Miryre Meets Modigliani-Miller: Optimal Individual/Corporate Taxes and Capital Structure*

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Abstract

This paper shows why the corporate tax (as a form of double taxation) is crucial for achieving socially optimal allocations in the Mirrlees framework with an additional but realistic assumption that the tax authority cannot tax unrealized capital income at the individual level. Use of the corporate tax requires adjustment of the individual capital tax. This sophisticated tax system is designed to influence the individual agent’s portfolio choice of debt and equity, which in turn endogenizes the leverage ratio. The corporate tax is indeterminate, but there is a minimal level. The socially optimal allocation can be achieved by any corporate tax and properly adjusted individual income taxes. An immediate question is what happens to capital structure if we increase or decrease the level of the corporate tax. In this optimal tax code, surprisingly, the firm’s leverage ratio is independent of the corporate tax rate, unlike in classical capital structure theories. With respect to the labor income tax, without intertemporal resource transfer, if the tax system provides more (less) insurance against low skill shocks, then the leverage ratio increases (decreases).

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Introduction

Corporate taxation has been widely criticized for several reasons. First of all, the corporate income tax is one type of tax on capital income. It is well-known in conventional macroeconomics literature that capital income taxes should be zero immediately or at least in the long-run (Judd (1985), Chamley (1986), Jones, Manuelli, and Rossi (1997), etc).

Thus, the corporate tax should be avoided as well. Second, but more important, the corporate taxation is considered as double taxation: corporations are owned by individual investors who are already subject to capital income taxes. Although many countries have decreased the corporate tax rates during the last couple of decades, they are still significant. Weichenrieder (2005) reported that the average corporate tax rate was around 30% among the OECD countries in 2004.

This paper studies under what environment the corporate tax is essential in a Mirrlees economy. The introduction of the corporate tax requires proper adjustment of the individual capital tax. This sophisticated tax system is designed to influence the individual agent’s portfolio choice of debt and equity, which in turn endogenizes a firm’s capital structure as well. The corporate tax is generally indeterminate, but it must be greater than or equal to a positive minimal level. Thus, the tax authority can take the corporate tax rate flexibly, however, the other tax rates should be changed as well in accordance with the corporate tax rate. This co-movement property results in that the leverage ratio is independent of the change in the corporate tax rate, unlike to classical capital structure theories. Finally, we investigate the impact of labor tax on the leverage ratio, which is also new finding in this paper.

We first start by showing how a standard dynamic public tax system fails to achieve a socially optimal allocation in the third best world. The third best means the Mirrleesian world with one additional assumption that the tax authority cannot tax unrealized capital income in the individual level. Notice that U. S. households pay personal property tax if they hold vehicles, intangible assets (e.g., copyrights, patents, etc.), durable goods, and other assets. However, for example, capital gain taxes are not

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1There are a few exceptions: Conesa, Sagiri and Krueger (2009) argued that the optimal capital tax rate should be significantly positive in an overlapping generations model with idiosyncratic, uninsurable income shocks and borrowing constraints. Chen, Chen, and Wang (2010) also derived the similar conclusion in a human capital-based endogenous growth model with the frictional labor market. However, neither of them specified the role of corporate taxation.

2The typical assumption in the Mirrlees tax framework is that the skill of each agent is private information and stochastically move over time. See Section 1

3Broadly speaking, we represent the world without asymmetric information the first best and the common Mirrleesian taxation framework as the second best. See Section 3 for detail description.
paid unless assets are sold. Therefore, in reality some of unrealized capital income are taxed, but definitely not fully. Since we are interested in the assets that are being traded every second in the market, namely, debt and equity in this paper, we take the extreme, but realistic assumption that no tax is imposed on unrealized capital income, which means agents never pay individual taxes just by holding assets.

Under the above assumption, the agent, at each period, has the option of whether to realize his returns on investment. In order to understand the effect of a tax timing option, we should notice the regressiveness property of the capital taxation scheme in the dynamic public taxation models such as Kocherlakota (2005, 2009) and Albanesi and Sleet (2006). Let us describe the idea using a following simple example. Suppose that the economy has only two homogenous agents at time 0 and one becomes high skilled and the other becomes low skilled in the next period with some probability. According to Kocherlakota (2005), the low skill agent pays the capital income taxes while the high skill agent receive the capital subsidy. Then, the low skill agent does not want to realize the capital income or return at this period if possible. By not realizing the returns, he would not pay any capital income taxes. This deviation, in turn, devastates the socially optimal allocation. In order to remove this tax timing option of the low skill agent, the tax authority should set up an additional tax in a corporation level. In other words, they should put a tax on the corporate earnings, which lead to double taxation.

In the real practices, whether the tax authority can tax unrealized capital income, especially in an individual level, depends on how properly it can monitor all the asset transactions in the market among shareholders. The corporate taxation, in fact, is never required if the Internal Revenue Service (IRS) can easily keep track of shareholders of a corporation. The constrained optimum can be implemented simply by using an individual capital/labor income tax code (as in Kocherlakota (2005) or Albanesi and Sleet (2006)) without using an additional tax instrument such as the corporate tax. A real example is the existence of C corporations and S corporations in the US tax code: C corporations can have an unlimited number of shareholders, while S corporations are restricted to no more than 100 shareholders. C corporations can have non-US residents as shareholders, but S corporations cannot. Because S corporations have

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4In other words, the agent can evade taxes by deferring the realization. This idea may go back to Stiglitz (1973). Interested readers can refer to literature on tax timing options, for example, Constantinides (1983). The important contribution in this paper is to endogenize the optimal taxation as well as the optimal capital structure.

5The definition of corporate earnings is total output minus total wages and debt payments, which is what is left to equity holders.

6Other differences are as follows: S corporations cannot be owned by C corporations, other S corporations,
simple ownership structures which can be easily accessed by the tax authority, they are not required to face taxes at the corporate level. On the other hand, the owners of a C corporation are changing every second in the stock market, including even foreign investors who are out of the control of the IRS. Therefore, there is the role of corporate taxes for C corporations.

Perhaps the most important contribution of this paper is that the capital structure of the corporation is endogenously determined together with the optimal individual/corporate capital tax system. Only the introduction of the corporate tax is not sufficient to achieve the social optimum. Suppose the corporate tax, \( \tau_c \), is only designed so as to get rid of tax timing options of low skill agents. Then, similar to a common argument in the trade-off theory of capital structure, one might suspect that every agent chooses to hold corporate debt rather than equity just to avoid double taxation.\(^7\) This 100% debt financing choice also allows consumption of agents to deviate from the socially optimal allocation. However, we carefully design the individual capital tax in accordance with the corporate tax, which causes firms indifferent to any capital structure. This mechanism results in that each agent faces a portfolio selection problem between debt and equity. More precisely, the ex-post high skill agents will prefer to hold debt while the ex-post low skill agents will prefer to hold equity in the tax system. Thus, ex-ante, each agent should optimally choose the ratio of portfolios of debt and equity one-period ahead, which determines the aggregate leverage ratio in the economy.

An important property of the corporate tax is its indeterminacy. However, there is a minimal level. Any corporate tax rates greater than the minimal level can achieve the constrained allocation if the individual taxes are properly adjusted according to the level of the corporate tax. This flexibility of choosing corporate taxes also show that the historically fairly high corporate tax rates may not be puzzling. In addition, due to the existence of corporate taxes the aggregate capital tax is nonzero in this setting.\(^8\)

Due to its indeterminacy one can ask whether there is any impact of the change in the corporate tax on the leverage ratio. First notice that the leverage ratio is positively correlated with the level of corporate tax in conventional capital structure theories. However, in this optimal tax system, the change of the corporate tax need not

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\(^7\)We do not consider bankruptcy. Hence, there is no default risk on debt.

