Signaling by Blurring*

Sho Miyamoto†
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Abstract

A blurred instruction can have a signaling purpose. A sender wants to persuade a receiver to take an informed action. The sender can control the noise in the signal of an action recommendation. A precise signal makes the receiver worry about the high cost of actions, whereas an imprecise signal confuses the receiver about what action to take. We show that the sender uses the noise as a credible signal of the low cost of actions in order to persuade the receiver to take action. The less precise the signal, the less costly actions appear and the more likely the receiver takes action. The signal becomes noisier as the agents’ interests become more congruent: imprecision signals congruence.

JEL classification: C72, D82, D83

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† Department of Economics, Washington University in St. Louis. E-mail: miyamoto@wustl.edu.
1 Introduction

Imagine you are hiking a mountain. With a short distance left, the trail splits to North and South, which frustrates your wife to suggest turning back home. You value a view from the top of the mountain 1 util but she only $\frac{1}{2}$. Both you and she incur the same disutility from the walking distance to the top, which is the product of the straight distance to the top, long ($\frac{1}{4}$) or short ($\frac{1}{4}$), and the straightness of the route, straight (1) or winding (2). One of the routes is straight and the other winding. She believes all contingencies are equally likely, while you, the hiking planner, know the straight distance is short and the North trail is straight. If you tell her “We take North,” will she come with you? No. She will believe you about the straight route but not trust you internalize her preference to walk only the short distance. Her expected disutility is $\frac{1}{2} \cdot \frac{1}{4} \cdot 1 + \frac{1}{2} \cdot \frac{1}{4} \cdot 1 = \frac{5}{8} > \frac{1}{2}$. You fail to persuade her because she is anxious about the remaining straight distance. What if you tell her “We can take either way. Flip a coin”? You would not say this if there were still the long distance left; you would end up walking in a winding route with probability $\frac{1}{2}$ and the expected disutility $\frac{4}{4} \cdot \frac{1+2}{2} = \frac{3}{2} > 1$. Your blurring convinces her that the remaining straight distance is short and her expected disutility becomes $\frac{1}{4} \cdot \frac{1+2}{2} = \frac{3}{8} < \frac{1}{2}$. You succeed to persuade her!

In this paper we generalize this intuition that an informed agent blurs an instruction of an efficient action in order to signal that the cost of taking action is low for an uninformed agent. The informed agent also cares about the cost of the action taken by the uninformed agent, and blurring an action recommendation has the cost of inducing an inefficient action. Whether this cost is small or not is the informed agent’s another private information. The informed agent purposefully adds noise to the instruction in order to signal that he can afford the ex-post inefficient noise; a blurred action recommendation can signal the low cost of actions. In the above example, you are persuading your wife to keep on walking. You know that it is efficient to take the straight North trail and that there is only the short straight distance left. If you blur which is the straight route, it signals that the straight distance is short because it is only then that
you can risk taking a winding trail. When the straight distance is short, there is the congruence of interest between you and her, which is to keep on walking. Thus our observation can be summarized as: imprecision signals congruence.

In our formal model, a sender wants to persuade a receiver to take an informed action. The sender sends a signal of the efficient action. The receiver takes action if her private valuation of an action exceeds its expected cost. For the persuasion, the sender needs to inform the receiver that the cost multiplier of an action is low, but the sender’s persuasion motive prohibits any informative cheap talk communication about it. Instead, the sender uses the variance of the signal of the efficient action as a costly signal. The sender as well as the receiver cares about the match between the efficient action and an action taken by the receiver. When the sender blurs the efficient action by sending the signal of high variance, the mismatch will occur. The significance of this mismatch for the agents depends on the cost multiplier; when it is lower, the agents suffer smaller losses from the mismatch. Thus, the signal of high variance is less costly to the sender when the cost multiplier is lower.

Concisely, we examine a persuasion game that has two features: (1) a sender can control the noise in the communication channel, and (2) sender information is two-dimensional. We model the noise level as the variance of a signal of the efficient action and show that the sender’s choice of the noise level signals the cost multiplier. Our theoretical contribution is to provide evidence that the sender may purposefully add the noise to his message in order to signal that he can afford the ex-post inefficient noise.

The basic idea of this paper has a number of applications in real world, as the above example suggests. We can postulate that cooking recipes and school textbooks are sometimes presented tersely for a signaling purpose. A culinary expert may provide a minimal instruction without precise measurements in time, volume, and temperature for a dish that can be made nicely even if the instruction is followed very differently from what the expert would do. A school teacher may give only the essence of an idea

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1In the formal model, blurring an instruction means to increase the variance of an action taken by the receiver.
if the rough understanding of it is sufficiently valuable. Certainly, we need to take into account numerous other factors such as bounded cognition to fully explain why terse presentations are sometimes preferred. But I think the idea of this paper suggests one important factor.

As an economic example, we apply the idea to explain that the detailedness of a product instruction signals how much care is necessary to use the product. The assumptions in the formal model map to the following setting between firms and consumers. A firm has product information as to the best way to use the product and the significance for its users to follow the instruction. A consumer has an outside option as her private information. A firm as well as consumers cares about user satisfaction that increases if consumers make better use of the product. The main result of our model states that if a firm has such a product that consumers can get high user satisfaction without a detailed instruction, then the firm suppresses the provision of the detailed instruction to increase the probability of consumer purchase even if the information suppression is ex-post inefficient for both the firm and consumers.

Apple Inc. seems to execute this idea as a part of its branding strategies. Apple’s products are widely known for perfected user experience. The company’s emphasis on ease and simplicity as core values is consistent from product design to advertising as the original ads for the Macintosh and its revolutionary mouse in 1984 suggest: “If you can point, you can use a Macintosh.” As an integral part of the marketing strategies, all of its products boast minimalist user guides that make a sharp contrast to typical electronics’ manuals with lengthy product details. It has the effect of impressing early adopters that its products are easy to use, and this belief immediately spreads to a market and brings in a large fraction of consumers.
Related Literature

This paper connects to multiple strands of literature on communication between informed experts and uninformed decision makers. The idea that noise in the communication channel between a sender and a receiver can improve information transmission was discussed by Myerson [22] and Blume et al. [5]. Blume and Board [4] assume that any sender message is noisy due to vagueness inherent in natural language and study how the sender’s strategic choice of a message adds endogenous noise to the exogenous noise of vague language. Not only our model is related to theirs in that a sender’s choice in communication is a random variable, but two models also exhibit a contrasting result about when communication noise is amplified.

Blume and Board [4] study a variant of the Crawford and Sobel [10] model, in which sender information is either high or low. Under the assumption that the sender’s choice is the mean of a signal distribution, they analyze the distance between the choices of the two sender types. As the sender’s bias increases, the distance becomes smaller and information transmission becomes noisier. In contrast, we study a persuasion game with two-dimensional information and assume that the sender’s choice is the variance of a signal distribution about one piece of information. As the congruence between the agents’ preferences about a binary decision increases, the sender chooses a signal of higher variance and information transmission on one dimension becomes noisier.

