a}\bar{\theta}} \in [0,1] \text{ and } \rho_{aa} = \frac{\sigma_{\bar{a}}^2}{\sigma_{\bar{a}}^2 + \sigma_{\Delta a}^2}. \tag{35}$$

The above proposition leads us conclude that the equilibrium outcome is jointly determined by the exogenous data in form of the mean and variance of the valuation, μ_θ and σ_θ, the supply function, r and r_0, and three endogenous equilibrium variables $(m, \rho_{\bar{a}\bar{\theta}}, \rho_{\Delta\Delta})$. Moreover, these three endogenous variables are unrestricted by the exogenous data of the game. The payoff relevant exogenous variables of the game only enter into the determination of the mean and variance of the equilibrium actions next to endogenous variables $(m, \rho_{\bar{a}\bar{\theta}}, \rho_{\Delta\Delta})$.

Proposition 5 informs us that the first moment, μ_a, depends only on of the equilibrium variables, namely the price impact m. By contrast, the variance of the average action depends on the correlation $\rho_{\bar{a}\bar{\theta}}$ between average action and average payoff state, and analogously the variance of the idiosyncratic component depends on the correlation $\rho_{\Delta\Delta}$ between idiosyncratic component of action and payoff state. Interestingly, the correlation between the individual payoff state, $\rho_{\theta\theta}$, does not appear in the equilibrium description at all, and hence the set of equilibrium outcomes is independent of the correlation between individual states.

In Section 3, we analyzed the Bayes Nash equilibria under a specific class of noise free information structures. There, we established in Corollary 1 the realized demands. In particular we showed that the idiosyncratic and the common component of the demand, Δa_i and $\bar{a}$, were linear functions of the idiosyncratic and common component of the payoff shocks, $\Delta\theta_i$ and $\bar{\theta}$, respectively. It follows that the Bayes Nash equilibrium outcomes under the noise free information structure displayed maximal correlation of $\rho_{\bar{a}\bar{\theta}}$ and $\rho_{\Delta\Delta}$, namely $\rho_{\bar{a}\bar{\theta}} = \rho_{\Delta\Delta} = 1$. This is of course consistent with agents having ex-post complete information, as their action is perfectly measurable with respect to the payoff relevant shocks. Thus, the class of noise-free signals allows us to decentralize all feasible price impacts $m \in (-1/2, \infty)$ with maximal correlation between the action and payoff states.

Below we visualize the range of first and second moments of the individual demand and the aggregate demand, that can be attained across all Bayes correlated equilibria. As both the mean and the standard deviation of the aggregate demand are proportional to the mean and the standard deviation of the payoff state, we plot the normalized mean μ_a/μ_θ and the normalized standard deviation σ_a/σ_θ. We also normalize the marginal cost of supply to zero, $r_0 = 0$. The upper boundary of the moments in Figure 2 and 3 - the dark blue line - is generated by the noise free information structures. With maximal price impact, that is as $\lambda \to -1/N$ and hence $m \to \infty$, there is zero trade in equilibrium, and the mean (and realized trade is zero). As the price impact weakens, and more weight is placed on the common payoff state in the noise-free signal, that is as $\lambda \to \infty$ and $m \to 0$, the volume of trade is increasing, and so is the maximally feasible variance of trade.

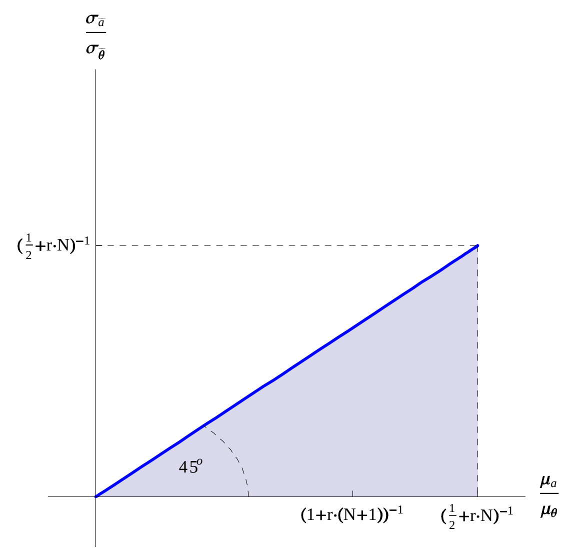

Figure 2: First and Second Moments of Average Demand

We can conclude that the mean volume of trade in equilibrium is very responsive to the information structure. The equilibrium trade volume displays a range that varies from the socially efficient trade in the absence of price impact, $m = 0$, to a complete shutdown in the market as the price impact increases without bounds as $m \to \infty$. By contrast, the equilibrium variance of trade is restricted in a linear fashion by the volume of trade.

The individual moments of trade behave similar to the aggregate moment. By the law of iterated expectation, the mean of the average trade is equivalent to the mean of the individual trade, and controlled by the equilibrium price impact m. The variance of the individual trade consists of the common and the idiosyncratic component. As the idiosyncratic component is not sensitive to the conditions of the aggregate supply function and is independent of r. In consequence, the relationship between the mean and the maximal variance of individual trade is not linear, and the maximal variance decreases more rapidly with an increase in price impact m as displayed in Figure 3.

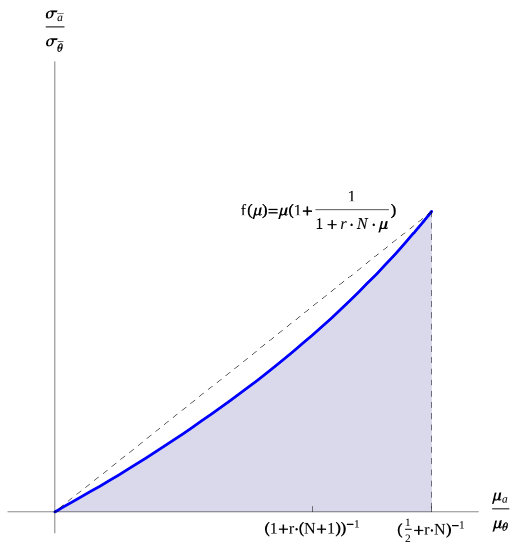

Figure 3: First and Second Moments of Individual Demand

4.3 Equivalence

We now provide a result that formally connects the solution concept of Bayes correlated equilibria to the Bayes Nash equilibrium under all possible (normal symmetric) information structure. We establish that the set of Bayes Nash equilibrium in demand functions for all (normal) information structure can be equivalently be described by the set of Bayes correlated equilibria. In the Bayes Nash equilibrium, we take as given exogenous data the information structure, the type space, that the agents have, and then derive the resulting equilibrium strategies. In turn, the equilibrium strategies generate a particular joint distribution of realized quantities, prices and payoff states. But instead of describing an equilibrium outcome through the process of finding the Bayes Nash equilibrium for every possible information structure, one can simply analyze which joint distributions of realized traded quantities, prices and payoff states, can be reconciled with the equilibrium conditions of best response and market clearing for some given equilibrium price impact. This latter description of an equilibrium exactly constitutes the notion of Bayes correlated equilibrium.

Proposition 6 (Equivalence)

A set of demand functions $\{x_i(s_i,p)\}_i$ and an information structures $\mathcal{I}$ form a Bayes Nash equilibrium if and only if the resulting equilibrium distribution of $(p,a_i,\bar{a},\theta_i,\bar{\theta})$ together with a price impact $m\in(-1/2,\infty)$ form a Bayes correlated equilibrium.

We see that Proposition 6 allow us to connect both solution concepts, and show that they describe the same set of outcomes. The Bayes Nash equilibrium in demand functions predicts not only an outcome, but also the demand function submitted by agents. In many cases this demand function can be observed in the data, so it is also empirically relevant. Yet, a Bayes correlated equilibrium also contains the information pertaining to the demand functions agents submit. Instead of describing the slope of the demand function an agent submits, a Bayes correlated equilibrium specifies the price impact each agent has. Yet, there is a bijection between price impact and the slope of the demand an agent submits in equilibrium. Thus, by specifying a Bayes correlated equilibrium we are not only specifying outcomes in terms of quantities traded, prices and types, but also the slope of the demands agents must have submitted in equilibrium.

4.4 Canonical Information Structures

With the equivalence result of Proposition 6, we know that the set of equilibrium outcomes as described by the Bayes correlated equilibrium in Proposition 5 is complete and exhaustive with respect to the set of all Bayes Nash equilibria. Yet, the equivalence does not tell us how complex the information structures have to be order in order to generate all possible information structures. We now provide an explicit characterization of the set of all equilibria by means of a class of one dimensional signal. This class of information structures is sufficiently rich to informationally decentralize all possible Bayes correlated equilibrium outcomes as Bayes Nash equilibria in demand functions. In this sense, this class can be seen as a canonical class of information structures.

The class of one-dimensional information structures that we consider are simply the noisy augmentation of the noise-free information structures that we considered earlier. Namely, let the one-dimensional signal be of the form:

$$s_i \triangleq \Delta\theta_i + \lambda\bar{\theta} + \varepsilon_i, \tag{36}$$

where, as before, $\lambda \in \mathbb{R}$ is the weight on the common component of the payoff state, and ε_i represents a normally distributed noise term with mean zero, variance σ_ε^2, and correlation coefficient $\rho_{\varepsilon\varepsilon}$ between the agents. As before, we shall use the decomposition into a common and an idiosyncratic component of the error term in the description of the equilibrium, and thus

$$\sigma_\varepsilon^2 \triangleq \sigma_{\bar{\varepsilon}}^2 + \sigma_{\Delta\varepsilon}^2,$$

which decomposes the variance of the error term into a common variance term $\sigma_{\bar{\varepsilon}}^2$ and an idiosyncratic variance term $\sigma_{\Delta\varepsilon}^2$. Thus, we now allow for noisy signals and the error terms are allowed to be correlated across the agents. This class of one-dimensional information structures is thus complete described by the triple $(\lambda, \sigma^2_{\bar{\varepsilon}}, \sigma^2_{\Delta\varepsilon}) \in \mathbb{R} \times \mathbb{R}_+ \times \mathbb{R}_+$.