\(^8\)Notice that the aggregate capital tax (or the conditional expectation of the next period tax) is zero in Kocherlakota (2005).
change the leverage ratio. The corporate tax rates in U.S. have decreased constantly and significantly from around 50% in the 1950s to around 35% in the 2000s. According to the traditional capital structure theory, the leverage ratio should have significantly decreased as well. However, a stylized empirical fact on capital structure is that the aggregate market-based leverage ratio is fairly stationary during the last century with surprisingly small fluctuations (See Section 3.1 in Frank and Goyal (2007)). Our theory may build a potential bridge to understand this anomaly.

Finally we also investigate the impact of the labor tax on the leverage ratio. The correlation between the leverage and the labor tax code can be hardly considered in the literature of capital structure. In our tax mechanism, an agent at time 0 chooses between debt and equity for insurance motive against future skill shocks. Thus, how much subsidy (tax) an agent will receive (pay) for each state should affect the portfolio choice of each agent. By the comparative statics analysis, we show that if the tax system provides more (less) insurance against low skill shocks for the case of no period-by-period resource transfer, then the leverage ratio increases (decreases). Recall that ex-post low skill agents prefer equity to debt. Thus, more insurance against low skill shocks gives agents incentives to hold more debts. Similarly if the intertemporal resource transfer is allowed, the leverage ratio is positively correlated with the expected present value of labor subsidies conditional on being a low skill agent.

Literature Review

Capital Structure Theory Literature on capital structure is so large that we will not even try to summarize it. Roughly speaking there are two widely held views. One is the trade-off theory and the other is the pecking order theory. The main driving force determining the use of debt in the trade-off theory is the trade-off between tax benefits and bankruptcy costs. In the pecking order theory, information asymmetry provides the strict order of financing: due to adverse selection, internal funds are used first, debt is issued if internal funds are depleted, and equity is a last resort if it is not sensible to issue more debt. Each theory can explain many features of corporate financing. As mentioned before, however, neither of them are satisfactory in terms of the stylized empirical long run stability of the leverage ratio and the downward trend

\[ \text{The market-based leverage ratio is defined by } \frac{\text{debt}}{\text{debt} + \text{market value of equity}}. \]

\[ \text{Notice that not only the corporate tax but also the labor tax code are indeterminate. The indeterminacy of the labor tax is basically due to the Ricardian equivalence. See Bassetto and Kocherlakota (2004) and chapter 4 of Kocherlakota (2009).} \]
in the corporate tax rates.\footnote{11}

Notice that it is not an entirely new view to explain the capital structure in the general equilibrium context, in particular, using the difference between individual and corporate taxes. Miller (1977) first proposed the idea that the aggregate leverage ratio results from different individual tax rates among investors given the corporate taxes. DeAngelo and Masulis (1980) and Auerbach and King (1983) formalize more micro-founded models. They, in addition, find that the individual short-sale constraints are necessary for the existence of the equilibrium.\footnote{12} The Miller equilibrium, however, should be quite sensitive to the relative ratio of the corporate to the highest individual tax rates.\footnote{13} The investors are separated into two groups: Those agents whose individual tax rate is greater than the corporate tax rate should be specialized in equities and the others in debts.\footnote{14} Therefore, a change in corporate taxes, \textit{ceteris paribus}, should directly affect the leverage ratios. This is also counterfactual to the stability of the leverage ratio given the very large changes in the corporate tax rates during the last century.\footnote{15} Furthermore, the agents are not completely separated in either equity or debt in this paper.

\textbf{New Dynamic Public Taxation} One of notable progresses in recent taxation theory is so called \textit{the new dynamic public finance}, which developed the optimal tax system by extending the seminal work of Mirrlees (1971) to a dynamic setting.\footnote{16} The main

\footnote{11}The pecking order theory is empirically rejected since firms often issue equities in wrong times. The two most common critiques on the standard trade-off theory are that (i) measured bankruptcy costs are too small, and moreover (ii) firms use too little debt. Dynamic versions of the trade-off theory seem to successfully explain that the observed levels of debt are not surprising (See Fischer et al. (1989), Hennessy and Whited (2005), Goldstein et al (2001), etc). In this sense, the recent dynamic trade-off theory becomes more compelling although any judgement on the results is still tentative. However, the amount of bankruptcy costs is still questionable and the long-term stability of the leverage ratio is another concern. See Frank and Goyal (2007) for excellent empirical surveys.

\footnote{12}The short-shale constraints are not necessary in our model.

\footnote{13}On the other hand, Graham (2003) and McDonald (2006) point out that the Miller equilibrium in the 1970’s was plausible, when the highest personal tax rates exceeded the highest corporate rates, but, in the 1980’s, the relative tax rates for corporations increased, making the Miller equilibrium implausible.

\footnote{14}Miller (1977), DeAngelo and Masulis (1980), and Auerbach and King (1983) all predict that high income people (with high tax bracket) hold equity whereas low income people (with low tax bracket) hold debt.

\footnote{15}Even before these models appeared, Stiglitz (1973) stated “Empirical studies of the effects of taxation on corporate financial structure suggest that taxation has not had a very significant effect on corporate financial structure, let alone the dramatic change that one might have anticipated given the very large increases in the corporate tax rates in the last fifty years.”

assumption in this literature is that agents in the economy have private information about their skills, which evolve stochastically over time. Our paper follows this spirit and especially build on Kocherlakota (2005).

Other papers closely related to this one are Golosov and Tsyvinski (2007) and Albanesi (2006). Golosov and Tsyvinski (2007) study asset testing mechanisms in the disability insurance system in which a disability transfer is paid only if an agent has assets below a specified threshold. An asset test deters false claims by penalizing the strategy of oversaving and not working. This idea can be applied the mechanism where the high type agent should be prevented from oversaving in order to not work. However, in our model oversaving is not the essential problem. Whether the agent deviates does not directly hinge on the the amount of agent’s current wealth, but on the fact that he has chance to be a high type worker in the future. Albanesi (2006) considers the dynamic moral hazard problem of entrepreneurs facing idiosyncratic capital risk. She investigates differential asset taxation to implement the optimal allocation. She also shows that the double taxation is optimal if entrepreneurs sell the ownership of their firms and buy the ownership of other firms. The corporate tax in Albanesi (2006) is levied only on outside investors, but not on the entrepreneur who also has the ownership. The corporate tax, however, is the tax imposed on earnings of a firm. To our knowledge, our model is the closest one that explains the real world double taxation mechanism. More importantly, the capital structure and optimal tax system are endogenously determined in our paper.

The rest of the paper is organized as following. Section 1 introduces a simple model. We first pin down the constrained optimum of the planner’s problem in Section 2. In Section 3 we review how to decentralize the constrained optimum using the capital/labor tax system developed in Kocherlakota (2005). Then, we study how this result can be distorted if we assume that the firm do not have to pay the capital rent within the period. Section 4 explains why we need to consider the corporate tax and we show how to endogenize the capital structure as well as the optimal tax system. We describe Comparative statics results in the leverage ratio with respect to labor taxes in Section 5. Section 6 extends the model for more than 3 periods and explains the key properties of the corporate tax: (i) indeterminacy and (ii) no impact on the leverage. Section 7 considers other generalizations: (i) with more than three periods and and (ii) with (aggregate) uncertainty. Section 8 concludes.

and Werning (2008 a, b) and the references theirin.