A sender in our model tries to persuade a receiver to take a binary choice by choosing the variance of the signal of verifiable information. Kamenica and Gentzkow [15] and Rayo and Segal [25] study the persuasion problems in which an informed sender can commit to more general signal technologies. Their results illustrate that the sender optimally suppresses information by providing noisy signals. Ostrovsky and Schwarz [23] show that a school may coarsen the grades of the students in order to improve

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2 Sobel [26] provides a systematic survey of this literature.
3 The endogenous choice of the variance of a signal distribution is also modeled by Dewan and Myatt [12] and Johnson and Myatt [14] in different contexts. Aside from this technical similarity, there exists little connection between their models and ours, since their models do not have our signaling feature.
their average job placement. A school’s incentive to create coarse grades results from the competition among multiple schools in assortative matching. Our result also has a competitive element in that each sender type sends a noisy signal so as to separate themselves from the worse types for the receiver.

In this paper we characterize strategic communication in a particular game of persuasion. Several other papers report related results in alternative economic situations. Che et al. [8] and Chakraborty and Harbaugh [7] show that an expert has a state-independent bias toward a binary choice and gives comparative cheap talk to persuade a decision maker to make the choice. In comparison, we show that the expert blurs an action recommendation in order to persuade the decision maker to take action.

In the setup of certification, Perez-Richet and Prady [24] show that a sender may complicate product information to persuade a receiver to certify the product. Their result that complexity inflation may lead to high rates of validation is similar to our finding that a less detailed product instruction leads to high chances of consumer purchase. The underlying intuitions are rather different. In their model, the degree of complexity induces less scrutiny by the receiver, which in turn lowers the certification criterion for a signal of product quality. In our model, the detailedness of a product instruction reveals the level of user satisfaction and the purchase decision depends on a consumer’s outside option, which is unknown to a firm.

Sender information in our model is two-dimensional: one is the efficient action and the other is the cost multiplier. If the private information is just the cost multiplier, the sender’s state-independent bias on this dimension prohibits any informative communication[^3]. If the private information is two-dimensional, however, the cost multiplier is fully revealed since information transmission on the efficient action works as a signaling device. In other words, the sender sacrifices information clarity on the efficient action to convey information on the cost multiplier[^5]. This result is consonant with the in-

[^4]: If the private information is just the efficient action, the sender reveals it as there is no conflict of interest on this dimension.

[^5]: In a distinct class of communication game where both the sender and the receiver must incur
sights found in Chakraborty and Harbaugh [6] and Levy and Razin [19] that information transmission in one dimension affects that in another dimension.

Finally, our result has an implication for the relationship between cheap talk and costly signaling. Austen-Smith and Banks [2] and Kartik [16] show that if costly signaling is available in a cheap talk environment, it greatly improves information transmission compared to pure cheap talk communication. If the idea of “burning money” is an euphemism for costly signaling, our natural question is whether we observe a costly signal in communication in reality. Kartik et al. [18] and Kartik [17] provide two rationales for the existence of costly signals: (1) the sender suffers costs from lying and (2) naive receivers take the sender’s message literally. Although our analysis is limited in a particular game of persuasion, we suggest a blurred instruction for a signaling purpose as another possibility.

2 The Model

2.1 Setup

We model a situation where a sender (S) wants a receiver (R) to take an informed action. The way in which the sender transmits his private information and its implications for the receiver’s action are the focus of our analysis.

The receiver chooses between an action and her outside option. If the receiver takes her outside option, both agents get zero utility. If the receiver takes action $a \in \mathbb{R}$, the utility of agent $i \in \{S, R\}$ is

$$v_i - z \left[ (a - x)^2 + c \right],$$

where $v_i$ is agent $i$’s valuation of an action and the second term expresses the total costs efforts for information transmission, Dewatripont and Tirole [13] discuss how cue communication about the congruence of the agents’ interests affects the subsequent issue-relevant communication that is indispensable for the receiver to implement action. If we translate our results into their language, we can say the sender finds cue communication in the way issue-relevant communication is delivered.

To ease the notation of personal pronouns, we assume that a sender is male and a receiver is female.
of action $a$. The total costs have informational and non-informational components, which share a cost multiplier $z \in Z \equiv [0, 1]$. The informational cost, $z(a - x)^2$, increases quadratically with the distance between a taken action $a \in \mathbb{R}$ and an efficient action $x \in \mathbb{R}$. The non-informational cost, $zc$, comes from the difficulty of an action outside the sender’s knowledge and the receiver’s decision, which is parameterized by $c \in (0, 1)$.

A conflict of interest arises as the sender has a vested interest in the receiver’s taking action. We assume $v_S$ is always higher than $v_R$. In particular we let $v_S = 1$ and $v_R = v \in [0, 1]$. The receiver’s valuation $v$ is her private information and is distributed according to $G$ that is twice continuously differentiable with full support on $[0, 1]$.

Both $x$ and $z$ are the sender’s private information. About $x$, the receiver initially has no information and assumes a flat prior on $\mathbb{R}$. About $z$, her initial information is a distribution $F$ that has density and full support on $Z$. In addition to different roles played by $x$ and $z$, we assume that $x$ is verifiable while $z$ is not. We do not require, however, that the sender must fully reveal $x$ to the receiver; the sender is allowed to partially reveal $x$ but he cannot lie in expectation. Formerly, we assume the sender can send a signal of $x$ with noise $b \in \mathbb{R}_+ \cup \{+\infty\}$ such that a signal realization $s \in \mathbb{R}$ is a random draw from a normal distribution $N$ with mean $x$ and variance $b^2$. We say a signal is noisy if $b > 0$. In contrast, the sender can send any signal of $z$. Since the sender has incentives to downplay $z$ regardless of its actual value, no information can be transmitted through the direct signal about $z$ in equilibrium. Therefore, we restrict our attention to a signal of $x$.

Timing of events is as follows. First, a sender type $(x, z)$ and a receiver type $v$ are

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7As a source of conflict, we can alternatively assume the utility difference from outside options instead of the difference in the action valuation. This point can be seen in the example of Section 2.2.

8The assumption that the receiver’s valuation is private information is not necessary for the essence of this paper, which is Proposition 1. The reason we adopt this assumption is because we can obtain a stronger result expressed in Proposition 2 by adding a realistic element to the model.

9Notice that any distribution whose first two moments are finite works here.

10Note that our analysis is not entirely general as we do not allow the sender to send a joint signal about $(x, z)$. 

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shown to the sender and the receiver, respectively. Second, the sender chooses a noise level $b$. Third, a signal realization $s$ is drawn from $N(x, b)$. Last, the receiver takes action $a$ or her outside option.

We conclude this section by presenting our interpretation of this two-agent game. Our view is that this is a game played among a large population of senders and receivers, whose types are their private information. Our motivation is to make a prediction of this game. We do not pursue a mechanism design approach, though we present welfare analysis in Section 4 to discuss policy implications.