Proposition 7 (Demand Function Equilibrium with Noisy Signals)
In the Bayes Nash equilibrium with the one-dimensional signals given by (36):

1. *the realized demand quantities are:*

$$\bar{a} = \frac{\mathbb{E}[\bar{\theta}|\bar{s}]}{1+m+Nr}, \ \Delta a_i = \frac{\mathbb{E}[\Delta\theta_i|\Delta s_i]}{1+m};$$

2. *the conditional expectations are:*

$$\mathbb{E}[\bar{\theta}|\bar{s}] = B\frac{\bar{s}}{\lambda} + (1-B)\mu_\theta, \qquad \mathbb{E}[\Delta\theta_i|\Delta s_i] = b\Delta s_i,$$

with

$$B \triangleq \frac{\sigma^2_{\bar{\theta}}\lambda^2}{\sigma^2_{\bar{\theta}}\lambda^2 + \sigma^2_{\bar{\varepsilon}}}, \quad b \triangleq \frac{\sigma^2_{\Delta\theta}}{\sigma^2_{\Delta\theta} + \sigma^2_{\Delta\varepsilon}}; \tag{37}$$

3. *the price impact is:*

$$m = \frac{1}{2}\left(-Nr\frac{(N-1)\Lambda - 1}{(N-1)\Lambda + 1} + \sqrt{(Nr\frac{(N-1)\Lambda - 1}{(N-1)\Lambda + 1})^2 + 2Nr + 1} - 1\right) \tag{38}$$

where

$$\Lambda \triangleq \lambda\frac{b}{B}.$$

The characterization of a Bayes Nash equilibrium in demand functions when agents get signals of the form (36) has three elements. The first part of the Proposition describes the realized traded quantity, in terms of the average and the idiosyncratic part of the trade, as a function of the conditional expectation. The average demand, $\bar{a}$, is the expectation of the average payoff state with respect to the average signal, normalized by taking into account the price impact m. The idiosyncratic component of the demand Δa_i is the expected idiosyncratic payoff state $\Delta\theta_i$, given the idiosyncratic component of the signal, Δs_i.

The second part of the Proposition provides the usual formula for the updating of the conditional expectation given the joint normality of the random variables. Each ratio, $b \in [0,1]$ and $B \in [0,1]$, represents the signal to noise ratio of the idiosyncratic and the common component in the signal s.

The third part of the Proposition provides the expression of the price impact m in terms of the informations structure. We should highlight how closely the equilibrium in this class of noisy information structures tracks the corresponding results for the noise free information structure. The realized demand with the noisy signals corresponds exactly to the demands under the noise free signals, see Corollary 1, after we replace the realized payoff state with the conditional expectation of the payoff state. Similarly, the equilibrium price impact m with noisy signals corresponds precisely to the expression in the noise-free equilibrium, see Proposition 1, after we replace λ by Λ which simply adjusts λ by the informativeness of the idiosyncratic component of the signal relative to the common component of the signal, the ratio b/B. Thus if the noise in the idiosyncratic component is large, then effective weight Λ attached to the common component is lowered, and similarly if the noise in the common component is large, then the effective weight Λ attached to the common component increases.

We should note that we can obtain the entire set of Bayes correlated equilibria with this class of one-dimensional signals. But, clearly, the actual information structure of the agents could be multi-dimensional. Proposition 6 and 7 jointly establish that in terms of equilibrium outcomes the equilibrium behavior under arbitrary (normal) multi-dimensional signal structures would nonetheless lead to behavior that can be completely described by a class of one-dimensional signals. The statistical characterization obtained in Proposition 5 informed us that the equilibrium set is completely described by the three equilibrium variables $\rho_{\bar{a}\bar{\theta}}, \rho_{\Delta\Delta}$ and m. The class of noise information structures is similarly described by a triple, namely $\sigma^2_{\bar{\varepsilon}}, \sigma^2_{\Delta\varepsilon}$, and λ. And while there is not a one-to-one mapping from one of the variables to the other, a comparison of Proposition 5 and 7 indicates how they perform similar tasks in terms of controlling the mean and variance-covariance of the equilibrium distribution.

Vives (2011b) considers an environment in which each agent receives a one-dimensional signal such that $\lambda = 1$ and $\rho_{\varepsilon\varepsilon} = 0$. Thus the class of information structures of the form given by (36) have two additional dimensions.

4.5 Information Revelation by Prices

We presented a sharp characterization of the equilibrium outcomes when the equilibrium prices aggregates information. However, we have not yet argued whether the equilibrium price aggregates the private information of the agents completely, and therefore might be fully revealing. In an environment with one-dimensional, symmetric and normally distributed signals Vives (2011b) establishes that the equilibrium price provides a sufficient statistic of the decentralized private information. By contrast, Rostek and Weretka (2012) maintain the one-dimensional normal information structure, but allow for asymmetric correlation coefficients across agents and then show that the equilibriums can fail to aggregate the private information. The sensitivity of the informativeness property of the price with respect to the information structure suggested by these contrasting results suggests that information revelation may frequently fail to occur.

Indeed, we shall now provide a simple example to illustrate the sensitivity of the information revelation property. The example has the feature that the distribution of the payoff state is one-dimensional, symmetric and normal across the agents as in Vives (2011b). Moreover, we maintain the same distribution of the payoff relevant environment and only vary the information structure. And in fact, the equilibrium distribution of the outcomes will be identical across the information structures. Nonetheless, in one information structure the equilibrium price will be fully revealing, and in the other, it will fail to fully reveal the private information of the agents. Importantly, the sensitivity of the aggregation property is not specific to the strategic environment here with a finite number of traders, and arises already with a continuum of traders. We shall therefore present the example momentarily in the simple setting with a continuum of traders, and consider the numerically more elaborate version with finite traders in the Appendix.

The payoff states are still described by the idiosyncratic and the common component respectively:

$$\theta_i \triangleq \bar{\theta} + \Delta\theta_i = \theta_1 + \theta_2 + \Delta\theta_i.$$

The common component is now represented as the sum of two random variables, θ_1, θ_2, that by definition add up to $\bar{\theta}$. All random variables are independent of each other, and for simplicity we assume that they are all *standard* normally distributed with mean 0 and variance 1. We specify the supply function to be $r_0 = 0$ and $r = 1$. In the first information structure, each agent gets two signals, s_1 and s_2 of the form:

$$s_1 = \theta_1 + \varepsilon_i; \;\; s_2 = \theta_2 + \Delta\theta_i. \tag{39}$$

Signal s_1 is a noisy signal of the common payoff state θ_1 and the error term ε_i is a standard normal as well. Signal s_2 is noise-free but confounds the idiosyncratic and the common payoff state. Manzano and Vives (2011) consider a similar class of two-dimensional information structures in a financial asset pricing model where the value of the asset is given by the sum of a liquidity shock and a payoff shock. There, each agent is supposed to know his own liquidity shock given by s_2. Moreover the liquidity shock displays some correlation across agents, but each agents does not know the average liquidity shock. The common payoff shock is given by θ_1 and each agent receives a private signal on the payoff of the asset with idiosyncratic noise given by s_1.

With a continuum of agents, no agent will have a price impact. In equilibrium, the best response condition of each agent has to be satisfied:

$$a_i = \mathbb{E}[\theta_i | s_1, s_2, p] - p,$$

and the equilibrium price clears the market:

$$p = r\mathbb{E}_i[a_i] = r\bar{a}.$$

Given the information structure, it is convenient to adopt the following change of variables:

$$\hat{a}_i \triangleq a_i - s_2,$$

and solve for an equilibrium in which the first order condition of each agent is given by:

$$\hat{a}_i = \mathbb{E}[\theta_1 | s_1, s_2, p] - p,$$

and the equilibrium price satisfies:

$$p = r\bar{a} = r(\mathbb{E}_i[\hat{a}_i] + \theta_2).$$

After the change of variable, we can reinterpret the equilibrium as one in which the agents have a common value for the good which is represented by θ_1, but there are "noise traders" represented by θ_2. After some standard algebraic operations, we find that in equilibrium the demand functions of each agents is given by

$$x_i\left(s_1, s_2, p\right) = \frac{2}{5}s_1 + \frac{4}{5}s_2 - \frac{1}{3}p,$$

and in terms of the payoff shocks:

$$x_i\left(s_1, s_2, p\right) = \frac{3}{10}\theta_1 + \frac{3}{5}\theta_2 + \frac{2}{5}\varepsilon_i + \frac{4}{5}\Delta\theta_i.$$

The resulting equilibrium is evidently not fully revealing. After all, in any fully revealing equilibrium, all payoff states would be revealed by looking at the signals of all players pooled together. Under complete information, the equilibrium demands would be given by:

$$a_i = \frac{\theta_1 + \theta_2}{2} + \Delta\theta_i.$$

We can compute the volatility of the individual demand a_i and decompose it in terms of the payoff relevant fundamentals and the noise:

$$\text{var}(a_i) = (\frac{3}{10})^2 + (\frac{3}{5})^2 + (\frac{2}{5})^2 + (\frac{4}{5})^2 = \frac{5}{4}, \;\; \text{var}(\bar{a}) = (\frac{3}{10})^2 + (\frac{3}{5})^2 = \frac{9}{20},$$

and the covariance is given by:

$$\text{cov}(a_i, \Delta\theta_i) = \frac{4}{5} \; ; \; \text{cov}(a_i, \bar{\theta}) = \frac{9}{10}.$$

Now consider a second information structure. Here, each agent is receiving a two-dimensional signal as well:

$$\tilde{s}_1 = \Delta\theta_i + \varepsilon_{i1}, \;\; \tilde{s}_2 = \bar{\theta} + \varepsilon_2. \tag{40}$$

If we calculate the equilibrium under this alternative information structure we get the following equilibrium actions:

$$a_i' = \mathbb{E}[\Delta\theta_i|\tilde{s}_1] + \mathbb{E}[\bar{\theta}|\tilde{s}_2] = \frac{4}{5}(\Delta\theta_i + \varepsilon_{i1}) + \frac{9}{20}(\bar{\theta} + \varepsilon_2).$$

If we calculate the variance of a_i', $\bar{a}'$ and decompose a_i' in terms of the payoff relevant fundamentals and noise, we get the following:

$$\text{var}(a_i') = (\frac{4}{5})^2(1 + \frac{1}{4}) = \frac{5}{4}, \;\; \text{var}(\bar{a}') = (\frac{9}{20})(2 + \frac{2}{9}) = \frac{9}{20},$$

and

$$\text{cov}(a_i', \Delta\theta_i) = \frac{4}{5}, \;\; \text{cov}(a_i', \bar{\theta}) = \text{cov}(a_i, \bar{\theta}) = \frac{9}{10}.$$

We thus find that the equilibrium outcome distribution, and in particular the second moments are identical under both information structures. However in the second information structure, the trades and the equilibrium price are fully revealing of the private information whereas they are not fully informative in the first information structure.