17The disability shock in Golosov and Tsyvinski is an absorbing state; once the agent declares disability, he/she can never come back to work.
1 A Simple Model

Here we first consider a simple model. Later, we also extend the model to a general case. The fundamental idea, however, is the same as the simple model introduced here. Suppose there are ex-ante identical unit measure of agents living for three periods with
the following undiscounted utility function. Then,

\[ \sum_{t=0}^{2} [u(c_t) - v(y_t)], \]

where \( c_t \) is consumption and \( y_t \) is labor provided by the agent in time \( t \). In period 0, there is no uncertainty in types and all agents are homogeneous. In the beginning of each period, each agent privately learns his/her type. The agent has a high skill with probability \( \pi \) and a low skill with probability \( 1 - \pi \). This distribution is i.i.d. over time and across agents. If a high skill agent works, we get disutility \( v(y) \) from labor \( y \). We assume that the low skill agents cannot provide labor, i.e., \( y = 0 \). It is rather an extreme case: An agent is either able or disable at period 1 and 2. This is for simplicity, thus we only need to consider incentives for the high skill agents to work. Later we will extend the setup where there are more than two types and all types of agents can work in Section 6. The production technology is given by

\[ F(K, Y) = rK + wY, \]

where \( K \) is aggregate capital and \( Y \) is aggregate labor. Capital is depreciated at the rate \( \delta \) in each period and must be installed one-period ahead. Here without loss of generality we replace \( r + (1 - \delta) \) with \( r \). The initial capital endowment is \( K_0 \). Every agent is assumed to have the same initial endowment \( k_0 \), so that \( k_0 = K_0 \). Assume that there is no government spending required.

We have the following convention for notations. A small letter represents individual choice or allocation and a large letter represents an aggregate variable (a firm’s choice if there is a single firm). The superscript, \( \ast \), represents optimality, i.e., solutions to the planner’s problem. For example, \( k_t \ast \) is investment of an agent at \( t = 1, 2 \) and \( K_t \ast \) is the aggregate investment or capital raised by the representative firm. \( k_t \ast \) and \( K_t \ast \) are the optimal values of \( k_t \) and \( K_t \), respectively.

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\( ^{18} \)It is easy to generalize the model with many (possibly infinite) periods. But, there should be more than two periods. Without loss of generality we assume there are three periods.

\( ^{19} \)The i.i.d. assumption is for simplicity. All results are robust to the extension to a general stochastic environment rather than the i.i.d. case.

\( ^{20} \)The results are also preserved for a variety of constant returns to scale production functions.
2 Constrained Optimum

The planner’s problem is to choose 
\[ (c_0, c_h, c_l, c_{hh}, c_{hl}, c_{lh}, c_{ll}, y_0, y_h, y_{hh}, y_{lh}, K_1, K_2) \],

each component of which is nonnegative to maximize an expected life time payoff

\[
\max \ u(c_0) - v(y_0) + \pi (u(c_h) - v(y_h)) + (1 - \pi)u(c_l) \\
+ \pi^2 (u(c_{hh}) - v(y_{hh})) + \pi(1 - \pi)u(c_{hd}) + \pi(1 - \pi)(u(c_{hh}) - v(y_{hh})) + (1 - \pi)^2u(c_{lh})
\]

subject to the resource constraints

\[
c_0 + K_1 = rK_0 + wy_0, \\
\pi c_h + (1 - \pi)c_l + K_2 = rK_1 + w\pi y_h, \\
\pi^2 c_{hh} + \pi(1 - \pi)c_{hl} + \pi(1 - \pi)c_{lh} + (1 - \pi)^2c_{ll} \\
= rK_2 + w\left(\pi^2 y_{hh} + \pi(1 - \pi)y_{lh}\right),
\]

and the incentive constraints

\[
u(c_{hh}) - v(y_{hh}) \geq u(c_{hl}), \quad (2.1) \\
u(c_{lh}) - v(y_{lh}) \geq u(c_{ll}), \quad (2.2) \\
u(c_h) - v(y_h) \geq u(c_l), \quad (2.3)
\]

Note \( c := \{c_0, c_h, c_l, c_{hh}, c_{hl}, c_{lh}, c_{ll}\} \) is the consumption plan of an agent at time 0, working in period 1, non-working in period 1, working in both periods 1 and 2, working in period 1 and non-working in period 2, non-working in period 1 and working in period 2, and non-working in both periods 1 and 2, respectively. \( y := \{y_0, y_h, y_{hh}, y_{lh}\} \) is the amount of labor provided by corresponding agents. Note that the disables at each period never work, i.e., \( y_l = y_{ll} = y_{hl} = 0 \).

First notice the low type agents cannot work, so that they do not lie. Only high types can pretend to be low types. Then, in fact there should be 5 incentive constraints even if (2.1), (2.2), and (2.3) are sufficient to summarize all the truthful telling constraints. (2.1) is the truth-telling constraint for the high skill agents in period 2. (2.2) is the truth-telling constraint for the low skill agent in period 2. In period 1, we should
have considered the following three truth-telling constraints.

\[ u(c_h) - v(y_h) + \pi(u(c_{hh}) - v(y_{hh}) + (1 - \pi)u(c_{hl}) \]
\[ \geq u(c_l) + \pi(u(c_{hh}) - v(y_{hh}) + (1 - \pi)u(c_{hl})) \] (2.4)
\[ u(c_h) - v(y_h) + \pi(u(c_{hh}) - v(y_{hh}) + (1 - \pi)u(c_{hl}) \]
\[ \geq u(c_h) - v(y_h) + \pi u(c_{hl}) + (1 - \pi)u(c_{hl}) \] (2.5)
\[ u(c_h) - v(y_h) + \pi(u(c_{hh}) - v(y_{hh}) + (1 - \pi)u(c_{hl}) \]
\[ \geq u(c_l) + \pi u(c_{hl}) + (1 - \pi)u(c_{hl}) \] (2.6)

(2.4) means that the payoff from lying only in period 1 is not better than the truth-telling payoff. (2.5) means that the payoff from lying only in period 2 is not better than the truth-telling payoff. Finally, (2.5) means that the payoff from lying in both period 1 and 2 is not better than the truth-telling payoff. Then, (2.4) is reduced to (2.3). By simple algebra, it is easy to see that (2.5) and (2.6) are redundant by using (2.1), (2.2), and (2.3).

Let \( (c^*, y^*, K^*) := \{c_0^*, c_h^*, c_l^*, c_{hh}^*, c_{hl}^*, c_{lh}^*, c_{ll}^*\}, \{y_0^*, y_h^*, y_{hh}^*, y_{lh}^*, y_{ll}^*\}, \{K_1^*, K_2^*\}\) be the constrained optimum. Then, it is easy to see from the first order necessary conditions that the constrained optimum satisfies

\[
\begin{align*}
    u'(c_0^*) &= \frac{r}{u'(c_h^*) + u'(c_l^*)}, \\
    u'(c_h^*) &= \frac{r}{u'(c_{hh}^*) + u'(c_{hl}^*)}, \\
    u'(c_l^*) &= \frac{r}{u'(c_{lh}^*) + u'(c_{ll}^*)}, \\
    v'(y_0^*) &= wu'(c_0^*), \quad v'(y_h^*) = wu'(c_h^*) \\
    v'(y_{hh}^*) &= wu'(c_{hh}^*), \quad v'(y_{lh}^*) = wu'(c_{lh}^*) \\
\end{align*}
\]

\[
\begin{align*}
    c_0^* + K_1^* &= rK_0 + wy_0^* \\
    \pi c_h^* + (1 - \pi)c_l^* + K_2^* &= rK_0 + w\pi y_h^* \\
    \pi^2 c_{hh}^* + \pi(1 - \pi)c_{hl}^* + \pi(1 - \pi)c_{lh}^* + (1 - \pi)^2 c_{ll}^* &= rK_2^* + w(\pi^2 y_{hh}^* + \pi(1 - \pi)y_{lh}^*)
\end{align*}
\] (2.8)

and

\[
\begin{align*}
    u(c_h^*) - v(y_h^*) &= u(c_l^*) \\
    u(c_{hh}^*) - v(y_{hh}^*) &= u(c_{hl}^*) \\
    u(c_{lh}^*) - v(y_{lh}^*) &= u(c_{ll}^*)
\end{align*}
\] (2.9)

The above conditions are also sufficient since the solution is in the interior and unique. First notice that it is easy to show that all three incentive constraints (2.1), (2.2), and
(2.3) are binding, which results in (2.9). For example, suppose \( u(c_h) - v(y_h) > u(c_l) \). Then, by the concavity of \( u \), the welfare goes up by increasing \( c_l \) a little bit and decreasing \( c_h \) a little bit without violating the resource constraint. The same argument applies to the second and the third equality.