2.2 Example of Product Instruction

Our model is useful in understanding how a firm may suppress a detailed user instruction to promise high user satisfaction to consumers. Consider a firm is trying to sell products to consumers. We abstract prices and assume that a trade yields a unit profit to a firm and a unit value to a consumer. A consumer, if she purchases the product, uses it in her own way $a$. The user satisfaction is $1 - z[(a - x)^2 + c]$, where $z(a - x)^2$ is the loss from not using the product in the best way $x$ and $zc$ is the inevitable loss in the product usage. The low value of $z$ means that the product provides high user satisfaction even without the precise knowledge of the best usage $x$. We think of the inverse of $z$ as the level of user satisfaction.

Product information $(x, z)$ is the firm’s private information. The firm cannot directly communicate about $z$ as it always promises high user satisfaction. The firm informs consumers about the best usage $x$ of the product through a package description, a user guide, and an advertisement. The firm can describe it thoroughly ($b = 0$) so that a consumer learns it precisely ($s = x$), or briefly ($b > 0$) so that a consumer learns it imprecisely ($s \neq x$).

Given the instruction $(s, b)$ about $x$, a consumer purchases the product if the expected user satisfaction $1 - E_{x,z} [z [(s - x)^2 + c] | s, b]$ is larger than her utility $1 - v$ from not purchasing it, where $v$ is her private information. The firm puts a weight $1 - \rho$ on earning
a profit and and $\rho$ on increasing the user satisfaction so it maximizes the realization of $1 - E_{x,z} [\rho z [(s - x)^2 + c] | s, b]$ \[1\] Interest of the firm and consumers is not aligned because of consumers’ outside option.

### 2.3 Benchmark Results

To see most clearly the roles of our model assumptions, we consider the three benchmark cases in which only one assumption is modified.

#### 2.3.1 Absence of non-informational cost

The first benchmark result is that if the cost of action is just informational, the cost multiplier $z$ becomes irrelevant when the informational cost is totally eliminated through communication. In this case, there is no need to signal the cost multiplier.

**Benchmark 1.** If $c = 0$, the most informative equilibrium is that the sender sends a precise ($b = 0$) signal of $x$ and the receiver always takes action $a = x$.

#### 2.3.2 Public knowledge of cost multiplier

The second benchmark result is that if sender information is just the efficient action $x$, the sender reveals it as their interest is aligned on this dimension.

**Benchmark 2.** If $z$ is publicly known, the most informative equilibrium is that the sender sends a precise ($b = 0$) signal of $x$ and the receiver takes action $a = x$ if $v > zc$ and her outside option otherwise.

We observe that the lower the cost multiplier $z$, the more likely the receiver takes action. Hence we interpret the inverse of $z$ as the degree of congruence between the agents’ interest.\[12\]

\[1\] The firm’s objective function is slightly differently from that of the sender in the model because of the additional parameter $\rho$. Proposition \[4\] shows that our argument is robust to this difference.

\[12\] Note that our definition of congruence is conceptually different from the degree of conflict used in
2.3.3 Public knowledge of efficient action

The third benchmark result is that if sender information is just the cost multiplier \( z \), then the sender cannot transmit any information. The sender’s bias on the dimension of \( z \) is independent of its value; the sender wants the receiver to believe \( z = 0 \) regardless of its actual value. This strong bias prohibits any informative communication about \( z \).

**Benchmark 3.** *If \( x \) is publicly known, there cannot be any informative communication in a unique equilibrium, where the receiver takes action \( a = x \) if \( v > E[z]c \) and the outside option otherwise.*

3 Equilibrium Analysis

3.1 Equilibrium

We study perfect Bayesian equilibria that are *symmetric* in a sense that the sender’s choice of noise \( b \) only depends on \( z \). Given the quadratic form of informational cost, the receiver’s expected utility from action \( a \) after she received a realization \( s \) of the signal with noise \( b \) is

\[
v - E_{x,z} \left[ z \left( (a - x)^2 + c \right) \bigg| s, b \right] = v - E_z \left[ z \left( (a - s)^2 + b + c \right) \bigg| s, b \right].
\]

This implies that the receiver’s optimal action \( a \) is equal to her signal realization \( s \). Since the receiver extracts no information about the cost multiplier \( z \) from her signal realization \( s \), let \( \gamma(z|b) \) denote the receiver’s posterior belief of \( z \) given the observed noise level \( b \). Note that \( \gamma \) is derived by Bayes’ rule on the equilibrium path. The above argument implies that the receiver takes action if

\[
v > \int_0^1 z(b + c)\gamma(z|b) \, dz. \tag{1}
\]

variants of the Crawford and Sobel [10] model, which is the inverse of the distance of the agents’ bliss points in action space. We adopt this terminology from Dewatripont and Tirole [13].

13 This is a natural restriction in our environment, where the space of \( x \) does not have boundaries and hence no \( x \) is special.

14 We assume that the receiver forms the same belief regardless of her type \( v \).
This defines the action probability \( \hat{q}(b) \) that the receiver takes action when the sender sends a signal of noise \( b \).

Given the receiver’s strategy and the quadratic form of informational cost, the sender’s expected utility from sending a signal of noise \( b \) is

\[
\hat{q}(b) \left( 1 - E_s \left[ z \left[ (s - x)^2 + c \right] \mid b \right] \right) = \hat{q}(b) \left[ 1 - z(b + c) \right].
\]

We observe that a noisy signal is costly. The noise \( b \) in a signal comes into the sender’s expected utility, while it does not appear in his ex-post utility. Although a signal itself is costless to send, a noisy signal incurs the expected informational cost, \( zb \), as an endogenous cost. I call it interpretive cost as it arises from the receiver’s (expected) misinterpretation of the sender’s imprecise message about \( x \). Essentially, a noisy signal lumps together good actions (\( a \) close to \( x \)) and bad actions (\( a \) away from \( x \)), and it is costly since bad actions are ex-post inefficient. Notice that we have transformed our model into the setting with a costly signal \( b \).

Let \( \beta(z) \) be the sender’s pure-strategy choice of noise \( b \) and let \( q(z) = \hat{q}(\beta(z)) \) be the action probability when the sender emulates the equilibrium behavior of type \( z \). From (2), the sender’s incentive compatibility conditions are: for all \( z \in \mathbb{Z} \),

\[
z \in \arg \max_{\tilde{z} \in \mathbb{Z}} q(\tilde{z}) \left[ 1 - z(\beta(\tilde{z}) + c) \right].
\]

Let \( U(b, z) = \hat{q}(b) \left( 1 - z(b + c) \right) \) be the expected utility of type \( z \) when he chooses \( b \). \( U^*(z) = U(\beta(z), z) \) is the expected utility of type \( z \) in equilibrium. Lemma 1 states the behavior of \( U^* \).

**Lemma 1.** \( U^* \) is positive and continuous on \( \mathbb{Z} \) and strictly decreasing in \( z \).

This lemma says that any sender type \( z \) can expect the receiver to take action with positive probability and that his expected utility is higher if the degree of congruence \( z^{-1} \)

\footnote{We observe noise plays different roles for the two agents. For the receiver, it is the variance of \( x \) as she is uncertain about the efficient action. For the sender, it is the variance of \( s \) as he is uncertain about a signal realization.}
is higher. Given this result, Proposition 1 provides us with the essential characteristics of equilibria.