Interestingly, an outside analyst, or the econometrician, who could observe the joint distribution of traded quantities, prices and payoff states, would find it impossible to distinguish between these two information structures on the basis of the observable outcomes. However, this doesn't mean that the game is perceived as equivalent by the agents in terms of their beliefs and higher order beliefs. In the second information structure, the average expectation of $\bar{\theta}$ is common knowledge. Moreover, all higher order beliefs on $\bar{\theta}$ are common knowledge. By contrast, in the first information structure the average expectation of $\bar{\theta}$ is not common knowledge.

We established in Proposition **??** that the set of outcomes can always be described in terms of a canonical one-dimensional information structure in which the average expectation of $\bar{\theta}$ is common knowledge. Thus, we can verify that the outcome under these two-dimensional information structures can also be obtained with a one-dimensional signal $\tilde{s}_i$:

$$\tilde{s}_i \triangleq \Delta\theta_i + \varepsilon_{i1} + \bar{\theta} + \varepsilon_2, \tag{41}$$

where ε_{i1} is an idiosyncratic noise term with variance of $1/4$, ε_2 is a common noise term with variance of $2/9$. We thus find that very different information structures, with very different informational properties, may yet allow us to decentralize the same equilibrium outcome distribution. The basic intuition about how different information structure can yet map into the same outcome distribution remains in a model with a finite number of players. The only additional difficulty with a finite number of players is that the information structure also affects the price impact of each agent, and hence it is a somewhat more subtle exercise to find outcome equivalent information structures and we provide the details in the Appendix.

5 Market Power

We will now study how price impact behaves in a demand function equilibrium. We first provide a qualitative description on how price impact changes with the informations structure and provide the intuitions behind the drivers of price impact. We then provide a analysis on the profit maximizing level of price impact.

5.1 Comparative Statics

From Proposition 7, we can see that all the effect of the information structure in the price impact is summarized in the parameter ι. In Figure 4 we show an schematic figure to illustrate how the price impact changes as a function of ι. We can identify two important cases, $\iota \to \infty$ and $\iota \to 0$. When $\iota \to \infty$ an agent has no price impact, which implies that any additional increase in the demand function he submits does not change the equilibrium prices. This implies that a change in the submitted quantity must be offset by the demand functions of the other agents. On the other hand, as $\iota \to 0$ we have that the price impact of an agent converges to Nr, which implements the collusive price level (we are more specific on how to calculate the collusive price when we discuss the profit maximizing price impact). This is as if any additional increase in the demand function he submits

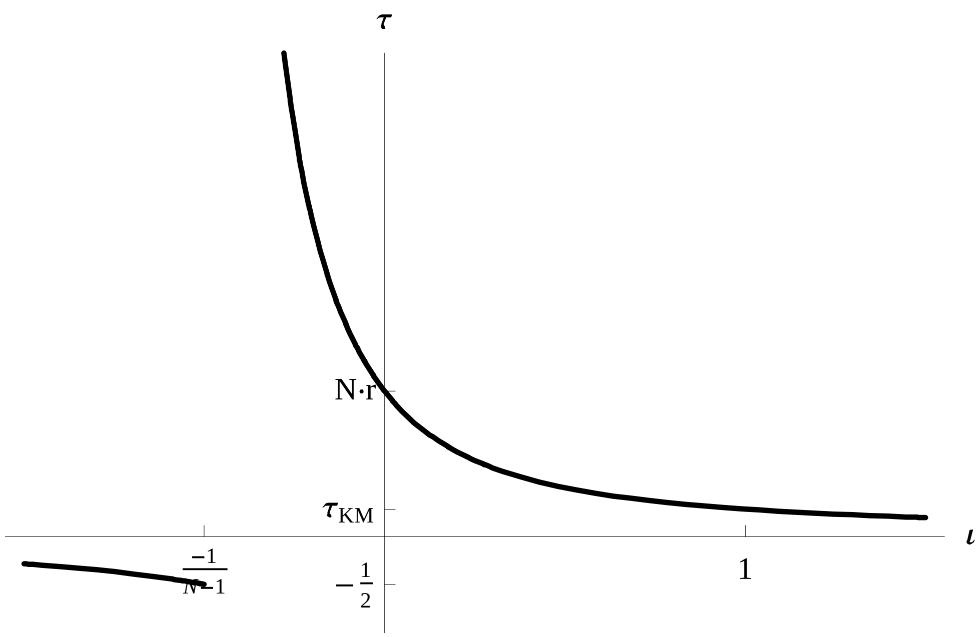

Figure 4: Information and Market Power

is perfectly replicated by all other agents. Thus, this yields the collusive level of output. Moreover, we can see the price impact is decreasing in ι almost everywhere, except for a discontinuity at $\lambda = -1/(N-1)$. The price impact converges to infinity if ι approaches $-1/(N-1)$ from the right, while it converges to $-1/2$ if it approaches $-1/(N-1)$ from the left.

If we impose $B = b = \lambda = 1$, then we recover the (robust) equilibrium with complete information, which is studied in Klemperer and Meyer (1989). In this case we have that,

$$\iota_{KM} \triangleq 1.$$

This implies that in the model studied by Klemperer and Meyer (1989), price impact is given by:

$$m_{KM} = \frac{1}{2}\left(-Nr\frac{N-2}{N} + \sqrt{(Nr\frac{N-2}{N})^2 + 2Nr + 1} - 1\right).$$

We can also recover the model studied by Vives (2011b).[3] This model corresponds to imposing $\lambda = 1$ and $\rho_{\varepsilon\varepsilon} = 0$, in which case we get:

$$\iota_V \triangleq \frac{(1-\rho_{\theta\theta})((n-1)\rho_{\theta\theta} + \frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + 1)}{((n-1)\rho_{\theta\theta} + 1)(1 + \frac{\sigma_\varepsilon^2}{\sigma_\theta^2} - \rho_{\theta\theta})} \in [\frac{1-\rho_{\theta\theta}}{(1-\rho_{\theta\theta}) + n\rho_{\theta\theta}}, 1].$$

[3]This is for the baseline model. He later studies the effect of introducing a public signal, which we also discuss later.

This implies that in the model studied by Vives (2011b), price impact is given in the following bounds:

$$m_V \in [\frac{1}{2}\left(-Nr\frac{(N-2)(1-\rho_{\theta\theta})-n\rho_{\theta\theta}}{N(1-\rho_{\theta\theta})+n\rho_{\theta\theta}}+\sqrt{(Nr\frac{(N-2)(1-\rho_{\theta\theta})-n\rho_{\theta\theta}}{N(1-\rho_{\theta\theta})+n\rho_{\theta\theta}})^2+2Nr+1}-1\right),$$
$$\frac{1}{2}\left(-Nr\frac{N-2}{N}+\sqrt{(Nr\frac{N-2}{N})^2+2Nr+1}-1\right)].$$

Note that in any equilibrium, the information agents have on $\Delta\theta_i$ and $\bar{\theta}$ is measured by b and B respectively, while λ only affects the price impact. To understand why λ affects price impact, we begin by studying the case in which $b = B = 1$. Thus, in equilibrium agents perfectly know the realization of $\Delta\theta_i$ and $\bar{\theta}$. We now proceed to provide an intuition behind the determination of the price impact.

There are two forces which determines the price impact under any information structure. To understand the drivers of price impact we look at the response of other players if player i decides to submit a higher demand than the one dictated by the equilibrium. First, agent observe higher prices, and thus they interpret this as a higher realization of $\bar{\theta}$ than they would have originally estimated. This results in all agents increasing their demand as well through the supply function they submitted. On the other hand, a unexpected increase in price also results in agents interpreting this as a lower realization of $\Delta\theta_i$ than they would have originally estimated. Thus, they reduce their demand through the supply function they submitted. We now explain which of these forces dominates depending on the information structure. For this we look at the two limit case of $\lambda = 0$ and $\lambda = \infty$, and analyze the equilibrium in this case (with the obvious interpretation when $\lambda = \infty$).

We begin by looking at the case $\lambda = 0$. In this case the signals agents have are purely idiosyncratic and by definition sum up to 0. Therefore, agents have no information on what the equilibrium price should be based on the signal they have. The equilibrium in this case consists in agents submitting a demand function that s perfectly collinear with the supply function, and with prices adjusting to the average signal (which for $\lambda = 0$ is actually equal to 0, but we take the obvious limit). This equilibrium suffers from the classic Grossman-Stiglitz paradox that prices are not measurable with respect to the information agent have. Yet, in the limit we get the same intuitions without suffering from the paradox. As $\lambda \to 0$, all information agents have is concerning their idiosyncratic shocks. Thus, they forecast the average type from the equilibrium price they observe. Thus, if an agent deviates and decreases the quantity he submits this is responded by agents forecasting a lower average type and thus also decreasing the quantity they submit. Thus, the deviations of an agent are reinforced by the best response of other agents, thus as $\lambda \to 0$ if an agent increases the price by increasing the quantity he submits, he expects that all other agents should do the same. Therefore, in the limit case we get the collusive price level (we are more specific on the precise meaning of collusive price level later).

On the other hand, the case $\lambda = \infty$ suffers from a similar paradox. In this case agents know exactly what is the equilibrium price based only on their signals, and thus they submit a perfectly elastic supply. The traditional approach would be to split the equilibrium demand equally among all players, yet this is not necessary. In this case the Walrasian auctioneer could split the demand between agents according to the information contained in Δs_i (which is equal to 0 when $\lambda = \infty$, but one can take the limit). In this case we would have a paradox as the Walrasian auctioneer would be splitting the demand between agents according to information no agent has. Note that a prior we could allow for $m = 0$ in the definition of Bayes correlated equilibrium, but this would not be implementable in demand functions. This is just because a perfectly elastic demand function cannot be submitted as a limit order function. Nevertheless, as we take the limit $\lambda \to \infty$ we get the same intuitions without the paradox. As $\lambda \to \infty$ the private signal of agents is a very good predictor of the average signal. Thus, they can correctly anticipate what will be the equilibrium prices. Thus, if agents see a higher equilibrium price than the one they anticipated, they attribute this to having a negative shock to the value of $\Delta\theta_i$. Therefore, if they see a high price they reduce their equilibrium demand which reduces prices, which implies a very elastic demand. Hence, if an agent deviated and attempts to increases the prices, this is responded by the best response of other players which offsets the deviation.