The first three equations in (2.7) are so called the inverse Euler equations. Golosov, Kocherlakota, and Tsyvinski (2003) first pinned down the intertemporal wedge in a Pareto optimum between an individual's marginal benefit of investing in capital and his marginal cost of doing so, which suggests the positive tax on capital income. Since then and contemporaneously, several optimal taxation mechanisms have been developed. Among them, Kocherlakota (2005) first proposed how to implement a market economy that is closest to the classical workhorse dynamic general equilibrium models. He shows that the constrained optimum cannot be decentralized by simply imposing homogenous capital income equal to the (ex-ante) wedge. Instead he proposed capital income taxes equal to the \textit{ex-post} wedge, which makes agents with different skills face different capital tax rates. The optimal capital income tax is zero in aggregate (or in the ex-ante expectation sense), but nonzero for individuals (in the ex-post sense). For example, people who are relatively low skilled in the next period pay a wealth tax; people who are relatively high skilled receive a wealth subsidy.

Before going further, we introduce the following lemma that will be used several times later to pin down size of optimal capital taxes.

\textbf{Lemma 1.} \textit{The optimal allocation satisfies}

\[ u'(c_0^*) < ru'(c_l^*). \]

\textit{Proof.} See the Appendix.

Lemma 1 still holds for a general case where there are many types of agents: When there are more than two types of agents, \( l \) should mean the lowest skill agents. The following corollary of Lemma 1 is also used later.

\textbf{Corollary 1.} \textit{The optimal allocation satisfies}

\[ u'(c_0^*) > ru'(c_h^*). \]

\textit{Proof.} See the Appendix.

\section{The First and The Second Best World}

Two decentralization methods are examined in this section. In Section 3.1 we briefly introduce the first best (or Ramsey) taxation scheme and explain briefly why it does not
work when there is private information (in the Mirrlees world or the second best world). Section 3.2 describes the second best taxation scheme as in Kocherlakota (2005). Then, in Section 3.3 we explain why this second best taxation method also fails to decentralize the constrained optimal allocation. In particular, this section explicitly describes the assumption of this paper and presents the intuition of how to use the tax timing option.

We refer the first best world as the economy without information asymmetry, and the second best world as the standard Mirrlees world as in Kocherlakota (2005). The third best world means the Mirrlees framework with an additional assumption that the tax authority cannot impose tax on unrealized returns in an individual level. In this manner, in Section 3.1 we define the first best taxation scheme by the tax system including the capital income tax that matches the wedge in the (ex-ante) Euler equation. The next period capital income tax rate should be contingent on the information available at the current period. Next, in Section 3.2 we define the second best taxation scheme by the tax system including the capital income tax that matches the wedge in the ex-post Euler equation. The next period capital income tax rates should be contingent on the full labor history including the next period (not even the current period).

Suppose there is a single firm that owns the technology. The firm rents capital and labor in each period to produce output. In period 0 and 1, the household decides how much to consume and work and how much capital to save (or accumulate). In period 2, agents decide how much to consume and work.

3.1 The First Best Taxation Scheme

First consider a tax system \( \{ \tau, \alpha_h, \alpha_l \} \) in period 1 where \( \tau \) is a capital tax rate and \((\alpha_h, \alpha_l)\) are lump-sum taxes on the labor income of working/non-working agents. The key point here is that the capital tax rate imposed on all types of agents are the same. In particular, let us to set up \( \tau \) such that

\[
 u'(c^*_0) = E[r(1-\tau)u(c^*_1)] = \pi r(1-\tau)u'(c^*_h) + (1-\pi) r(1-\tau)u'(c^*_l).
\] (3.1)

This tax system works if there is no information asymmetry (in a Ramsey taxation world). With private information it fails to achieve the constrained optimum allocation. In particular, it fail to satisfy the incentive constraint of the high skill agent. The high skill agent will deviate by oversaving and pretending to be low skilled (See the two-period example in Kocherlakota (2005)).
3.2 The Second Best Tax Scheme

Secondly, we consider a tax system \( \{\tau_i, \alpha_i\}_{i=l,h} \) for period 1 and \( \{\tau_{ij}, \alpha_{ij}\}_{i,j=h,l} \) for period 2 proposed by Kocherlakota (2005). Note that \( l \) means that the agent does not work and \( h \) means that the agent works. For example, \( \tau_h \) is the (period 1) capital tax on the agent who works in period 1, \( \alpha_{lh} \) is the (period 2) labor income tax on the agent who does not work in period 1 and works in period 2. Notice that the tax mechanism has the full labor-history dependence up to the period when the corresponding capital tax is imposed. In essence, differentiating the tax rates on capital is required to achieve a constrained optimal allocation.

Given the tax plan \( \{\tau_i, \alpha_i\}_{i=l,h} \) and \( \{\tau_{ij}, \alpha_{ij}\}_{i,j=1,2} \), an agent’s problem is to choose consumption \( (c_0, c_h, c_l, c_{hh}, c_{hl}, c_{lh}, c_{ll}) \), labor \( (y_0, y_h, y_{hh}, y_{lh}) \), and investment \( (k_1, k_{2h}, k_{2l}) \) to maximize

\[
    u(c_0) - v(y_0) + \pi (u(c_h) - v(y_h)) + (1 - \pi)u(c_l) \\
    + \pi^2 (u(c_{hh}) - v(y_{hh})) + \pi (1 - \pi)u(c_{hl}) + \pi (1 - \pi) (u(c_{lh}) - v(y_{lh})) + (1 - \pi)^2u(c_{ll})
\]

subject to the following budget constraints. The constraint in \( t = 0 \) is

\[ c_0 = r k_0 - k_1 + w y_0, \]

the constraint in \( t = 1 \) is

\[ c_h = r (1 - \tau_h) k_1 - k_{2h} + w y_h + \alpha_h, \quad \text{if } y_h > 0 \]
\[ c_l = r (1 - \tau_l) k_1 - k_{2l} + \alpha_l, \quad \text{otherwise,} \]

the constraint in \( t = 2 \) when the agent works in \( t = 1 \) is

\[ c_{hh} = r (1 - \tau_{hh}) k_{2h} + w y_{hh} + \alpha_{hh}, \quad \text{if } y_{hh} > 0 \]
\[ c_{hl} = r (1 - \tau_{hl}) k_{2h} + \alpha_{hl}, \quad \text{otherwise,} \]

and finally the constraint in \( t = 2 \) when the agent does not work in \( t = 1 \) is

\[ c_{lh} = r (1 - \tau_{lh}) k_{2l} + w y_{lh} + \alpha_{lh}, \quad \text{if } y_{lh} > 0 \]
\[ c_{ll} = r (1 - \tau_{ll}) k_{2l} + \alpha_{ll}, \quad \text{otherwise.} \]