**Proposition 1.** $\beta$ and $q$ are weakly decreasing in $z$.

This proposition says that the sender blurs the efficient action $x$ in order to signal that the degree of congruence $z^{-1}$ is high and persuade the receiver to take action. Put simply, imprecision signals congruence. It is less costly for the sender with low cost multiplier $z$ to lump together good and bad actions. Thus, the lower the cost multiplier $z$, the more interpretive cost $zb$ the sender can afford and the noisier his signal becomes. From the receiver’s point of view, the noisier the signal, the lower the cost multiplier she anticipates. Whether she takes action depends on her valuation of an action, but in expectation the noisier the signal, the more likely the receiver takes action.

This result has a straightforward implication in our application. The detailedness of a product instruction signals the level of user satisfaction. If a product has a low level of user satisfaction, the instruction would have to be specific to indicate the best usage. Otherwise, an imprecise instruction could be followed very differently from the firm’s intension. From the specific instruction, consumers take a cue about the level of user satisfaction. In contrast, if a product has a high level of user satisfaction, the firm would provide little, possibly no, user instruction. This conveys to consumers the firm’s confidence that users can enjoy the product without knowing the precise details. Although providing little instruction is costly in terms of maximizing the gain from the trade, the firm needs to persuade consumers to purchase the product for the realization of that gain, and the signaling of the level of user satisfaction $z$ plays a crucial role in this persuasion.

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16 As we discussed in introduction, this positive relationship contrasts to the finding of Blume and Board [4] that the noise in the communication channel increases with the degree of conflict between the sender and the receiver.
3.2 Separating Equilibrium

Without any restrictions on the receiver’s beliefs when a signal of off-equilibrium noise is observed, there exist every class of equilibria: pooling if $|\beta(Z)| = 1$, separating if $\beta$ is one-to-one, and partially pooling otherwise. To obtain a sharp result, we refine off-equilibrium beliefs by the concept of universal divinity, developed by Banks and Sobel [3], which requires that the sender’s deviation to an off-equilibrium signal should be believed to be taken by the sender type who is most likely to deviate in the sense that he has the biggest set of the receiver’s best responses that make him prefer the deviation to his equilibrium strategy.\footnote{Technically, our refinement is different from theirs because there are multiple receiver types in our model. They compare the set of mixed-strategy best responses to a deviation for a single receiver that make a sender type willing to deviate. We instead compare the set of profiles of pure-strategy best response for all receiver types that make a sender type willing to deviate. In light of forward induction reasoning, our refinement is still that of universal divinity. Note that in the game we study universal divinity is equivalent to the D1 refinement of Cho and Kreps [9].}

Given any off-equilibrium noise $b \not\in \beta(Z)$, let
\[
\xi(z|b) = \frac{q(z) \left[ 1 - z(\beta(z) + c) \right]}{1 - z(b + c)}.
\]
be the action probability that makes type $z$ indifferent between choosing his equilibrium noise $\beta(z)$ and deviating to the off-equilibrium noise $b$. Universal divinity requires that for any off-equilibrium noise $b \not\in \beta(Z)$, $\gamma(z'|b) > 0$ if and only if $z' \in \arg\min_{z \in Z} \xi(z|b)$. This refinement is motivated by the following forward induction reasoning. If, on observing an off-equilibrium noise, all receiver types took action, every sender type would want to deviate to send this noisy signal.\footnote{It follows that the intuitive criterion of Cho and Kreps [9] has no bite in our model.} If instead fewer receiver types were expected to take action, fewer sender types would find the deviation profitable. As the action probability goes down, the sender type most likely to deviate is to be identified. The receiver believes that this sender type sent that noisy signal.

With this refinement, we obtain the existence of a unique separating equilibrium among symmetric perfect Bayesian equilibria.
Proposition 2. Among symmetric perfect Bayesian equilibria, a universally divine equilibrium exists, is unique, and is separating with $\beta(1) = 0$.

The noise $b$ in the signal strictly increases as the degree of congruence $z^{-1}$ increases, and hence the receiver can identify the degree of congruence from the observed level of noise. This perfect knowledge of $z$ in and of itself is desirable for the receiver, but it comes with the loss of information clarity on the efficient action $x$. Given his type $z$, the sender has no incentive to emulate some other type $\tilde{z} \in Z$ by sending a signal of noise $\beta(\tilde{z})$. Either the interpretive cost $z\beta(\tilde{z})$ is too high when $\tilde{z} < z$, or the action probability $q(\tilde{z})$ is too low when $\tilde{z} > z$. When the sender fully reveals $x$, the receiver believes that congruence is lowest.

In our application of product instruction, the firm decreases the volume of instruction as the level of user satisfaction increases. By inspecting the detailedness of the instruction, a consumer gets to know the exact level of user satisfaction. A small fraction of consumers purchase a product whose instruction is full of precise details, whereas a large fraction of consumers purchase a product that has little instruction.

In general, the equilibrium noise $\beta$ is the solution of a differential equation. The solution does not depend on $F$ but it depends on $G$ and $c$. When the distribution of the receiver’s valuation is uniform ($G(v) = v$), we can derive the closed-form solution:

$$\beta(z) = c(z^{-\frac{1}{2}} - 1), \quad (3)$$

which is pictured in Figure 1 for $c = 0.4$\footnote{See Appendix B for the derivation.}

Even though the equilibrium noise $\beta(z)$ increases unboundedly as $z$ approaches 0, the expected total costs $z(\beta(z) + c)$ are increasing in $z$. In contrast, the interpretive cost $z\beta(z)$ is non-monotonic in $z$\footnote{See Appendix A.4 for these results.} Figure 2 draws the three equilibrium costs in the case of a uniform distribution with $c = 0.4$.\footnote{See Appendix B for the derivation.}
Figure 1: the equilibrium noise $\beta(z)$

Figure 2: total costs $z(\beta(z) + c)$, non-informational cost $zc$, and interpretive cost $z\beta(z)$


3.3 Comparative Statics

We observe in (3) that as the non-informational cost parameter \( c \) goes to 0, the sender gets to blur an action recommendation less for any cost multiplier \( z \) and the equilibrium noise \( \beta \) vanishes everywhere in the limit except at \( z = 0 \). This observation holds for a general distribution.

**Proposition 3.** As \( c \) goes to 0, \( \beta(z; c) \) converges to 0 everywhere except \( z = 0 \).

As the non-informational cost \( zc \) of action decreases, the receiver becomes willing to take action with lower valuation \( v \). This weakens the sender’s incentive for persuasion, and consequently less noise is observed at any cost multiplier \( z \). Although it is outside our model analysis, this is most illustrated in the limit case \( c = 0 \), which is the first benchmark result in Section 2.3. If the sender reveals \( x \), then the receiver incurs no cost and she always takes action. Although our model does not include this limit case, we can observe the continuity of our equilibrium at \( c = 0 \). As \( c \) approaches 0, the expected total costs \( z(\beta(z) + c) \) vanish everywhere and the action probability \( q(z) \) approaches 1 for any \( z \).