For negative values of λ we have that both forces reinforce each other. Depending on the value of λ, agents interpret a high price as a lower shock to $\bar{\theta}$ and $\Delta\theta$, in which case the price impact gets to be above Nr. The other situation is that agents interpret a high price as a higher shock to $\bar{\theta}$ and $\Delta\theta$, in which case the price impact gets to be below 0, as agents decrease the demand of other agents by increasing the price level.

We can finally understand what are the effects in the price impact of having $b, B \neq 1$. Having noise in signals has the only effect that it adds residual uncertainty to the information of agents and thus dampens the response of the agents to the signals. For each agent i we can define a modified type as follows,

$$\varphi_i \triangleq \mathbb{E}[\Delta\theta_i|\Delta s_i] + \mathbb{E}[\bar{\theta}|\bar{s}] = b\Delta s_i + B\bar{s} + (1-B)\mu_\theta.$$

We can rewrite the signal an agent receives as follows,

$$s_i = \frac{1}{b}(b\Delta s_i + \frac{b}{B}B\bar{s}) = \frac{1}{b}(\Delta\varphi_i + \iota\bar{\varphi} - (1-B)\mu_\theta).$$

Yet, this signal is informationally equivalent to the following signal:

$$s_i' = \Delta\varphi_i + \frac{\lambda b}{B}\bar{\varphi}.$$

Thus, it is easy to see that we can repeat the previous analysis but using the definition already made $\iota = \frac{\lambda b}{B}$. Therefore, the errors in the signals affect the price impact by dampening the response to the average and idiosyncratic part of the signal, which serves as a reweighing of the informational content of the signal.

Finally, it is worth highlighting from the previous analysis that the actual correlation in agents type ρ is irrelevant for price impact, beyond the impact it has on the information structure. Keeping ι fixed, varying the correlation in agents type does not change price impact. This implies that a priori, the lemons problem can be exacerbated or dampen without the need for a change in the fundamentals. A change in the information structure agents have may have arbitrary large effects on the amount of trade in equilibrium, going from a fully competitive efficient allocation, to a situation of market shutdown.

5.2 Profit Maximizing Market Power

Before we provide a characterization of the profit maximizing price impact it is useful to provide a characterization of what would agents pick as traded quantities if they could collude. That is, what is the maximum added profits that agents could get under any realization of types. For this we solve the following maximization problem,

$$\{a_1^*, ..., a_N^*\} = \arg\max_{\{a_1,...,a_N\}} \sum_{i\in N} \theta_i a_i - \frac{1}{2}a_i^2 - a_i p;$$

$$\text{subject to } p = r_0 + r\sum_{i\in N} a_i.$$

Calculating the optimal traded quantities we get:

$$\bar{a}^* = \frac{\bar{\theta}}{1+2rN} \; ; \; \Delta a_i^* = \Delta\theta_i.$$

If we compare the optimal traded quantities with the ones provided in Proposition 7, we can see that for any information structure the profits in equilibrium under demand function competition will always be below the maximum possible that can be achieved. If $m = Nr$ then the price is always equal to the collusive price, and thus agents get the maximum profits from the variations in $\bar{\theta}$. Yet, in this case the trade between agents is too low, and thus the profits are lower than the maximum achievable. On the other hand, as $m \to 0$ the trade between agents approaches the optimal one, but in this case the average price is too responsive to the average type. Thus, the competition lowers the profits agents get from the exogenous supply. It will be clear that there is an optimal price impact which will be between $m = Nr$ and $m = 0$.

We can easily find the optimal price impact for the case in which agents receive noiseless signals ($\sigma_\varepsilon = 0$). If agents receive noiseless signals, then the profits of agents can be written in terms of the price impact as follows:

$$\mathbb{E}[\pi] = (1/2 + m)\left(\frac{(\mu_\theta - r_0)^2 + \sigma^2_{\bar{\theta}}}{(1 + m + Nr)^2} + \frac{\sigma^2_{\Delta\theta}}{(1 + m)^2}\right) \tag{42}$$

If we maximize (42) with respect to m, we get the optimal price impact, which we denote m^*.

Proposition 8 (Optimal Market Power)
The function $\mathbb{E}[\pi]$ has a unique maximum m^*, moreover $m^* \in [0, N \cdot r]$.

Proof. We first prove that $\mathbb{E}[\pi]$ has a unique maximum in m, and the maximum is in $(0, Nr)$. For this, first note that the function $(1/2 + x)/(1 + \beta + x)^2$ is quasi-concave in x, with a unique maximum at $x = \beta$. Second, note that the function $(1/2 + x)/(1 + \beta + x)^2$ is strictly concave in x for $x < 1/2 + \beta$. Since the sum of concave functions is concave, it is easy to see that $\mathbb{E}[\pi]$ is strictly concave for $m \leq 1/2 + N \cdot r$. Moreover, we have that:

$$0 = \arg\max_m \; (1/2 + m)\frac{\sigma^2_{\Delta\theta}}{(1 + m)^2} \; ; \; Nr = \arg\max_m \; (1/2 + m)\frac{(\mu_\theta - r_0)^2 + \sigma^2_{\bar{\theta}}}{(1 + m + Nr)^2}.$$

Thus, it is easy to see that $\mathbb{E}[\pi]$ is decreasing for $m \geq 1/2 + N \cdot r$ (which is the part we cannot check it is concave) and has a unique maximum in $[0, Nr]$.

Using the previous result we can also check the m^* is monotonic increasing in $\rho_{\theta\theta}$. We note that:

$$\frac{\partial}{\partial m}\left((1/2 + m)\frac{\sigma^2_{\Delta\theta}}{(1 + m)^2}\right)\Bigg|_{m=m^*} < 0 \; ; \; \frac{\partial}{\partial m}\left((1/2 + m)\frac{(\mu_\theta - r_0)^2 + \sigma^2_{\bar{\theta}}}{(1 + m + Nr)^2}\right)\Bigg|_{m=m^*} > 0.$$

Using the fact that $\sigma^2_{\Delta\theta} = \frac{N-1}{N}(1 - \rho)\sigma_\theta$ and $\sigma^2_{\bar{\theta}} = \frac{(N-1)\rho+1}{N}\sigma^2_\theta$ we have that m^* is increasing in $\rho_{\theta\theta}$.
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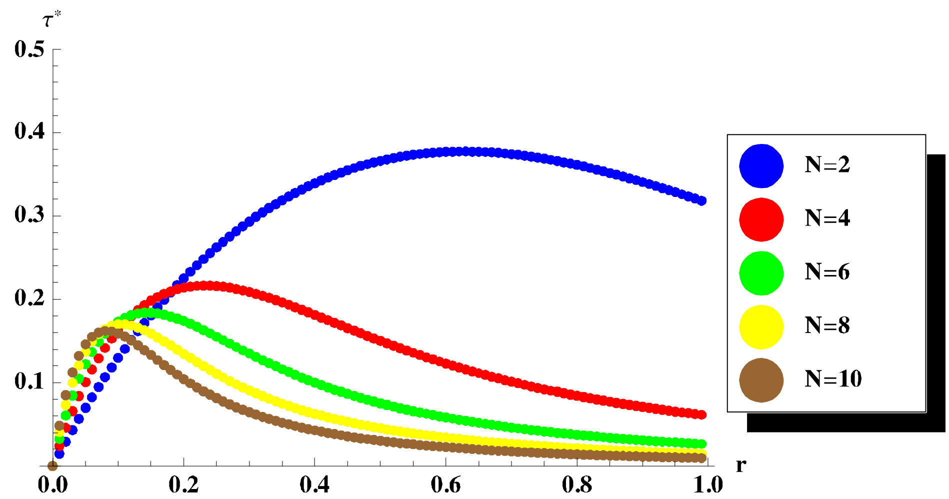

Figure 5: Information and Market Power

To understand the bounds provided in Lemma 8 we find the optimal price impact in some extreme cases. If $\sigma_{\Delta\theta} = 0$, then it is easy to see that it is optimal to impose $m^* = Nr$. Intuitively, when there are no gains from trade between agents, it is best to impose the optimal price level. If $(\mu_\theta - r_0)^2 + \sigma^2_{\bar{\theta}} = 0$, then it is optimal to maximize trade between agents and impose $m^* = 0$.

Although the solution is algebraically cumbersome, and thus it is not worth providing it, some comparative statics are easy to find and the problem is also numerically easy to solve. In the proof of Lemma 8 we checked that m^* is increasing in $\rho_{\theta\theta}$. On the other hand, the comparative statics with respect to r is non-monotonic. To illustrate this, in Figure 5 we plot m^* as a function of r.

The intuition on why m^* is non-monotonic in r is simple. For a very small r the exogenous supply is very elastic, and thus the optimal price level is almost constant. This implies that agents would like to maximize the gains from trade between each other, as the aggregate price level is already constant. Thus, agents would like a small price impact. On the other hand, for a very large r, once again the maximum profits from the exogenous supply are unresponsive to shocks in $\bar{\theta}$. Thus, agents rather maximize the profits from trade between each other. For intermediate levels of r it is the case it is most important for agents to pin-down the optimal price level to maximize profits from the exogenous supply, and thus it is optimal to have a higher price impact. Of course, as N becomes large, the gains from trade between agents becomes larger. This can be seen from the fact that $\sigma^2_{\Delta\theta}$ increases. Thus, agents have bigger gains from trading within each other, which decreases the optimal level of price impact.