Notice that positive \( \alpha \)'s represent subsidy and negative \( \alpha \)'s represent tax while positive \( \tau \)'s represent tax and negative \( \tau \)'s represent subsidy. The market clearing conditions
are given by

\[
\begin{align*}
(t = 0) \quad & c_0 + k_1 = r k_0 + w y_0, \\
(t = 1) \quad & \pi c_h + (1 - \pi) c_l + \pi k_{2h} + (1 - \pi) k_{2l} = r k_1 + w \pi y_h, \\
(t = 2) \quad & \pi^2 c_{hh} + \pi (1 - \pi) c_{hl} + \pi (1 - \pi) c_{lh} + (1 - \pi)^2 c_{ll} \\
& \quad = r \pi k_{2h} + (1 - \pi) k_{2l} + w (\pi^2 y_{hh} + \pi (1 - \pi) y_{hl}).
\end{align*}
\]

Suppose the government does not period-by-period transfer resources, i.e., the government does not issue bonds. Then, the budget constraint of an agent and the market clearing condition imply the following government budget constraint in each period.

\[
\begin{align*}
(t = 1) \quad & [\pi \tau_h + (1 - \pi) \tau_l] r k_1 = \pi \alpha_h + (1 - \pi) \alpha_l, \quad (3.2) \\
(t = 2) \quad & \pi [\pi \tau_{hh} + (1 - \pi) \tau_{hl}] k_{2h} + (1 - \pi) [\pi \tau_{lh} + (1 - \pi) \tau_{ll}] r k_{2l} \\
& \quad = \pi \alpha_{hh} + \pi (1 - \pi) \alpha_{hl} + (1 - \pi) \pi \alpha_{lh} + (1 - \pi)^2 \alpha_{ll}. \quad (3.3)
\end{align*}
\]

If we enable the government to finance their budget through government bonds, then the labor income tax should be indeterminate.\textsuperscript{21} In this section we keep (3.2) and (3.3) for simplicity. However, from the next section on we will see the case where the government does issue bonds or does period-by-period transfer resources.

In order to achieve the constrained optimal competitive allocation, any tax system must be consistent with the ex-post Euler equation (not ex-ante Euler equation). Given the constrained optimum allocation \((c_0^*, c_h^*, c_l^*, c_{hh}^*, c_{hl}^*, c_{lh}^*, c_{ll}^*), (y_0^*, y_h^*, y_{hh}^*, y_{hl}^*),\) and \((k_1^*, k_{2h}^*, k_{2l}^*),\) we require the capital tax system \(\{\tau_h, \tau_l\}\) and \(\{\tau_{hh}, \tau_{hl}, \tau_{lh}, \tau_{ll}\}\) to be defined so that the ex-post Euler equation is satisfied with equality at each period and require the labor tax system \(\{\alpha_h, \alpha_l\}\) and \(\{\alpha_{hh}, \alpha_{hl}, \alpha_{lh}, \alpha_{ll}\}\) to satisfy the budget

\textsuperscript{21}Interested readers can see the arguments in Section 4.4.3 in Kocherlakota (2009).
constraint as follows.

\[
\begin{align*}
    r(1 - \tau_h)u'(c_h^*) &= u'(c_h^*), \quad r(1 - \tau_l)u'(c_l^*) = u'(c_l^*) \\
    \alpha_h &= c_h^* + k_{2h}^* - r(1 - \tau_h)k_1^* - wy_h^* \\
    \alpha_l &= c_l^* + k_{2l}^* - r(1 - \tau_l)k_1^* \\
\end{align*}
\]

(3.4)

with \( \pi k_{2h}^* + (1 - \pi)k_{2l}^* = K_2^* \), and

\[
\begin{align*}
    r(1 - \tau_{hh})u'(c_{hh}^*) &= u'(c_{hh}^*), \quad r(1 - \tau_{hl})u'(c_{hl}^*) = u'(c_{hl}^*) \\
    r(1 - \tau_{lh})u'(c_{lh}^*) &= u'(c_{lh}^*), \quad r(1 - \tau_{ll})u'(c_{ll}^*) = u'(c_{ll}^*) \\
    \alpha_{hh} &= c_{hh}^* - r(1 - \tau_{hh})k_{2h}^* - wy_{hh}^* \\
    \alpha_{hl} &= c_{hl}^* - r(1 - \tau_{hl})k_{2h}^* \\
    \alpha_{lh} &= c_{lh}^* - r(1 - \tau_{lh})k_{2l}^* - wy_{lh}^* \\
    \alpha_{ll} &= c_{ll}^* - r(1 - \tau_{ll})k_{2l}^* \\
\end{align*}
\]

(3.5)

Then, it is not hard to see that the agent’s optimal choice \((c, y, k)\) is equal to the constrained optimum, i.e., \((c_0, c_h, c_l, c_{hh}, c_{hl}, c_{lh}, c_{ll}) = (c_0^*, c_h^*, c_l^*, c_{hh}^*, c_{hl}^*, c_{lh}^*, c_{ll}^*)\), \((y_0, y_h, y_{hh}, y_{lh}) = (y_0^*, y_h^*, y_{hh}^*, y_{lh}^*)\), and \(k_1 = K_1^*\), conditional on \((y_h, y_{hh}, y_{lh}) >> 0\).

Then, we have the following proposition.

**Proposition 1** (Kocherlakota (2005)). The competitive equilibrium is the constrained optimum allocation if the tax system satisfies (3.4) and (3.5).

**Proof.** See the appendix.

Let us summarize the properties of the tax system of (3.4) and (3.5) in the following proposition.

**Proposition 2.** The tax system defined in (3.4) and (3.5) also satisfies \(E_t[\tau_{t+1}] = 0\), in other words,

(a) \(\pi\tau_h + (1 - \pi)\tau_l = 0\) and \(\tau_h < 0 < \tau_l\).

(b) \(\pi\tau_{hh} + (1 - \pi)\tau_{hl} = 0 = \pi\tau_{lh} + (1 - \pi)\tau_{ll}\). Furthermore, \(\tau_{hh} < 0 < \tau_{hl}\) and \(\tau_{lh} < 0 < \tau_{ll}\).

Proposition 2 implies that the expected ex-ante capital tax is zero although the ex-post capital tax is never zero. Notice that the expected labor income tax is not necessarily zero.\(^{22}\) It is zero under the assumption that there is no intertemporal transfer of consumption by the planner, i.e., the government does not generate any debt repaid in the future. The working agents pay the labor income tax and the

\(^{22}\)If we assume that government never creates any bonds, we have \(\pi\alpha_h + (1 - \pi)\alpha_l = 0\) with \(\alpha_h < 0 < \alpha_l\) and \(\pi^2\alpha_{hh} + \pi(1 - \pi)\alpha_{hl} + (1 - \pi)\pi\alpha_{lh} + (1 - \pi)^2\alpha_{ll} = 0\).
disable agents receive labor subsidy, which means the government insures the agents against the skill shocks. However, in order to give able agents incentives to work, the government should award the working agents the capital income (or wealth) subsidy instead of making them pay the labor income taxes for the disabled.

3.3 Tax Timing Options

The analysis in Section 3.2 is based on a standard framework of the dynamic Mirrless taxation models. In fact, the main contribution of this paper start from here. From this section on, we add real world features of the tax code into the model as in assumption 1 below. With this assumption, an agent is entitled with so called a tax timing option that is the option to realize capital incomes in each period. Then, as will be shown later, the decentralization method in the previous section fails to achieve the constrained optimum allocation. Notice that we are not criticizing dynamic Mirrlees taxation models by citing practical problems. The idea of tax timing options can also be applied to break down any dynamic Ramsey models as well. What we want to focus on is how to correct this failure in the Mirrlees framework, which eventually justifies corporate taxation. Now we introduce the following simplifying assumptions.

Assumption 1.  
(i) The government cannot impose tax on any unrealized capital returns of individual agents. 