We conclude our analysis by reporting that our results do not depend on the assumption that the sender cares about the total costs of action as much as the receiver does.

**Proposition 4.** If the sender’s utility from action \( a \) is instead \( 1 - \rho z [(a - x)^2 + c] \) for \( \rho \in (0, \frac{1}{c}) \), then our qualitative results remain unchanged. As \( \rho \) goes to 0, \( \beta(z) \to \frac{c(1-z)}{z} \) and \( q(z) \to 1 - G(c) \).

If the sender cares less about the costs of action, the equilibrium noise increases because now it is less costly to blur an action recommendation. But all the qualitative features of our analysis remain unchanged. With this inflated noise, the expected total costs increase and the action probability decreases. If the sender totally neglects costs, however, our analysis will not work since then a noisy signal is no longer a costly signal.
For our results to hold, the sender cannot be completely ignorant about the costs of action.

4 Discussion

We observed that the sender sends a noisy signal in attempt to persuade the receiver to take an informed action. The analysis never claimed that the equilibrium is desirable for the agents. Interpreting that the game is not played literally between two agents but between two representatives, we analyzed the strategic behavior of a sender in a social meeting with a receiver. Here we put aside this equilibrium viewpoint to examine how the equilibrium performs in terms of the receiver’s ex-ante welfare in comparison to two other case scenarios.\footnote{See Appendix A.7 for computation.}

4.1 Full Disclosure of Verifiable Information

The separating equilibrium can be worse than the pooling equilibrium in which the sender fully reveals $x$ with no informative communication about $z$. The results are mixed and depend on the model specification. If $F$ and $G$ are both uniform, the receiver is better-off in the pooling equilibrium when the non-informational cost parameter $c$ is small. When non-informational cost is significant, the receiver would prefer to learn the cost multiplier $z$ through a noisy signal.

This has an important policy implication for our application of product instruction. If a government regulation requires firms to reveal the best usage $x$ of the product with all the detailed spelled out, then consumers learn no information about the level of user satisfaction. The above result says that the full disclosure policy can be detrimental to consumer welfare if consumers have the high needs to know the level of user satisfaction.\footnote{Anderson and Renault [1] obtains a similar result in a different model framework.} Since our model captures only a part of reality, we refrain ourselves from making a suggestion about any regulatory policy such as the government regulation on drug.
instruction. Our results, however, imply that consumers may not learn which drug has stronger side-effects if all drugs have the same layers of detailed instructions.

4.2 Receiver Commitment

A blurred instruction results from the sender’s persuasion motive. If the receiver can eliminate her outside option and commit to take an informed action, then the conflict of interest will disappear and the sender will fully reveal $x$ without any noise. If $F$ and $G$ are both uniform, this is welfare-improving when $c$ is small. This is reminiscent of a result in Che et al. that the presence of outside options can harm the decision maker.

5 Conclusion

Many game theoretic studies of communication assume a noiseless communication channel between a sender and a receiver. Without the noise, the sender is certain of the receiver’s response to his message. This is not always the case in our actual communication. This paper shows that the sender may purposefully add noise to his message in order to signal that he can afford the ex-post inefficient noise.

We model the noise in the communication channel as the variance of a signal distribution and examine the sender’s endogenous choice of noise. Our central result is nothing but the simple intuition illustrated in the introductory example: an informed agent blurs an instruction of an efficient action in order to signal that the cost of taking action is low for an uninformed agent. This intuition can be hidden in many situations in reality. It possibly occurs in a less conscious way than we have argued thus far. When the informed agent tries to persuade the the uninformed agent, his communication might be constrained by the subconsciously shared notion that a precise instruction warns the uninformed agent of an adverse environment. This paper sheds light on the shade in our mind. If a detailed instruction tires us, that has something to do with our cognition. If it instead scares us, that is signaling that we had better be careful.
A Proofs

A.1 Proof of Lemma 1

First, observe that for all \( z \in Z \) there exists \( \bar{v} \in (0,1) \) such that the sender takes action if \( v \in (\bar{v},1) \), since for any belief \( \gamma(\cdot|0) \), \( \int_0^1 zc \gamma(z|0)dz \leq c < 1 \). Since the sender can always choose \( b = 0 \), it must be that for any \( z \in Z \), \( q(z) > 0 \) and \( U^*(z) > 0 \). Then, note that if \( z < z' \) then \( U^*(z) \geq U(\beta(z'),z) > U^*(z') \). Last, suppose that there exist \( \hat{z} \in Z \) and \( \epsilon > 0 \) such that \( U^*(\hat{z}-) - U^*(\hat{z}+) = \epsilon \). From the uniform continuity of \( U(b,z) \) in \( z \), there exists \( \delta > 0 \) such that if \( |z - z'| < \delta \) then \( |U(\beta(z),z) - U(\beta(z),z')| < \epsilon \).

But, for \( z < \hat{z} < z' \), \( U(\beta(z),z) = U^*(z) > U^*(\hat{z}) > U^*(z') \geq U(\beta(z),z') \) and hence \( U(\beta(z),z) - U(\beta(z),z') > \epsilon \), a contradiction.

A.2 Proof of Proposition 1

Let \( z < z' \). From (IC) for \( z \) and \( z' \),

\[
q(z)[1 - z(\beta(z) + c)] \geq q(z')[1 - z'(\beta(z') + c)]
\]

\[
q(z')[1 - z'(\beta(z') + c)] \geq q(z)[1 - z'(\beta(z) + c)]
\]

Since \( \forall z \in Z, U_A^*(z) > 0 \), we can arrange the above inequalities as:

\[
\frac{1 - z(\beta(z') + c)}{1 - z(\beta(z) + c)} \leq \frac{q(z)}{q(z')} \leq \frac{1 - z'(\beta(z') + c)}{1 - z'(\beta(z) + c)}
\]

(4)

Define \( \eta(z) = \frac{1 - z(\beta(z) + c)}{1 - z(\beta(z) + c)} \) and suppose \( \beta(z) < \beta(z') \). Since

\[
\eta'(z) = \frac{\beta(z) - \beta(z')}{[1 - z(\beta(z) + c)]^2} < 0,
\]

it follows that \( \eta(z) > \eta(z') \), which contradicts the inequalities in (4). Therefore \( \beta(z) \geq \beta(z') \). Then, from the first inequality in (4), we obtain \( \frac{q(z)}{q(z')} \geq \eta(z) \geq 1 \) and hence \( q(z) \geq q(z') \).
A.3 Proof of Proposition 2

I prove it in steps.

Step 1. Look at the universal divinity requirement for off-equilibrium beliefs.

Lemma 2 summarizes the universal divinity requirement for the sender’s belief $\gamma(\cdot|b)$ at the receiver’s off-equilibrium signal $b \notin \beta(Z)$.