Since the complete information price impact (m_{KM}) is also non-monotonic in r, it is not clear

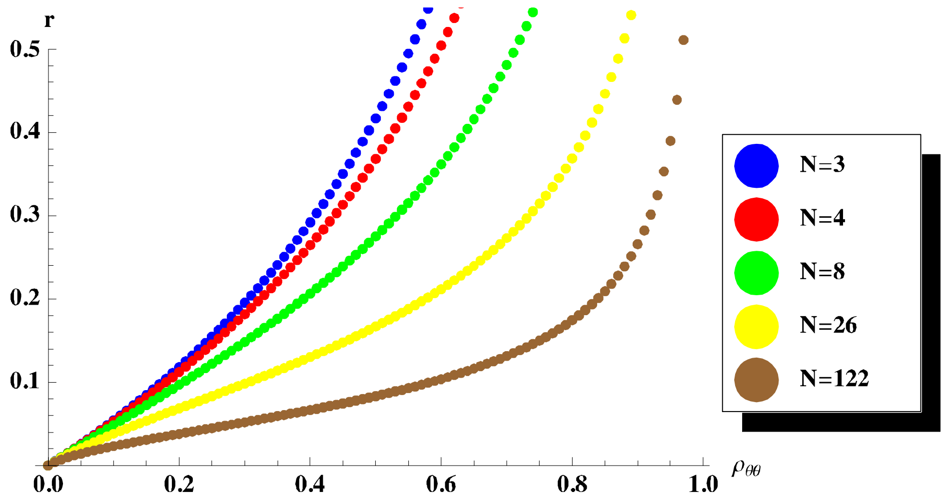

Figure 6: Interdependence and Interaction

from Figure 5 whether the optimal price impact is above or below the complete information one. Thus, we compare the optimal equilibrium price impact compare to the complete information price impact, given by m_{KM}. In Figure 6 we show for what values of $(r, \rho_{\theta\theta})$, the optimal price impact is equal to m_{KM}. Since m^* is monotonic increasing with respect to $\rho_{\theta\theta}$ and m_{KM} is constant with respect to $\rho_{\theta\theta}$, we know that the points above the line corresponds to values of $(r, \rho_{\theta\theta})$ for which m^{KM} is too low, while below the line corresponds to values $(r, \rho_{\theta\theta})$ for which m^{KM} is too high.

We can see that as $\rho_{\theta\theta}$ increases, it is more likely that agents will rather have the optimal price level, and thus the complete information price impact will be too low. On the other hand, as $\rho_{\theta\theta}$ decreases, agents would like to increase trade between each other, which implies that agents would like a lower price impact than the complete information one.

6 Market Mechanisms and Outcomes

The competition in demand functions provides a market mechanism that balances demand and supply with a *uniform price* across traders. The preceding analysis established that the market allocation is sensitive, in particular with respect the mean volume of trade, to the information structure that the agents possess. As the competition in demand function is only one of many mechanism that match demand and supply on the basis of a uniform price, it is natural to compare the outcome under demand function competition with other uniform price market mechanisms.

The immediately relevant mechanism is the competition in quantities, the Cournot oligopoly model. We observed earlier, see (30) that the best response condition under quantity competition differs from the demand function competition in two important respects. First in the demand function competition, the agents can make their trade contingent on the equilibrium price, whereas in the quantity competition, the demand has to be stated unconditional. Second, the price impact of each agent depends on the submitted demand function of all other agents, whereas by contrast, in the quantity competition, the price impact is constant and simply given by the supply conditions.

It follows that the equilibrium moments in the demand function competition are restricted by the fact that the endogenous (equilibrium) information that is conveyed by the equilibrium price is shared and in this sense synchronized across the agents. By contrast, in the quantity competition, each trader only forms an expectation of the equilibrium price, and this expectation may differ widely across the agents as it is formed on the basis of their individual private information. This may indicate that the variance of the trade volume is lower (and in tighter bounds) under demand function competition than it would be under quantity competition. However, as the volume of trade is strongly affected by the price impact under demand function competition, and as quantity competition results in a constant price impact, the average volume of trade may be more restricted under quantity competition.

6.1 Cournot Competition

We maintain the same payoff and information environment as in the demand function competition. The only change that arises with the quantity competition is that each agent i submits a demanded quantity q_i. As before, the market clearing prices is given by balancing demand and supply:

$$p = r_0 + r \sum_{i \in N} q_i.$$

In the present section, we denoted the demand variable from by q_i (rather than a_i) to emphasize that the market mechanism that we are considering has changed. The strategy of each trader is therefore a mapping from the private signal s_i into the demanded quantity, thus

$$q_i : \mathbb{R}^K \to \mathbb{R}.$$

The best response of agent i is:

$$q_i = \frac{1}{1+r}\mathbb{E}[\theta_i - (r_0 + rN\bar{q})|s_i]. \tag{43}$$

Definition 3 (Symmetric Bayes Nash Equilibrium with Cournot Competition)
The random variables $\{q_i\}_{i \in N}$ *form a symmetric normal Bayes Nash equilibrium under competition in quantities if for all* i *and* s_i *the best response condition (43) holds.*

The market outcome under Cournot competition can also be analyzed for all information structures at once. In other words, we can define the Bayes correlated equilibrium in quantity competition and then offer a characterization of the equilibrium moments for all information structures.

Definition 4 (Bayes Correlated Equilibrium with Cournot Competition)
A Bayes correlated equilibrium is a joint (normal) distribution of $(p, a_i, \bar{a}, \theta_i, \bar{\theta})$ *(as given by (27)) such that best response condition holds for all* i, a_i*:*

$$\mathbb{E}[\theta_i - p|a_i] - a_i - ra_i = 0; \tag{44}$$

and the market clears:

$$p = r_0 + rN\bar{a}. \tag{45}$$

The equivalence between the Bayes correlated equilibrium and the Bayes Nash equilibrium for all information structures remains valid in the present game, see Bergemann and Morris (2014) for a canonical argument, and we can again provide a statistical characterization of the Bayes correlated equilibrium in terms of the moments of the equilibrium distribution.

Proposition 9 (Moments of Cournot Competition)
The normal random variables $(\theta_i, \bar{\theta}, q_i, \bar{q})$ *form a Bayes correlated equilibrium if and only if:*

1. *the joint distribution of variables is given by* (32)*;*

2. *the mean of the individual action is*

$$\mu_q = \frac{\mu_\theta - r_0}{1 + (N+1)r}; \tag{46}$$

3. *the variance of the individual action is*

$$\sigma_q = \frac{\rho_{q\theta}\sigma_\theta}{1 + r - Nr\rho_{qq}};$$

4. the correlations satisfy the following inequalities:

$$\left(\rho_{a\theta}-\rho_{a\phi}\right)^2 \leq \frac{(N-1)^2}{N^2}\left(1-\rho_{aa}\right)\left(1-\rho_{\theta\theta}\right), \quad \rho_{a\phi}^2 \leq \frac{\left((N-1)\rho_{aa}+1\right)\left((N-1)\rho_{\theta\theta}+1\right)}{N^2},$$

and

$$\rho_{aa} \in [-\frac{1}{N-1}, 1].$$

We thus find that with Cournot competition, the mean demand of each agent is constant across Bayes correlated equilibrium, and thus constant across all information structures. As the price impact is given by the parameter r of the supply function, the expected volume of trade is invariant with respect to the information structure. The correlation coefficients, $\rho_{a\theta}$ and $\rho_{a\phi}$, are restricted by inequalities that relative to the scale of the competition, N, and the correlation in the payoff states of the agents. By contrast, in the demand function competition, the corresponding correlation coefficients, $\rho_{\overline{a}\overline{\theta}}$ and $\rho_{\Delta\Delta}$, were unrestricted. These observations would suggest that we arrive at a tighter bound on the equilibrium moments with quantity competition. However, the variance of the individual demand is less restricted under quantity competition as the conditioning information on the price is missing, and as such the individual demand can respond strongly due the private signal, and lacks the discipline that the common conditioning on the equilibrium price under demand function competition guarantees. After all, when the agents compete in quantities, each agent attempts to anticipate the quantities that the other traders will submit, but the quantity submitted by any agent does not depend directly on the quantity submitted by the other agents.

In Bergemann, Heumann, and Morris (2015) we analyzed the equilibrium behavior with a continuum of agents in the same linear-quadratic payoff environment as analyzed here. There, the resulting equilibrium analysis is much simplified by the continuum assumption. Most importantly, no single agent can influence the aggregate outcome, and in particular no single agent can have a positive price impact through his infinitesimal small demand. Given the above characterization with a finite number of agents, it is elementary to show that the equilibrium set here converges to the equilibrium set obtained with a continuum of agents if we let N grow large and compute the limiting values as $N \to \infty$. As one might expect, the price impact of each agent with a continuum of agents is zero, and therefore in the comparison between demand function competiton and quantity competition, the only difference that remains is the observability of equilibrium price (or conditioning on the equilibrium price). The additional information that the agents then have in the demand function competition then only restricts the set of equilibrium correlations that can arise relative to the quantity competition. In consequence, the set of equilibrium outcomes under demand function competition turns into a strict subset of the outcomes under quantity competition.

6.2 Comparing Equilibrium Outcomes across Market Designs

The lack of conditioning information under quantity competition might be overcome by giving each agent additional information relative to the information provided in the competition with demand function. On the other hand, the price impact in the quantity competition is constant and equal to r, and so if we would like to replicate the outcome of the demand function competition, it would appear that one would have to adjust the responsively of the demand conditions as represented by concavity of the utility function.

Proposition 10 (Demand Function and Quantity Competition)
Let $(a_i, \bar{a}, \theta_i, \bar{\theta})$ be the outcome of the Bayes Nash equilibrium in demand functions with signals $\{s_i\}_{i\in N}$, then $(a_i, \bar{a}, \theta_i, \bar{\theta})$ is the outcome of the Bayes Nash Equilibrium under quantity competition if each agent receives the two-dimensional signal $\{(\Delta s_i, \bar{s})\}_{i\in N}$ and the best response of agents is given by,

$$q_i = \frac{1}{1+m}(\mathbb{E}[\theta_i - rN\bar{a}|\mathcal{I}_i] - r_0),$$

where m is given by (38).

Thus, the outcome of any Bayes Nash equilibrium in demand function competition can be described as the equilibrium outcome of a competition in quantities after changing two distinct elements in the decision problem of each agent.

First, each agent requires more information about the payoff state. The necessary additional information is attained by splitting the original signal s_i into two signals, namely the common and the idiosyncratic component of the signal that each agent received in the demand function competition. The additional information conveyed by the two components allows the agents to improve their estimate about the demands by the others through $\bar{s}$, and yet adjust the individual demand through the knowledge of the idiosyncratic signal component Δs_i. It is perhaps worth noting that the set of signals of $\mathcal{I}_i = (\Delta s_i, \bar{s})$ is equivalent to the informational environment in which each agent receives a private signal s_i, and then agents pooled and shared their signals to obtain $\bar{s}$ before they submit their individual demand. It is then easy to see that the resulting Bayes Nash equilibrium is *privately revealing* to each agent, as defined by Vives (2011b).

Second, each agent's marginal willingness to pay needs to be attenuated relative to the demand function competition. This is achieved by increasing the concavity in the utility function of each agent, namely instead of $1/2$, the quadratic in the payoff function of the trader is required to be $1/2\,(1+m)$ to account for the price impact in the demand function competition.