(ii) In period 1, an agent can resell equities to the corporation (or firms repurchase equities from the shareholders.) When they do so, they must pay the individual capital taxes. Otherwise, they do not pay the taxes just by holding equities. 

(iii) There is no long-term debt, in other words, only one-period bonds are available in the market. Debt issued in period \( i \) must be paid in period \( i + 1, i = 0, 1 \). Then, the individual taxes are imposed as well.

The most important is Assumption 1-(i). As mentioned before in the introduction, in the real world, people annually need to pay taxes on some capital holding regardless of the capital gain realization, for example, real estates, vehicles, intangible assets (copyrights, patents, etc) and durable goods. However, these assets are not traded often and here we are interested in stocks and bonds that are being traded every second in the market. In addition, it is factual that the capital gain taxes are paid when stocks and bonds are sold.\(^{23}\) Therefore, we take assumption 1-(i).

\(^{23}\)The capital gain taxes are asymmetric. There are tax credits for capital loss. In this paper, we do not consider the tax credits. This is for simplicity. In fact, since the model has no uncertainty in production, we do not have to take capital loss into account.
Assumption 1-(ii) implies that dividend distribution and share repurchase are identical. Practically, dividend payout usually has tax disadvantages relative to share repurchase. In particular, in the current U. S. tax code, the effective tax rates on dividends are slightly higher than those on share repurchases. Then, an immediate question is why firms are distributing dividends. However, those topics related to this dividend puzzle are beyond the scope of this paper (See Black (1976) and Miller (1986) for the dividend puzzle). Therefore, for simplicity, we assume that the share repurchasing is equivalent to dividends distribution. It also means that the agents realize capital gains by receiving cash in exchange for all or some fraction of the firm’s outstanding equity that they hold or by selling all or some equity to any individual agent or the firm. In addition, we assume that there is no floatation cost and no friction in issuing equity and debt.

Assumption 1-(iii) identifies the difference between debt and equity. There are two differences. First, debt is corporate tax-free while equity is not. Second, debt has a maturity so that it should be paid on a specified time while equity does not. Therefore, we assume for simplicity there is only one-period debt.

Now the intuition of the tax timing option is as follows. Although we have a three-period model, the model can be easily extended to a general case. Therefore, let us imagine that there are many periods and individual skills are arbitrarily evolving (potentially very persistently). Suppose the tax system is given by equations (3.4) and (3.5). If an agent sees that the capital income tax is high enough at the current period, then she can postpone realization of her capital income to the next period. In this case, the unrealized returns are left in the firm, which is automatically transferred to reinvestment without taxes under assumption 1. In particular, the agent who has surprisingly low skill in the current period, therefore is facing positive capital taxes, will have the incentive to defer her capital income realization in order to evade the taxes. If she realizes her capital income at the time she becomes (surprisingly) high skilled in some periods later, she can receive even more subsidy proportional to the wealth accumulated without having paid taxes than what she would get if she realized her capital income earlier. In particular, currently low skill agents choose to exercise the tax timing option whereas the currently high type agents do not. Therefore, tax timing options provide typical arbitrage opportunities. Now we are ready to show the following proposition which is the starting point for the whole analysis in the remaining part of the paper.

---

24 These unpaid retained earnings are sometimes called internal equities in the capital structure literature. Then, common stocks traded in the market are called external equities.
Proposition 3. Suppose Assumption 1 holds. Then, the socially optimal allocation cannot be implemented by the tax system \( \{\tau_i, \alpha_i\}_{i=1,h} \) and \( \{\tau_{ij}, \alpha_{ij}\}_{i,j=h,l} \) in (3.4) and (3.5).

Proof. See the Appendix.

We have two remarks on Proposition 3. First, we focus only on the behavior of the low skill agents in period 1. The high skill agents already do not have incentives to deviate under the the second best world tax scheme. Second, although in the second best world we only investigated the case where there is no intertemporal transfer of resources, one should notice that, in general, the labor taxation is indeterminate. Therefore, the agent’s investment (or saving) strategy depends on how much labor taxes will be assigned in period 1, in particular, how big \( (\alpha_h, \alpha_l) \) in (3.4) are. Proposition 3 is true for any labor tax system, in other words, it is valid regardless of whether the government period-by-period transfers resources.

Now, using the argument in Section 3.1 and the argument in proposition 3, we can establish the following corollary.

Corollary 2. Suppose Assumption 1 holds. The constrained optimum cannot be decentralized by any tax systems using the capital income tax defined (i) to be equal to the ex-post wedge of the intertemporal Euler equation or (ii) to be equal to the ex-ante wedge in the intertemporal Euler equation.

Corollary 2 gives a hint of how to design a optimal tax scheme in order to avoid the tax timing option. If the market would fail to achieve the optimal allocation by using only one of (i) and (ii) in the corollary, then one can think of a proper mixture of them as a solution. The next section shows an alternative way.

4 The Third Best World: Optimal Debt/Equity and Individual/Corporate Capital Income Tax

How does the government prevent agents from this deviation as in the proof of Proposition 3? For logical simplicity, we consider the following two cases step by step: (1) when firms do not issue debts and (2) when firms issue both equities and debts. In conclusion, the government should be required to tax unrealized returns or earnings in the firm level (as well as in the individual level), which is so called corporate taxation.
4.1 When No Debt, But Only Equity is Available

Assume firms are not allowed to issue debts. Then, corporate earnings in this case is equal to output minus labor shares. If the government sets any taxes in the corporate level, then it makes all the agents pay capital income taxes although they do not realize their capital income. In particular, if this tax is set to be same as \( \tau_l \) in (3.4), then the low skill agents cannot defer to pay the capital income taxes to the next period, which means that they lose their tax timing options. More precisely, consider the following tax system \((\tau^e_c, \tau^e_l, \tau^e_h)\) in period 1 where \(\tau^e_c\) is the corporate tax rate, \(\tau^e_l\) is the individual capital tax rate for non-working agents in period 1, and \(\tau^e_h\) is the individual capital tax rate for working agents in period 1 such that

\[
\tau^e_c := \tau_l, \quad \tau^e_l := 0, \quad \tau^e_h := \tau_h - \tau^e_c. \tag{4.1}
\]

where \(\tau_l\) and \(\tau_h\) are defined by (3.4). Notice that the low skill agents are now indifferent between realizing the return on capital investment and non-realizing. The high skill agents should pay the corporate tax \(\tau^e_c\), but they can get back tax benefits \(\tau_h - \tau_l\) when they realized their capital income. Therefore, the net capital income is \([{(1 - \tau_h + \tau_l) - \tau_l}]rk^*_1 = (1 - \tau_h)rk^*_1\), which is the same as that under the previous tax system (3.4) and (3.5).

4.2 When Both Debt and Equity are Available

Notice the tax system (4.1) is the optimal tax only if debt is unavailable. If debt is available and the individual capital taxes are given by \((\tau^e_l, \tau^e_h)\), then the agents in period 0 have no reason to buy equity since there is a positive corporate tax \(\tau_c > 0\). Then, corporations raise 100% debt financing since we do not assume bankruptcy costs. Therefore, the optimal allocation cannot be obtained under (4.1).

Now suppose both debt and equity are available in the market. We need to introduce more precise individual taxes as well as the corporate tax. Let us define \(\tau^*_c\) by the corporate tax rate and \((\tau^*_l, \tau^*_h)\) by the individual capital income taxes of non-working \((l)\) and working \((h)\) agents, respectively. Superscript \(B\) represents debt and \(E\) represents equity. Then, we formalize the problem as follows: find the optimal tax system \((\tau^*_c, \tau^*_l, \tau^*_h)\) such that given the agents tell the truth, the tax system guarantees that the agents choose the socially optimal allocations and given the agents optimally chooses their allocation, the agents choose to tell the truth.