**Lemma 2.** In a universal divine equilibrium, $\gamma$ must satisfy:

(i) if $b \in [0, \beta(1))$, then $\gamma(z|b) > 0$ iff $z = 1$;

(ii) if $b \in (\beta(0), \infty]$, then $\gamma(z|b) > 0$ iff $z = 0$; and

(iii) if $b \in (\beta(\bar{z}+), \beta(\bar{z}-))$ for $\bar{z} \in Z$, then $\gamma(z|b) > 0$ iff $z = \bar{z}$,

where for $\bar{z} = 0$ or 1, we let $\beta(0-) = \beta(0)$ and $\beta(1+) = \beta(1)$.

**Proof.** Let $z < z'$ and $b \notin \beta(Z)$. Define $\phi(b) = \frac{1-z(b+c)}{1-z'(b+c)}$ and observe that

$$\phi'(b) = \frac{z' - z}{[1 - z'(b + c)]^2} > 0.$$ 

Consider the following cases.

(1) When $b < \beta(z') < \beta(z)$, we have

$$\frac{\xi(z|b)}{\xi(z'|b)} = \frac{q(z)}{q(z')} \cdot \frac{1 - z (\beta(z) + c)}{1 - z' (\beta(z') + c)} \cdot \frac{1 - z'(b + c)}{1 - z (b + c)} \geq \frac{1 - z (\beta(z) + c)}{1 - z (\beta(z) + c)} \cdot \frac{1 - z' (\beta(z') + c)}{1 - z (b + c)} \cdot \frac{1 - z'(b + c)}{1 - z (b + c)} \tag{\because (i)}$$

$$= \frac{\phi (\beta(z'))}{\phi (b)} > 1.$$ 

(2) When $b < \beta(z') = \beta(z)$, we have $q(z) = q(z')$ and

$$\frac{\xi(z|b)}{\xi(z'|b)} = \frac{q(z) \cdot 1 - z (\beta(z) + c)}{q(z') \cdot 1 - z' (\beta(z') + c)} \cdot \frac{1 - z'(b + c)}{1 - z (b + c)}$$

$$= \frac{\phi (\beta(z'))}{\phi (b)} > 1.$$
(3) When \( \beta(z') < \beta(z) < b \), we have
\[
\frac{\xi(z|b)}{\xi(z'|b)} = \frac{q(z)}{q(z')} \cdot \frac{1 - z (\beta(z) + c)}{1 - z' (\beta(z') + c)} \cdot \frac{1 - z' (b + c)}{1 - z (b + c)} \\
\leq \frac{1 - z' (\beta(z') + c)}{1 - z' (\beta(z') + c)} \cdot \frac{1 - z (\beta(z) + c)}{1 - z' (\beta(z') + c)} \cdot \frac{1 - z' (b + c)}{1 - z (b + c)} \\
= \frac{\phi(\beta(z))}{\phi(b)} < 1.
\]

(4) When \( \beta(z') = \beta(z) < b \), we have \( q(z) = q(z') \) and
\[
\frac{\xi(z|b)}{\xi(z'|b)} = \frac{q(z)}{q(z')} \cdot \frac{1 - z (\beta(z) + c)}{1 - z' (\beta(z') + c)} \cdot \frac{1 - z' (b + c)}{1 - z (b + c)} \\
= \frac{\phi(\beta(z))}{\phi(b)} < 1.
\]

Since \( \gamma(z'|b) > 0 \) iff \( z' \in \arg \min_{z \in Z} \xi(z|b) \), the result (i) follows from (1) and (2), and (ii) from (3) and (4). To see (iii), note that \( \beta(z) \geq \beta(\tilde{z}^-) \) for all \( z < \tilde{z} \) and \( \beta(\tilde{z}+) \geq \beta(z) \) for all \( z > \tilde{z} \), which implies that (iii) follows from the similar arguments to the ones for (i) and (ii).

\[\square\]

**Step 2. Look at the behavior of \( \beta \).**

Since \( \beta \) is monotone, if it has discontinuities then they are jump discontinuities and the points of discontinuities are countable. If \( \beta \) has a jump discontinuity at \( z \) then either \( \beta(z) = \beta(z^-) \) or \( \beta(z) = \beta(z^+) \), since otherwise \( \beta(z) \in (\beta(z^+), \beta(z^-)) \) and the receiver identifies type \( z \) on signal \( \beta(z) \), but then Lemma 2 (iii) implies that type \( z \) deviates to any \( b \in (\beta(z^+), \beta(z)) \). Let \( J \) be the set of discontinuities of \( \beta \) and let \( B \) be the set of off-equilibrium noise levels.\[^{23}\] From the above argument,
\[
B = [0, \beta(1)) \cup (\beta(0), \infty) \cup \cup_{z \in J} (\beta(z^+), \beta(z^-)).
\]

Next, define \( T(\tilde{z}) = \{ z \in Z : \beta(z) = \beta(\tilde{z}) \} \) for each \( \tilde{z} \in Z \). We say that \( \beta \) is separating at \( \tilde{z} \) if \( |T(\tilde{z})| = 1 \) and pooling at \( \tilde{z} \) otherwise.\[^{24}\] Also, define \( z^m(\tilde{z}) = \int_{T(\tilde{z})} z \, dF(z) \) and note that if \( b = \beta(\tilde{z}) \) for some \( \tilde{z} \in Z \) then \( \tilde{q}(b) = 1 - z^m(\tilde{z})(b + c) \).

\[^{23}\]Note that \( J \) can be dense in \( Z \).

\[^{24}\]Note that if \( |T(\tilde{z})| > 1 \) then the monotonicity of \( \beta \) implies that \( T(\tilde{z}) \) is an interval.
Step 3. Prove that $\beta$ is separating everywhere and $\beta(1) = 0$.

First, it is easy to see that $\beta$ is separating at $z = 0$. Otherwise $z^m(0) > 0$, which implies that type $z = 0$ deviates to $b > \beta(0)$.

Now, by way of contradiction, suppose that $J \neq \emptyset$. Let $z'$ be an arbitrary point in $J$. If $\beta$ is pooling at $z'$, $\beta$ must have a jump at $\inf T(z')$ as otherwise $U^*$ cannot be continuous. Then, $z'$ deviates to $b = \beta(z') + \epsilon$ for small $\epsilon > 0$ since $\inf T(z') < z^m(z')$. It follows that $\beta$ is separating at $z'$. But this implies that $U^* (\beta(z' -)) < U^* (\beta(z' +))$ and contradicts the continuity of $U^*$. We thus conclude that $J = \emptyset$ and $\beta$ is continuous everywhere.

Recall that $\beta$ is separating at $z = 0$. If $\beta$ is pooling somewhere, then $\beta$ must change from separating to pooling at some point, say $z'$, at which $U^*$ cannot be continuous since $z^m(z') > z'$. Therefore, $\beta$ is separating everywhere.

Finally, note that $\beta(1) = 0$ as otherwise a deviation is possible for $z = 1$.

Step 4. Find a unique separating equilibrium from the necessary condition.