Earlier, we represented the first and second moments under demand function competition. With Proposition 9, we can now contrast the equilibrium moments across the market mechanisms. We describe the set of equilibrium moments under demand competition in red. The most immediate contrast is the first moment. Under Cournot competition, the information structure has no influence on the price impact, and hence the nature of the best response function does not vary across information structures. In consequence, the law of iterated expectation pins down the mean of the individual demand, and of the average demand across all information structure to a unique value. However, the variance and the covariance of the demand is strongly influenced by the information structure in the Cournot competition. Moreover, as the traders cannot condition their demand on the realized prices, they lack in an important instrument to synchronize their demand. In consequence the variance of the aggregate demand can be much larger in Cournot competition than in the demand function competition as displayed in Figure 7. In other words, the observability (or conditionality) of the prices in the demand function competition imposes constraints on the responses of the agents to their private information that in turn imposes constraints both on the individual variance, but more importantly on the variance of the aggregate demand.

Interestingly, the set of the feasible first and second (normalized) moments for the aggregate demand under demand function competition (blue shaded area) does not depend on the correlations of types. In Cournot competition this is not the case. As we take $\rho_{\theta\theta} \to 0$, we know that the maximum aggregate demand variance remains positive and bounded away from 0. Thus, the ratio between the variance of the aggregate action to the aggregate shock goes to infinity as $\rho_{\theta\theta} \to 0$. With Cournot competition each agent acts on a signal that is confounding the idiosyncratic and the aggregate component of the payoff state. By contrast„ in the demand function competition, in equilibrium there is always a separation between the aggregate and idiosyncratic payoff state revealed through the price, and thus the set of feasible first and second (normalized) moments of the aggregate demand do not depend on the correlation of the shocks.

In Figure 8 we compare the set of feasible normalized mean and normalized variance of the individual action under Cournot and demand function competition. The main difference is that the restrictions imposed on the individual demand by the observability of the prices are less stringent, and hence the maximum individual demand variance can now be larger in the demand function competition than in the Cournot competition, even though this could never happened for the variance of the aggregate demand.

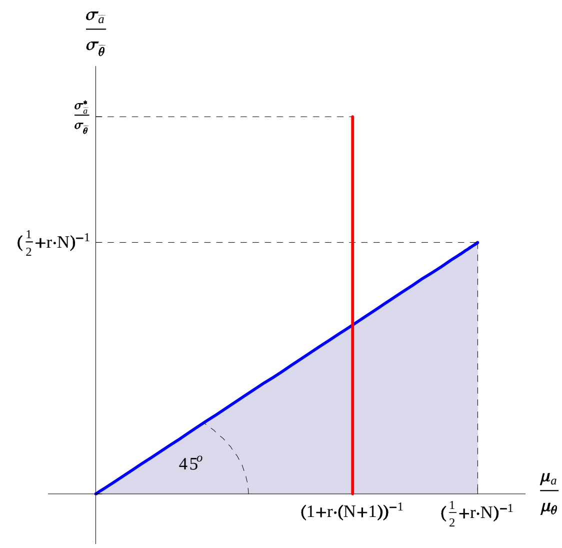

Figure 7: First and Second Equilibrium Moments of Aggregate Demand in Demand Function and Cournot Competition

The equilibrium price is a linear function of the aggregate demand. The volatility of the equilibrium price therefore follows the volatility of the aggregate demand. In particular, as the observability of the equilibrium price limits the variance of the aggregate demand, it also limits the volatility of the price. By contrast, in the Cournot competition, the demand by the agents is less synchronized, and there are information structure that decentralizes a Bayes Nash equilibrium with arbitrary large price volatility. As the determination of the individual demands are made prior to the determination of the equilibrium price, idiosyncratic uncertainty can lead to large aggregate volatility.

Proposition 11 (Price Volatility)

1. *The price volatility with demand function competition is bounded by the variance of the aggregate shock:*
$$\sigma_p^2 \leq \sigma_{\overline{\theta}}^2;$$
2. *the maximal price volatility with demand function is strictly below the Cournot competition;*
3. *the maximal price volatility in Cournot competition is unbounded as the variance of the idiosyncratic payoff state, $\sigma_{\Delta\theta}$, increases:*
$$\lim_{\sigma_{\Delta\theta}\to\infty} \max \sigma_p = \infty.$$

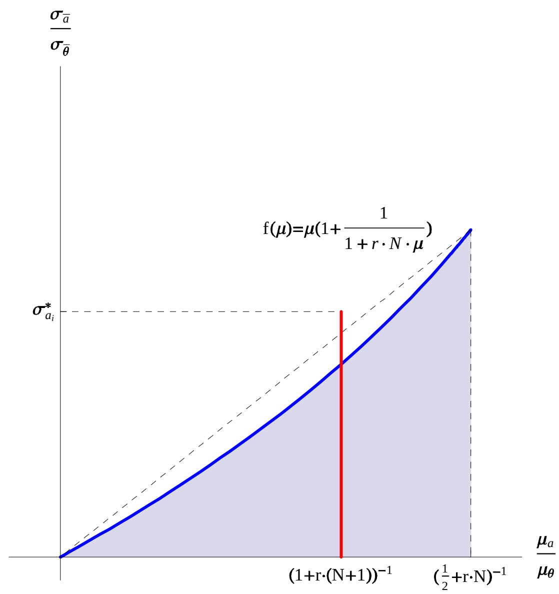

Figure 8: First and Second Equilibrium Moments of Individual Demand

Thus with demand function competition, the equilibrium price volatility is bounded by the volatility of aggregate shocks. By contrast, with Cournot competition, the volatility of the price can grow without bounds for given aggregate shock as long as the variance of the idiosyncratic payoff shock increases. The volatility of the price in the absence of aggregate uncertainty is closely related the recent work that relates idiosyncratic uncertainty to aggregate volatility. For example, Angeletos and La'O (2013) provide a model of an economy in which there is no aggregate uncertainty, but there may be aggregate fluctuations. One of the key aspects is that in the economy the production decisions are done prior to the exchange phase, and thus there are no endogenous information through prices. The present analysis with demand function competition therefore gives us an understanding how the aggregate volatility may be dampened by the presence of endogenous market information as provided by the equilibrium price in demand function competition.

7 Conclusion

We studied how the information structure of agents affects the Bayes Nash equilibrium of a game in which agents compete in demand functions. We have shown that price impact strongly depends on the nature of the private information agents. The analysis provided a very clear understanding on how the information in prices affects the set of feasible outcomes. This allowed us to provide a sharp distinction between the set of feasible outcomes that can be achieved under demand function competition and under quantity competition.

8 Appendix

Proof of Proposition 6. (Only if) We first consider a price impact constant m and joint distribution of variables $(p, a, \bar{a}, \theta_i, \bar{\theta})$ that constitute a symmetric Bayes correlated equilibrium, and show there exists normal signals $\{s_i\}_{i\in N}$ and demand function $x(s_i, p)$ that constitute a symmetric Bayes Nash equilibrium in demand functions such that,

$$p = \hat{p} \text{ and } a_i = x_i(s_i, \hat{p}),$$

where $\hat{p}$ is the equilibrium price in the demand functions equilibrium.

Define a constant β as follows,

$$\beta \triangleq \frac{r - m}{mr(N-1)},$$

and suppose players receive signals $s_i = a_i + \beta p$. We will show that the demand functions

$$x(s_i, p) = s_i - \beta p \tag{47}$$

constitute a symmetric Bayes Nash equilibrium in linear demand functions. If all players submit demand functions as previously defined, then each player will face a residual demand given by,

$$p_i = \frac{1}{1 + r(N-1)\beta}(P_i + ra), \tag{48}$$

where

$$P_i \triangleq r_0 + r\sum_{j\neq N} \beta s_i.$$

Note that by definition, if $a = a_i$, then $p_i = p$.

We now consider the following fictitious game for player i. We assume all players different than i submit demand functions given by (47) first. Then player i observes P_i and decides how much quantity he wants to buy assuming the market clearing price will be given by (48). If we keep the demand functions of players different than i fixed, this fictitious game will obviously yield weakly better profits for agent i than the original game in which he submits demand functions simultaneously with the rest of the players.

In the fictitious game player i solves the following maximization problem:

$$\max_a \mathbb{E}[\theta_i a - \frac{1}{2}a^2 - p_i a | s_i, P_i].$$

The first order condition is given by (where a^* denotes the optimal demand),

$$\mathbb{E}[\theta_i|s_i, P] - a^* - p_i - \frac{\partial p_i}{\partial a}a^* = 0.$$

We can rewrite the first order condition as follows,

$$a^* = \frac{\mathbb{E}[\theta_i|s_i, P_i] - p_i}{1 + \frac{\partial p_i}{\partial a}}.$$

Also, note that,

$$\frac{\partial p_i}{\partial a} = \frac{r}{1 + r(N-1)\beta} = m.$$

Moreover, remember that if $a = a_i$ then $p_i = p$. This also implies that P_i is informationally equivalent to p. Thus, we have that if $a^* = a_i$ the first order condition is satisfied. Thus, $a_i^* = a_i$ is a solution to the optimization problem.

Finally, if agent i submits the demand function $x(s_i, p) = s_i - \beta p = a_i$ he would play in the original game in the same way as in the fictitious game. Thus, he will be playing optimally as well. Thus, the demand function $x(s_i, p)$ is a optimal response given that all other players submit the same demand. Thus, this constitutes a Bayes Nash equilibrium in demand functions.

(If) We now consider some information structure $\{\mathcal{I}_i\}_{i\in N}$ and some symmetric linear Bayes Nash equilibrium in demand functions given by $x(\mathcal{I}_i, p)$. We first note that we can always find a set of one dimensional signals $\{s_i\}_{i\in N}$ such that there exists demand functions, denoted by $x'(s_i, p)$, that constitute a Bayes Nash equilibrium and that are outcome equivalent to the Bayes Nash equilibrium given by $x(\mathcal{I}_i, p)$. For this, just define signal s_i as follows,

$$s_i \triangleq x(\mathcal{I}_i, p) - \beta_p p \text{ where } \beta_p \triangleq \frac{\partial x(\mathcal{I}_i, p)}{\partial p}.$$

We now define,

$$x'(s_i, p) \triangleq s_i + \beta_p p = x(\mathcal{I}_i, p).$$

By definition $x'(s_i, p)$ is measurable with respect to (s_i, p). We now check $x'(s_i, p)$ constitutes a Bayes Nash equilibrium. By definition, if all players $j \neq i$ submit demand functions $x'(s_j, p)$, then player i faces exactly the same problem as in the Bayes Nash equilibrium when players submit demand functions given by $x(\mathcal{I}_i, p)$, except he has information s_i instead of $\mathcal{I}_i$. From the way s_i is defined, it is clear that $\mathcal{I}_i$ is weakly more informative than s_i. Thus, if $x(\mathcal{I}_i, p)$ is a best response when player has information $\mathcal{I}_i$, then $x(\mathcal{I}_i, p)$ would also be a best response when player i has information s_i. Yet, if player submits demand function $x'(s_i, p)$ he will be submitting the same demand function as $x(\mathcal{I}_i, p)$, thus this is a best response. Thus, $x'(s_i, p)$ constitutes a Bayes Nash equilibrium that is outcome equivalent to $x(\mathcal{I}_i, p)$.