Let us describe the idea of taxation as follows. Above all, unlike (4.1), we impose positive individual capital taxes on both equity and debt holdings of the low skill agents and, in particular, we set the tax rate on the debt holding of the low type
agents greater than the corporate tax rate. It follows that the individual capital tax rates for the high skill agents should be adjusted to fit the Euler equation. Similarly to the above subsection, the tax rate on equity of the working agents should be negative. Then, the above idea is mathematically summarized as the following criterion.

\[ 0 < \tau^* < \tau_B^E \quad \text{and} \quad \tau_h^E < \tau_h < 0. \]  

(4.2)

In fact, we need more constraints, but they are rather less important than (4.2). They will be specified in the below. This minor importance is due to the fact that if we set \( \tau_B^E = \tau_E^B \) and \( \tau_h^B = \tau_h^E \), then the other criteria will trivially hold.

Let us consider in the ex-post sense who prefer debt and who prefer equity under (4.2). The high skill agents would be happier if they find themselves have more bonds. The low skill agents would be happier if they find themselves have more stocks. In other words, the high types prefer to be "debt holders" while the low types prefer to be "equity holders" in ex-post. Therefore, in the ex-ante sense, in period 0, the risk-averse agents are facing a non-trivial portfolio selection problem between equities and bonds given the tax system.

Notice that no corporate tax is required in period 2 since all the firms are liquidated in period 2. Therefore, \( \{ \tau_{ij} \}_{i,j=h,l} \) of (3.5) is still the optimal capital income tax in period 2. Define \( B_1 \) and \( E_1 \) by the amount of debt holdings and equity holdings, respectively. Then, given the tax system \( \{ \tau^*_c, \tau_B^E, \tau_B^B, \tau_h^B, \tau_h^E \} \), the agent’s budget constraint in each period is as follows. In period 2, we have the same constraints as in the second-best case:

\[ c_{hh} = r(1 - \tau_{hh})k_{2h} + wy_{hh} + \alpha_{hh}, \quad \text{if} \ y_{hh} > 0 \]  

(4.3)

\[ c_{hl} = r(1 - \tau_{hl})k_{2h} + \alpha_{hl}, \quad \text{otherwise} \]  

(4.4)

and

\[ c_{lh} = r(1 - \tau_{lh})k_{2l} + wy_{lh} + \alpha_{lh}, \quad \text{if} \ y_{lh} > 0 \]  

(4.5)

\[ c_{ll} = r(1 - \tau_{ll})k_{2l} + \alpha_{ll}, \quad \text{otherwise} \]  

(4.6)

In period 1, however, we have

\[ c_{h} = r(1 - \tau_h^B)B_1 + \max \{ \text{realize, not} \} ((1 - \tau^*_c)(1 - \tau^E_h), 1 - \tau^*_c) rE_1 \]

\[ - k_{2h} + wy_{h} + \alpha_{h}, \quad \text{if} \ y_{h} > 0 \]  

(4.7)

\[ c_{l} = r(1 - \tau_l^B)B_1 + \max \{ \text{realize, not} \} ((1 - \tau^*_c)(1 - \tau^E_l), 1 - \tau^*_c) rE_1 \]

\[ - k_{2l} + \alpha_{l}, \quad \text{otherwise} \]  

(4.8)
Assume first the criteria in (4.2) is true. Moreover, suppose an agent enter period 1 with positive amount of both debt and equity. If in period 1 the agent finds him high skilled, then he would realize his return on equity since $\tau_E^h < 0$. Here, we need another criterion: He would be better with more debt if the net return on debt is greater than the net return on equity if

$$1 - \tau_h^B > (1 - \tau_e^*) (1 - \tau_h^E).$$  \hspace{1cm} (4.9)$$

On the other hand, if in period 1 the agent finds him low skilled, then he would not realize his return on equity if we have

$$\tau_l^E > 0.$$  \hspace{1cm} (4.10)$$

Then, he also would be better if he only holds equity since the net return on equity is greater than the net return on debt:

$$1 - \tau_e^* > 1 - \tau_l^B,$$

which is true by (4.2). Therefore, in period 0, if the tax system satisfies (4.2), (4.9), and (4.10), the agent faces a portfolio selection between $(B_1, E_1)$ since he does not know which type he will be in period 1. The budget constraint in period 0 is as follows.

$$c_0 = rk_0 - (B_1 + E_1) + wy_0 \quad \text{with} \quad B_1 + E_1 = K_1^*.$$  \hspace{1cm} (4.11)$$

We now introduce the optimal tax system in period 1 as follows (The period 2 capital taxes are the same as (3.5)). Define $(\tau^*_e, \tau^*_l, \tau^*_h, \tau^*_l)$ and $(\tau_{hh}, \tau_{hl}, \tau_{lh}, \tau_{ll})$ by

$$\begin{cases}
    r(1 - \tau^*_e) = \frac{u'(c_0)}{u'(c_l)} \\
    r(1 - \tau^*_e)(1 - \tau_h^E) = \frac{u'(c_0)}{u'(c_h)} \\
    1 - \tau_l^B > (1 - \tau_e^*)(1 - \tau_h^E) \\
    \tau_l^E > 0 \\
    \pi r(1 - \tau_h^B)u'(c_h^*) + (1 - \pi) r(1 - \tau_l^B)u'(c_l^*) = u'(c_0^*) \\
    r(1 - \tau_{hh})u'(c_{hh}^*) = u'(c_h^*) \\
    r(1 - \tau_{hl})u'(c_{hl}^*) = u'(c_h^*) \\
    r(1 - \tau_{lh})u'(c_{lh}^*) = u'(c_l^*) \\
    r(1 - \tau_{ll})u'(c_{ll}^*) = u'(c_l^*) 
\end{cases}$$  \hspace{1cm} (4.12)$$

and define labor taxes $(\alpha_h, \alpha_l)$ and $(\alpha_{hh}, \alpha_{hl}, \alpha_{lh}, \alpha_{ll})$ such that their present values are
matched:

\[-\{\pi u'(c_h^*)c_h + (1 - \pi)u'(c_t^*)c_t + \pi u'(c_{hh}^*)c_{hh} + (1 - \pi)u'(c_{tt}^*)c_{tt}\} \]

\[= u'(c_h^*)(r\tau_k + wy_h^*) + \pi u'(c_h^*)wy_h^* + \pi u'(c_{hh}^*)wy_{hh}^* \]

\[= \{u'(c_0^*)c_0^* + \pi u'(c_h^*)c_h^* + (1 - \pi)u'(c_t^*)c_t^* + \pi u'(c_{hh}^*)c_{hh}^* + (1 - \pi)u'(c_{tt}^*)c_{tt}^*\} \quad (4.13) \]

Moreover, we have

\[
\begin{align*}
    u'(c_{hh}^*)\{c_{hh}^* - wy_{hh}^* - \alpha_{hh}\} &= u'(c_{tl}^*)\{c_{tl}^* - \alpha_{tl}\} \\
    u'(c_{th}^*)\{c_{th}^* - wy_{th}^* - \alpha_{th}\} &= u'(c_{lt}^*)\{c_{lt}^* - \alpha_{lt}\}
\end{align*}
\]

(4.14)

Equation (4.13) results from adding the budget constraints (4.3), (4.4), (4.7), (4.8), and (4.11), each of whom are multiplied by \(\pi u'(c_{hh}^*), (1 - \pi)u'(c_t^*), \pi u'(c_h^*), (1 - \pi)u'(c_t^*), \) and \(u'(c_h^*),\) respectively and using the definitions of the capital income tax code (4.12).