In a separating equilibrium, (1) implies that $q(z) = 1 - G[z (\beta(z) + c)]$. The incentive compatibility condition for $z \in Z$ is

$$z \in \arg\max_{\tilde{z} \in Z} [1 - G[\tilde{z} (\beta(\tilde{z}) + c)]] [1 - z (\beta(\tilde{z}) + c)].$$

As a monotonic function, $\beta$ is differentiable almost everywhere. The necessary first order condition at points of differentiability yields

$$\beta'(z) = \frac{-(\beta(z) + c) [1 - z (\beta(z) + c)] G'[\beta(z) + c]}{z [1 - z (\beta(z) + c)] G'[\beta(z) + c] + z [1 - G[\beta(z) + c]]}.$$

Let $\eta(z, \beta)$ denote the right hand side of (5) with $\beta(z)$ being replaced by $\beta$. We need to find a unique solution $\tilde{\beta}$ to the terminal value problem

$$\beta' = \eta(z, \beta), \ \beta(1) = 0.$$ 

Define $D = \{(z, \beta) : 0 < z \leq 1, 0 \leq \beta < \frac{1}{2} - c\}$. Since $\eta \in C(D)$ and $(1, 0) \in D$, there exists a local solution $\tilde{\beta}$ on $(z', 1]$ for some $z' \in (0, 1)$. Since $\eta(z, \beta)$ is Lipschitz continu-
ous in $\beta$ (as $\frac{\partial \eta}{\partial \beta} \in C(D)$), the uniqueness of solutions is guaranteed. It remains to show
that the local solution $\tilde{\beta}$ is continuable to $(0, 1]$, which requires Lemma 3.

**Lemma 3.** Let $\tilde{D}$ be a nonempty connected set in the $(z, \beta)$ domain and let $\eta$ be a bounded and continuous function on $\tilde{D}$. Suppose $\tilde{\beta}$ is a solution of $\beta' = \eta(z, \beta)$ on the interval $(\tilde{z}, 1]$. Then (i) the left-hand limit of $\tilde{\beta}$ at $\tilde{z}$, $\tilde{\beta}(\tilde{z} +)$ exists, and (ii) if $(\tilde{z}, \tilde{\beta}(\tilde{z} +)) \in \tilde{D}$ then the solution $\tilde{\beta}$ is continuable to the left past the point $z = \tilde{z}$.

If we can find $D_{\tilde{z}}$ for any $\tilde{z} \in (0, 1)$ such that $(\tilde{z}, \tilde{\beta}(\tilde{z} +)) \in D_{\tilde{z}}$, we can conclude that $\tilde{\beta}$ is continuable to $(0, 1]$. Let $D_{\tilde{z}} = \{ (z, \beta) : \tilde{z} \leq z \leq 1, 0 \leq \beta \leq \frac{1}{\tilde{z}} - c - \epsilon \} \subseteq D$ for some $\epsilon \in (0, 1 - c)$. For any $(z, \beta) \in D_{\tilde{z}}$, $z(\beta + c) < 1 - \tilde{z}\epsilon$ and $G[z(\beta + c)] < G[1 - \tilde{z}\epsilon]$.

Also $G'(z)$ is bounded away from 0. Thus $\eta(z, \beta)$ is bounded on $D_{\tilde{z}}$. Now I claim that $(\tilde{z}, \tilde{\beta}(\tilde{z} +)) \in D_{\tilde{z}}$ for any $\tilde{z} \in (0, 1)$. To see this, note that for any $(z, \beta(z)) \in D_{\tilde{z}} \subseteq D$,

$$|\beta'(z)| = \frac{\beta(z) + c}{z} \cdot \left[ 1 + \frac{1 - G[z(\beta(z) + c)]}{[1 - z(\beta(z) + c)]G'[z(\beta(z) + c)]} \right]^{-1} < \frac{1}{z^2}.$$  

Then we observe that for any $\tilde{z} \in (0, 1)$,

$$\tilde{\beta}(\tilde{z} +) = \tilde{\beta}(1) - \lim_{\tilde{z} \downarrow z} \int_{\tilde{z}}^{1} \tilde{\beta}'(z) \, dz$$

$$< 1 - c - \epsilon - \int_{\tilde{z}}^{1} \left( -\frac{1}{z^2} \right) \, dz$$

$$< 1 - c - \epsilon - \int_{\tilde{z}}^{1} \frac{d}{dz} \left( \frac{1}{z} - c \right) \, dz$$

$$= 1 - c - \epsilon.$$  

**Step 5.** Show that the necessary condition is sufficient.

We shall show that the strict incentive compatibility condition is satisfied for any $z \in Z$.\(^{26}\) Define $V(z, \tilde{z}, b) = [1 - G[\tilde{z}(b + c)]][1 - z(b + c)]$. We need to show that for any $z \in Z$,

$$\{ \beta(z) \} = \arg \max_{b \in \beta(Z)} V(z, \beta^{-1}(b), b). \quad (6)$$

\(^{24}\)This is the adjusted statement of Theorem 3.1 in Miller and Michel [21].

\(^{26}\)The technique found here is due to Mailath [20].
Note that (5) can be written as

$$\beta'(z) = -\frac{V_2(z, z, \beta(z))}{V_3(z, z, \beta(z))}. \tag{7}$$

Also, if $1 - z(b + c) > 0$, then for any $\tilde{z} \in Z$,

$$V_2(z, \tilde{z}, b) = -(b + c)G' [\tilde{z}(b + c)] [1 - z(b + c)] < 0. \tag{8}$$

Furthermore, for any $z, \tilde{z} \in Z$,

$$\frac{\partial}{\partial z} \left[ \frac{V_3(z, \tilde{z}, \beta(\tilde{z}))}{V_2(z, \tilde{z}, \beta(\tilde{z}))} \right] = \frac{1 - G [\tilde{z}(\beta(\tilde{z}) + c)]}{(\beta(\tilde{z}) + c)G' [\tilde{z}(\beta(\tilde{z}) + c)] [1 - z(\beta(\tilde{z}) + c)]} > 0. \tag{9}$$

By way of contradiction, suppose that there exists $z \in Z \setminus \{0\}$ such that (6) is not true. Let $b' \neq \infty$ be a maximizer of $V$ such that $b' \neq \beta(z)$. Note that $V_2(z, \tilde{z}, b') \neq 0$ for any $\tilde{z} \in Z$, since $1 - z(b' + c) > 0$ by Lemma 1.

(i) Suppose that $\beta^{-1}(b') = z' \in \text{int} Z$. By the first order condition for $z$ in (6),

$$V_2(z, z', b') \frac{d\beta^{-1}}{db} \bigg|_{b=\beta(z')} + V_3(z, z', b') = 0. \tag{7}$$

From (7),

$$V_2(z', z', b') \frac{d\beta^{-1}}{db} \bigg|_{b=\beta(z')} + V_3(z', z', b') = 0. \tag{8}$$

Combining these equations yields

$$\frac{V_3(z, z', b')}{V_2(z, z', b')} = \frac{V_3(z', z', b')}{V_2(z', z', b')},$$

which contradicts (9).