We now consider some one dimensional signals $\{s_i\}_{i \in N}$ and some symmetric linear Bayes Nash equilibrium in demand functions given by $x(s_i, p)$ that constitute a Bayes Nash equilibrium in demand functions and show that there exists a Bayes correlated equilibrium that is outcome equivalent. We know that we can write $x(s_i, p)$ as follows,

$$x(s_i, p) = \beta_0 + \beta_s s_i + \beta_p p$$

where $\beta_0, \beta_s, \beta_p$ are constant. In the Bayes Nash equilibrium in demand functions player i faces a residual demand given by,

$$p = P_i + \frac{r}{1 - (N-1)\beta_p} a_i,$$

where,

$$P_i = r_0 + r(N-1)\beta_0 + r \sum_{j \neq i} \beta_s s_j.$$

In the Bayes Nash equilibrium in demand functions player i cannot do better than if he knew what was the residual demand he was facing and he responded to this. In such a case, we would solve,

$$\max_a \mathbb{E}[\theta_i a - \frac{1}{2} a_i^2 - a_i p | P_i, s_i].$$

The best response to the previous maximization problem is given by:

$$\mathbb{E}[\theta_i | P_i, s_i] - a_i^* - (P_i + \frac{r}{1(N-1)\beta_p} a_i^*) - \frac{\partial p}{\partial a_i} = 0.$$

Note that conditioning on the intercept of the residual demand that agent faces is equivalent to conditioning on the equilibrium price

$$p = P_i + \frac{r}{1 - (N-1)\beta_p} a_i^*.$$

Thus, the first order condition can be written as follows,

$$\mathbb{E}[\theta_i | p, s_i] - a_i^* - p - \frac{\partial p}{\partial a_i} = 0.$$

But, note that agent i can get exactly the same outcome by submitting the demand function,

$$x(s_i, p) = \frac{\mathbb{E}[\theta_i | p, s_i] - p}{1 + \frac{\partial p}{\partial a_i}},$$

thus this must be the submitted demand function in equilibrium. Thus, in any Bayes Nash equilibrium the equilibrium realized quantities satisfy the following conditions,

$$\mathbb{E}[\theta_i|p, s_i] - a_i^* - p - \frac{\partial p}{\partial a_i} = 0.$$

Besides the market clearing condition $p = r_0 + rN\bar{a}$ is also obviously satisfied. Since in equilibrium all quantities are normally distributed, we have that $(p, \bar{a}, \Delta a_i, \bar{\theta}, \Delta\theta_i)$ form a Bayes correlated equilibrium. ■

Proof of Proposition 7. We assume agents receive a one dimensional signal of the form:

$$s_i = \Delta\theta_i + \Delta\varepsilon_i + \bar{\varepsilon} + \lambda\bar{\theta}.$$

We find explicitly the equilibrium in demand functions. We conjecture that agents submit demand functions of the form:

$$x(s_i, p) = \beta_0 + \beta_s s_i + \beta_p p. \tag{49}$$

Note that:

$$\frac{1}{N}\sum_{i\in N} x(s_i, p) = \beta_0 + \beta_s \bar{s} + \beta_p p,$$

and thus in equilibrium:

$$\hat{p} = r_0 + r\sum_{i\in N} x(s_i, \hat{p}) = r_0 + Nr(\beta_0 + \beta_s\bar{s} + \beta_p\hat{p}),$$

which leads to

$$\hat{p} = \frac{1}{1 - Nr\beta_p}(r_0 + Nr(\beta_0 + \beta_s\bar{s})).$$

Thus,

$$\bar{s} = \frac{(1 - Nr\beta_p)\hat{p} - r_0 - Nr\beta_0}{Nr\beta_s}.$$

Also, note that if all agents submit demand functions of the form (49), then agent $i \in N$ will face a residual demand with a slope given by,

$$\frac{\partial p_i}{\partial a} = m = \frac{r}{1 - r(N-1)\beta_p}.$$

As before, we use the variable p_i for the residual supply that agent i faces. We now note that:

$$\mathbb{E}[\theta_i|s_i, \hat{p}] = \mathbb{E}[\theta_i|\Delta s_i, \bar{s}] = \mathbb{E}[\Delta\theta_i|\Delta s_i] + \mathbb{E}[\bar{\theta}|\bar{s}].$$

Calculating each of the terms,

$$\mathbb{E}[\Delta\theta_i|\Delta s_i] = \frac{\sigma^2_{\Delta\theta}}{\sigma^2_{\Delta\theta}+\sigma^2_{\Delta\varepsilon}}\Delta s_i = \frac{\sigma^2_{\Delta\theta}}{\sigma^2_{\Delta\theta}+\sigma^2_{\Delta\varepsilon}}(s_i - \frac{(1-Nr\beta_p)\hat{p} - r_0 - Nr\beta_0}{Nr\beta_s});$$

$$\mathbb{E}[\bar{\theta}|\bar{s}] = \frac{\sigma^2_{\bar{\theta}}}{\sigma^2_{\bar{\theta}}+\sigma^2_{\bar{\varepsilon}}/\lambda^2}\frac{\bar{s}}{\lambda} + \frac{\sigma^2_{\bar{\varepsilon}}}{\sigma^2_{\bar{\theta}}+\sigma^2_{\bar{\varepsilon}}/\lambda^2}\mu_\theta = \frac{\sigma^2_{\bar{\theta}}}{\sigma^2_{\bar{\theta}}+\sigma^2_{\bar{\varepsilon}}/\lambda^2}\frac{(1-Nr\beta_p)\hat{p} - r_0 - Nr\beta_0}{\lambda Nr\beta_s} + \frac{\sigma^2_{\bar{\varepsilon}}}{\sigma^2_{\bar{\theta}}+\sigma^2_{\bar{\varepsilon}}/\lambda^2}\mu_\theta.$$

It is convenient to define,

$$\begin{aligned} b &\triangleq \frac{\sigma^2_{\Delta\theta}}{\sigma^2_{\Delta\theta}+\sigma^2_{\Delta\varepsilon}} = \frac{(1-\rho_{\theta\theta})\sigma^2_\theta}{(1-\rho_{\theta\theta})\sigma^2_\theta + (1-\rho_{\varepsilon\varepsilon})\sigma^2_\varepsilon}\ ; \\ B &\triangleq \frac{\sigma^2_{\bar{\theta}}}{\sigma^2_{\bar{\theta}}+\sigma^2_{\bar{\varepsilon}}/\lambda^2} = \frac{(1+(N-1)\rho_{\theta\theta})\sigma^2_\theta}{(1+(N-1)\rho_{\theta\theta})\sigma^2_\theta + (1+(N-1)\rho_{\varepsilon\varepsilon})\sigma^2_\varepsilon/\lambda^2}. \end{aligned}$$

We conjecture that the following demand functions form an equilibrium:

$$\begin{aligned} x(s_i,p) &= \frac{\mathbb{E}[\theta_i|s,p] - p}{1+m} \\ &= \frac{b(s_i - \frac{(1-Nr\beta_p)p - r_0 - Nr\beta_0}{Nr\beta_s}) + B\frac{(1-Nr\beta_p)p - r_0 - Nr\beta_0}{\lambda Nr\beta_s} + (1-B)\mu_\theta - p}{1+m} \end{aligned}$$

We can express the solution from matching the coefficients:

$$\begin{aligned} \beta_s &= \frac{b\left(\kappa + \sqrt{\kappa^2+2nr+1} - 1\right)}{\kappa + nr} \\ m &= \frac{1}{2}\left(-\kappa + \sqrt{\kappa^2+2nr+1} - 1\right) \\ \beta_0 &= \frac{(B-1)r\mu_\theta\left(\kappa + \sqrt{\kappa^2+2nr+1} - 1\right) + r_0((n-2)r-\kappa)}{r\left(-\kappa + n^2 r - n(\kappa+3r)\right)} \\ \beta_p &= -\frac{\kappa + r\left(\kappa + \sqrt{\kappa^2+2nr+1} - n + 1\right)}{(n-1)r(nr-\kappa)}, \end{aligned}$$

with

$$\kappa \triangleq nr\frac{b(n-1)\lambda - B}{b(n-1)\lambda + B}.$$

Note that the second root of the quadratic problem would lead us to $m \leq -1/2$ and thus this does not constitute a valid equilibrium. On the other hand, the first root delivers $m \geq -1/2$, and thus this constitutes a valid equilibrium. By rewriting the terms and using the definition of ι we get the result. ■

Proof of Lemma 1. The conditions of the first moments are direct from the symmetry assumption. To be more specific, let

$$\bar{\theta} = \frac{1}{N}\sum_{i\in N}\mathbb{E}[\theta_i].$$

Taking expectations of the previous equation:

$$\mu_{\bar{\theta}} = \mathbb{E}[\bar{\theta}] = \frac{1}{N}\sum_{i\in N}\mathbb{E}[\theta_i] = \frac{1}{N}\sum_{i\in n}\mu_{\theta_i} = \mu_{\theta}.$$

The same obviously holds for $\mu_{\bar{a}} = \mu_a$. To prove the results on the second moments, we first prove that,

$$\sum_{i\in N}\Delta\theta_i = 0$$

For this just note that:

$$\sum_{i\in N}\Delta\theta_i = \sum_{i\in N}(\theta_i - \bar{\theta}) = \sum_{i\in N}\theta_i - N\bar{\theta} = \sum_{i\in N}\theta_i - N(\frac{1}{N}\sum_{i\in}\theta_i) = 0$$

In any symmetric equilibrium, we must have that for all $i, j \in N$, $\text{cov}(\theta_i\bar{\theta}) = \text{cov}(\theta_j, \bar{\theta})$. Thus, we have that:

$$\text{cov}(\bar{\theta}, \theta_i) = \frac{1}{N}\text{cov}(\bar{\theta}, \sum_{i\in N}\theta_i) = \frac{1}{N}\text{cov}(\bar{\theta}, \sum_{i\in N}\bar{\theta} + \Delta\theta_i) = \text{var}(\bar{\theta}) + \frac{1}{N}\text{cov}(\bar{\theta}, \underbrace{\sum_{i\in N}\Delta\theta_i}_{=0}) = \text{var}(\bar{\theta}).$$

For the rest of the moments we obviously just proceed the same way. ■

Proof of Proposition 5. (Only if) We first prove that if normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ and the price impact parameter m form a Bayes correlated equilibrium then conditions 1-4 hold. Condition 1 is trivial from the fact that the definition of Bayes correlated equilibrium imposes normality. If the normal random variables are normally distributed, then their variance/covariance must be positive-semidefinite. But this is equivalent to imposing that the variance-covariance matrix of the random variables is positive semi-definite. Yet, this directly implies condition 4.