(ii) Suppose that $\beta^{-1}(b') = 1$. By the first order condition for $z$,

$$V_2(z, 1, b') \frac{d\beta^{-1}}{db} \bigg|_{b=\beta(1)} + V_3(z, 1, b') \leq 0. \tag{7}$$

Then, by (7),

$$V_2(z, 1, b') \left[ -\frac{V_3(1, 1, b')}{V_2(1, 1, b')} + \frac{V_3(z, 1, b')}{V_2(z, 1, b')} \right] \leq 0. \tag{8}$$

But (8) and (9) imply the opposite strict inequality, a contradiction.
A.4 Equilibrium Costs

For the total costs, use (5) to obtain

\[
\frac{\partial}{\partial z} [z(\beta(z) + c)] = \beta(z) + c + z\beta'(z)
\]

\[
= \frac{(\beta(z) + c) [1 - G[z(\beta(z) + c)]]}{[1 - z(\beta(z) + c)] G'[z(\beta(z) + c)] + [1 - G[z(\beta(z) + c)]]}.
\]

This value is positive everywhere and approaches \(\infty\) as \(z\) goes to 0. Since \(\beta(1) = 0\), the total costs are bounded by \(c\).

For the interpretive cost, use (5) to obtain

\[
\frac{\partial}{\partial z} [z\beta(z)] = \beta(z) + z\beta'(z)
\]

\[
= \frac{\beta(z) [1 - G[z(\beta(z) + c)]] - c [1 - z(\beta(z) + c)] G'[z(\beta(z) + c)]}{[1 - z(\beta(z) + c)] G'[z(\beta(z) + c)] + [1 - G[z(\beta(z) + c)]]}.
\]

This value is negative at close to \(z = 1\) and approaches \(\infty\) as \(z\) goes to 0.

A.5 Proof of Proposition 3

We observed in Appendix A.4 that \(z(\beta(z) + c) \leq c\) for all \(z \in Z\). Therefore, \(\beta(z) \leq \frac{c(1-z)}{z}\) for all \(z \in Z\). As \(c\) goes to 0, the term on the right vanishes everywhere and hence \(\beta(z; c)\) converges to zero.

A.6 Proof of Proposition 4

Our analysis goes through with obvious minor changes. The incentive compatibility condition for \(z \in Z\) is

\[
z \in \text{arg max}_{\tilde{z} \in Z} [1 - G[\tilde{z} (\tilde{\beta}(\tilde{z}) + c)]] [1 - \rho \tilde{z} (\tilde{\beta}(\tilde{z}) + c)],
\]

and the differential equation is

\[
\beta'(z) = \frac{- (\beta(z) + c) [1 - \rho z (\tilde{\beta}(\tilde{z}) + c)] G'[z(\tilde{\beta}(\tilde{z}) + c)]}{z [1 - \rho z (\tilde{\beta}(\tilde{z}) + c)] G'[z(\tilde{\beta}(\tilde{z}) + c)] + \rho \tilde{z} [1 - G[z(\tilde{\beta}(\tilde{z}) + c)]].
\]
In the limit $\rho = 0$, we have
\[
\beta'(z) = -\frac{\beta(z) + c}{z},
\]
from which we obtain
\[
\int_{z}^{1} \frac{\partial}{\partial z} \left[ z(\beta(z)) \right] \, dz = \int_{z}^{1} (-c) \, dz,
\]
and thus $\beta(z) = \frac{c(1-z)}{z}$, which is drawn in Figure $1$. In this limit, the total costs $z(\beta(z) + c)$ are $c$ for all $z \in Z$ and hence the action probability is $q(z) = 1 - G(c)$ for all $z \in Z$.

In the case of uniform distribution, we have
\[
\beta'(z) = -\frac{\beta(z) + c}{(1 + \rho)z},
\]
and the equilibrium noise is
\[
\beta(z) = \frac{cz \frac{1}{1 + \rho} (1 - z \frac{\rho}{1 + \rho})}{\rho}.
\]

A.7 Welfare Analysis

We assume that $F$ and $G$ are both uniform. In the separating equilibrium, given $\beta(z) = c(z^{-\frac{1}{2}} - 1)$, the receiver will take action if $v > z(\beta(z) + c) = cz^\frac{3}{2}$. Her ex-ante welfare $U^s_R$ is
\[
U^s_R = \int_{0}^{c} \int_{0}^{\left(\frac{c}{z}\right)^2} (v - cz^\frac{3}{2}) \, dz \, dv + \int_{c}^{1} \int_{0}^{1} (v - cz^\frac{3}{2}) \, dz \, dv
= \frac{1}{2} - \frac{2c}{3} + \frac{5c^2}{12} - \frac{c^5}{48}.
\]
In the pooling equilibrium, the receiver will take action if $v > E[z]c = \frac{c}{2}$. Her ex-ante welfare $U^p_R$ is
\[
U^p_R = \int_{\frac{c}{2}}^{1} \int_{0}^{1} (v - zc) \, dz \, dv
= \frac{1}{2} - \frac{c}{2} + \frac{c^2}{8}.
\]
Computation shows that $U^p_R > U^s_R$ if and only if $c \lesssim 0.6$.

If the receiver eliminates her outside option, her ex-ante welfare $U^n_R$ is

$$U^n_R = \int_0^1 \int_0^1 (v - zc) \, dz \, dv$$

$$= \frac{1}{2} - \frac{c}{2}.$$

Computation shows that $U^n_R > U^s_R$ if and only if $c \lesssim 0.4$.

### B Closed-form Solution

When $G(v) = v$, the differential equation [5] is

$$\beta'(z) = -\frac{\beta(z) + c}{2z}.$$

Rewrite it as

$$\beta'(z) + \frac{1}{2z} \beta(z) = -\frac{c}{2z}.$$

Multiply the integrating factor $e^{\int_1^z \frac{1}{2y} \, dy}$ throughout to obtain

$$\beta'(z)e^{\int_1^z \frac{1}{2y} \, dy} + \frac{1}{2z} \beta(z)e^{\int_1^z \frac{1}{2y} \, dy} = -\frac{c}{2z}e^{\int_1^z \frac{1}{2y} \, dy},$$

which simplifies due to the product rule to

$$\frac{d}{dz} \left[ \beta(z)e^{\int_1^z \frac{1}{2y} \, dy} \right] = -\frac{c}{2z}e^{\int_1^z \frac{1}{2y} \, dy}.$$

Integrate both sides over $[z, 1]$ to obtain

$$-\beta(z)e^{\int_1^z \frac{1}{2y} \, dy} = -\int_z^1 \frac{c}{2w}e^{\int_1^w \frac{1}{2y} \, dy} \, dw + l,$$

where $l$ is the constant of integration. If we note that $e^{\int_1^z \frac{1}{2y} \, dy} = z^{\frac{1}{2}}$, then easy algebra shows

$$\beta(z) = c(z^{-\frac{1}{2}} - 1) - lz^{-\frac{1}{2}}.$$

The boundary condition $\beta(1) = 0$ gives us $l = 0$, and the equilibrium noise is

$$\beta(z) = c(z^{-\frac{1}{2}} - 1).$$
References