If normal random variables $(\theta_i, \bar{\theta}, a_i, \bar{a})$ and the price impact parameter m form a BCE then we have that,

$$\mathbb{E}[\theta_i|\bar{a}, a_i] - a_i - p - ma_i = 0,$$

where we use that p and $\bar{a}$ are informationally equivalent. Taking expectations of the previous equality and using the Law of Iterated Expectations we get condition (33). If we multiply the previous equation by a_i we get:

$$\mathbb{E}[a_i\theta_i|\bar{a},a_i] - a_i^2 - a_i(r_0+Nr\bar{a}) - ma_i^2 - \mu_a\underbrace{(\mu_\theta - r_0 - \mu_a(1+m+Nr))}_{=0} = 0.$$

Grouping up terms, we get:

$$\mathrm{cov}(a_i\theta_i) - \mathrm{var}(a_i) - Nr\,\mathrm{cov}(a_i,\bar{a}) - m\,\mathrm{var}(a_i) = 0.$$

But, just by rewriting the value of the variances and covariances, the previous equality can be written as follows:

$$\sigma_a = \frac{\rho_{a\theta}\sigma_\theta}{1+m+r((N-1)\rho_{aa}+1)}.$$

Thus, we get (34). If we repeat the same as before but multiply by $\bar{a}$ instead of a_i we get:

$$\mathrm{cov}(\bar{a},\theta_i) - \mathrm{cov}(a_i,\bar{a}) - Nr\,\mathrm{var}(\bar{a}) - m\,\mathrm{cov}(\bar{a},a_i) = 0.$$

As before, by rewriting the value of the variances and covariances, the previous equality can be written as follows:

$$\sigma_a = \frac{N\rho_{a\phi}\sigma_\theta}{(1+m+rN)((1-\rho_{aa})(N-1)+1)}.$$

Using (34) we get (34).

(If) We now consider normal random variables $(\theta_i,\bar{\theta},a_i,\bar{a})$ such that conditions 1-4 are satisfied. First, note that condition 4 guarantees that the variance/covariance matrix is positive-semidefinite, and thus a well defined variance/covariance matrix. Moreover, if condition 1 is satisfied, we can obviously relabel the terms such that we can rewrite the distribution as in **??**. We just need to prove that restrictions (44) and (45) of the definition of Bayes correlated equilibrium are also satisfied. We will show that the following restriction holds,

$$\mathbb{E}[\theta_i|a_i,p] - (r_0+rN\bar{a}) - a_i - ma_i = 0. \tag{50}$$

Then obviously restriction (45) is just the determination of the price in terms of the average quantity $\bar{a}$ and it will be evidently satisfied by defining the price in this way.

We now show that conditions 2 and 3 imply that equation (50) is satisfied. We define the random variable

$$z \triangleq \mathbb{E}[\theta_i|a_i,p] - (r_0+rN\bar{a}) - a_i - ma_i.$$

Since $(\theta_i, \bar{\theta}, a_i, \bar{a})$ are jointly normal, we have that z is normally distributed. If we calculate the expected value of z we get:

$$\mathbb{E}[z] = \mu_\theta - (r_0 + rN\mu_a) - \mu_a - m\mu_a = 0,$$

where the second equality is from condition 2. If we calculate the variance of z we get,

$$\begin{aligned}
\text{var}(z) &= \text{var}(\mathbb{E}[\theta_i|a_i, p] - (r_0 + rN\bar{a}) - a_i - ma_i) \\
&= \text{cov}(z, \mathbb{E}[\theta_i|a_i, p] - (r_0 + rN\bar{a}) - a_i - ma_i) \\
&= \text{cov}(z, \mathbb{E}[\theta_i|a_i, p]) - (1+m)\,\text{cov}(z, a_i) - rN\,\text{cov}(z, \bar{a}) - r_0\,\text{cov}(z, 1).
\end{aligned}$$

Note that $\text{cov}(z, 1) = \mathbb{E}[z] = 0$ by condition 2. On the other hand, it is direct that $\text{cov}(z, a_i) = 0$ by (34) and $\text{cov}(z, \bar{a}) = 0$ by (34). On the other hand, we can find constants $\alpha, \beta, \gamma \in R$ such that,

$$\mathbb{E}[\theta_i|a_i, p] = \mathbb{E}[\theta_i|a_i, \bar{a}] = \alpha a_i + \beta\bar{a} + \gamma.$$

Thus, we have that,

$$\text{cov}(z, \mathbb{E}[\theta_i|a_i, p]) = \alpha\,\text{cov}(z, a_i) + \beta\,\text{cov}(z, \bar{a}) + \gamma\,\text{cov}(z, 1) = 0,$$

by the same argument as before. Thus, we have that $\mathbb{E}[z] = \text{var}(z) = 0$. Since z is normally distributed, this implies that $z = 0$. Thus , (50) is satisfied. Thus, by adequately defining p we have that restrictions (44) and (45) are satisfied. Hence, we get the result. ■

Proof of Proposition 9. See Bergemann, Heumann, and Morris (2015). ■

Proof of Proposition 10. First, note that $\text{cov}(\bar{s}, \Delta\theta_i) = \text{cov}(\bar{\theta}, \Delta s_i) = 0$. Thus,

$$\mathbb{E}[\theta_i|\bar{s}, \Delta s_i] = \mathbb{E}[\bar{\theta}|\bar{s}] + \mathbb{E}[\Delta\theta_i|\Delta s_i].$$

By definition $\sum_{i\in N} \Delta s_i = 0$, thus in equilibrium,

$$\mathbb{E}[\bar{a}|\bar{s}, \Delta s_i] = \mathbb{E}[\bar{a}|\bar{s}].$$

Thus, it is easy to see that the equilibrium actions will be given by,

$$q_i = \frac{\mathbb{E}[\Delta\theta_i|\Delta s_i]}{1+m} + \frac{\mathbb{E}[\bar{\theta}|\bar{s}]}{1+r+m}.$$

Yet, this is exactly the characterization provided in Proposition 7. ■

We briefly discuss a version of two information structures that are outcome equivalent, yet in the first case aggregate private information in equilibrium, and in the second case fail to aggregate private information. The present example complements the discussion in Section 4.5 in that it explicitly considers a finite number of agents. For the sake of simplicity, we maintain the specific information structures discussed in Section 4.5 and set the number of agents to $N = 9$. In this case, under the first information structure, the price impact of agents will be given by $m = 1/3$. On the other hand, the equilibrium action will be given by:

$$a_i = \frac{1}{1+m}(\frac{2}{5}s_1 + \frac{4}{5}s_2 - \frac{1}{3}p) = \frac{3}{4}(\frac{2}{5}s_1 + \frac{4}{5}s_2 - \frac{1}{3}p).$$

If we go back to the second information structure and consider $N = 9$, then the outcome is no longer independent of λ. To be more specific, the equilibrium demands are given by:

$$a_i' = \frac{4}{5(1+m)}(\Delta\theta_i + \varepsilon_{i1}) + \frac{1}{1+r+m}\frac{9}{10}(\bar{\theta} + \varepsilon_2),$$

where m is the price impact. In the specific example here, m will depend on λ and is:

$$m = \frac{-320\lambda + \sqrt{64\lambda(1600\lambda - 279) + 2025} + 36}{64\lambda + 9}.$$

It is easy to check that if we chose $\lambda = 117/248$, then we get $m = 1/3$. Thus, the noisy information structure is now:

$$s_i = \Delta\theta_i + \varepsilon_{i1} + \lambda(\bar{\theta} + \varepsilon_2),$$

with $\lambda = 117/248$ and the variances for the noise terms previously specified are outcome equivalent to the information structure (39) when $N = 9$. Thus, the analysis with a finite number of agents is similar as to the continuum of agents, except that we need to adjust λ to match for the equilibrium price impact.

References

Angeletos, G.-M., and J. La'O (2013): "Sentiments," *Econometrica*, 81, 739–779.

Ausubel, L., P. Cramton, M. Pycia, M. Rostek, and M. Weretka (2014): "Demand Reduction and Inefficiency in Multi-Unit Auctions," *Review of Economics Studies*, forthcoming.

Bergemann, D., T. Heumann, and S. Morris (2015): "Information and Volatility," *Journal of Economic Theory*, forthcoming.

Bergemann, D., and S. Morris (2013a): "Bayes Correlated Equilibrium and Comparison of Information Structures," Discussion Paper 1822R, Cowles Foundation for Research in Economics, Yale University.

——— (2013b): "Robust Predictions in Games with Incomplete Information," *Econometrica*, 81, 1251–1308.

——— (2014): "Bayes Correlated Equilibrium and the Comparison of Information Structures in Games," Cowles Foundation Discussion Paper 1909RR, Yale University and Princeton University.

Klemperer, P., and M. Meyer (1989): "Supply Function Equilibria in Oligopoly under Uncertainty," *Econometrica*, 57, 1243–1277.

Manzano, C., and X. Vives (2011): "Public and Private Learning from Prices, Strategic Substitutability and Complementarity, and Equilibrium Multiplicity," *Journal of Mathematical Economics*.

Rostek, M., and M. Weretka (2012): "Price Inference in Small Markets," *Econometrica*, 80, 687–711.

Vives, X. (2011a): "A Large Market Rational Expectations Equilibrium Model," Discussion paper, IESE Business School.

——— (2011b): "Strategic Supply Function Competition with Private Information," *Econometrica*, 79, 1919–1966.

Weretka, M. (2011): "Endogenous Market Power," *Journal of Economic Theory*, 146, 2281–2306.