Equation (4.14) is also derived using the definition of the capital income tax code (4.12) such that

\[u'(c_{hh}^*)r(1 - \tau_{hh}^*) = u'(c_{tl}^*)r(1 - \tau_{tl}^*)\]

\[u'(c_{th}^*)r(1 - \tau_{th}^*) = u'(c_{lt}^*)r(1 - \tau_{lt}^*).\]

Technically \(\tau_{c}^* \) and \(\tau_{h}^E\) in (4.12) are first set up to be equal to the ex-post wedge between the MRT and the MRS that appear in the first order condition (or Euler equation) for the equity holding choice \(E_1.\) Then, \((\tau_{t}^B, \tau_{t}^E)\) is determined in the first order condition for the debt holding choice \(B_1.\) There are two tax rates that can be flexibly chosen: \((\tau_{t}^E, \tau_{t}^B).\) \(\tau_{t}^E\) should be positive. Notice that in (4.12) we set

\[\tau_{t}^B < \tau_{t}^E + \tau_c^* \tau_{c}^E,\]

which is in fact from (4.9). By simple algebra we have, by Corollary 1,

\[\tau_{t}^E + \tau_c^* - \tau_{c}^E = 1 - \frac{u'(c_0^*)}{ru'(c_{h}^*)} < 0,\]

which implies that \(\tau_{t}^B < 0.\) It is notable that either \(0 > \tau_{t}^B > \tau_{t}^E\) or \(0 > \tau_{t}^E > \tau_{t}^B\) is possible.

Notice that \(B_1 + E_1 = K_{t}^*\) should be satisfied since the agents are homogenous in period 0. On the other hand, it is not necessary that \(\pi k_{2h}^* + (1 - \pi)k_{2t}^* = K_{2}^*\) if we allow resource transfer between period 1 and 2. It is also easy to verify that the tax system (4.12) satisfies the intuitive criteria given in (4.2).
period 1 of (4.12) can be rewritten as
\[ \tau^* = 1 - \frac{u'(c^*_0)}{ru'(c^*_1)}, \quad (4.15) \]
\[ \tau^E = 1 - \frac{u'(c^*_0)}{u'(c^*_h)}, \quad (4.16) \]
\[ \tau^B < \tau^E + \tau_c - \tau_c \tau^E = 1 - \frac{u'(c^*_0)}{ru'(c^*_h)}, \quad (4.17) \]
\[ \tau^E > 0 \quad (4.18) \]
\[ \tau^B = 1 - \frac{u'(c^*_0) - \pi r(1 - \tau^B)u'(c^*_h)}{(1 - \pi)ru'(c^*_h)} \quad (4.19) \]

Here, note again \( \tau^E \) is arbitrary. From equations (4.15), (4.16), (4.17), (4.18), and (4.19), we can directly confirm criteria (4.2), (4.9), and (4.10). We summarize this result as the following lemma that will be used later.

**Lemma 2.** The tax system (4.12) satisfies
\[ 0 < \tau^* < \tau^B \quad \text{and} \quad \tau^E < \tau_h < 0. \]

One may be interested in the case where \( \tau^B \approx \tau^E \). The following lemma tells about this special case.

**Lemma 3.** The tax system (4.12) is given. Then, \( \tau^B = \tau^E \) if and only if \( \tau^B = \frac{\tau^*}{1 - \pi} \).

**Proof.** This just results from (4.19).

Now we are ready to state our main theorem.

**Theorem 1.** Suppose the government can impose the corporate tax. Given the tax system (4.12), the consumption and labor allocation of the competitive equilibrium coincide with those of the constrained optimum allocation.

**Proof.** See the Appendix.

One may think that until now we have only considered the individual investors, so that the role of firms are ignored in debt and equity issuance. In fact, the effect of the corporate tax is offset by that of the individual capital taxes. Simple algebra shows that the expected tax rate on holding equity in \( t = 0 \) is
\[ \pi[1 - (1 - \tau^E)(1 - \tau^*_c)] + (1 - \pi)\tau^*_c = 0. \quad (4.20) \]
Therefore the tax system (4.12) makes firms indifferent to any capital structure as described in the proof of Theorem 1. In other words, the capital structure only results
from the aggregate debt and equity portfolio choice of individual agents. Therefore, in
the firm’s point of view, the Modigliani-Miller theorem still holds. This idea is quite
similar to that of Miller (1977).

**Corollary 3** (Modigliani-Miller Theorem Revisited). *The market value of any firm is
independent of its capital structure.*

One important remark is that Corollary 3 is not automatically true for the case of
more than two types. As will be explained in Section 6, if the number of types of agent
is more than two (the number of assets in the market, debt and equity), the expected
tax rate on equity is not necessarily equal to zero since we have more degree of freedom
to choose the tax rates. Therefore, the tax authority need to set the expected tax rate
to be zero. Otherwise, the capital market does not clear. Therefore, for the case of
more than two types of agents, Corollary 3 is not a property of the optimal tax system,
but it should be a condition when setting up the optimal tax rates. This is the only
one difference between the case where there are two types and the case where there are
more than two types of agents.

### 4.3 A Simple Example

This section provides a very simple example. For the case where the utility function is
logarithmic and the dis-utility function is linear, we describe some comparative statics
results. In particular, the corporate tax rate increases in $\pi$, the probability of being a
high skill agent. In this sense, we provide a simple regression result between the average
schooling years and the corporate tax rates among OECD countries. Although we need
carefully interpret the result due to the indeterminacy property of the corporate tax
when there are more than two types of agents (See Section 6).

Assume that the utility function is log and the disutility function is linear:

\[
\begin{align*}
  u(c) &= \log(c), \\
  v(y) &= \kappa y.
\end{align*}
\]

Then, the the first order conditions (2.7) yields

\[
\begin{align*}
  c_0^* &= c_h^* = c_{hh}^* = c_h^* = \frac{w}{\kappa}. \\
  \text{(4.22)}
\end{align*}
\]

Putting this into the inverse Euler equation in (2.7) to get

\[
\begin{align*}
  c_l^* &= c_{hl}^* = \frac{(r - \pi)w}{(1 - \pi)\kappa}, \\
  c_l^* &= \frac{1}{1 - \pi} \left( \frac{r(r - \pi)}{1 - \pi} - \pi \right) \frac{w}{\kappa}.
\end{align*}
\]

Notice that we need the following assumption to get the well-defined solution.

\[
\begin{align*}
  \pi < r < 1 \quad \text{and} \quad r(r - \pi) > \pi(1 - \pi). \\
  \text{(4.24)}
\end{align*}
\]
Recall the linear disutility function \( v(y) \) in (4.21). If (4.24) does not hold, then the agent will choose negative work (therefore negative disutility) in order to increase utility. Then, (2.9) gives

\[
y_h^* = \log(c_h^*) - \log(c_h^*) = \log\left(\frac{1 - \pi}{r - \pi}\right) > 0
\]

\[
y_{hh}^* = \log(c_{hh}^*) - \log(c_{hh}^*) = \log\left(\frac{1 - \pi}{r - \pi}\right) > 0
\]

\[
y_{lh}^* = \log(c_{lh}^*) - \log(c_{lh}^*) = \log\left(\frac{1 - \pi}{r(r - \pi) - \pi(1 - \pi)}\right) > 0,
\]

where all the equations are positive by (4.24). Then, finally we get \( y_0^*, K_1^*, \) and \( K_2^* \) from (2.4) as follows:

\[
rK_2^* = \left[ \pi + \pi(1 - \pi) + \pi(1 - \pi)\left(\frac{r - \pi}{1 - \pi}\right) + (1 - \pi)\left(\frac{r}{1 - \pi} - \pi\right) \right] \frac{w}{\kappa}
\]

\[
- w \left[ \pi^2 \log\left(\frac{1 - \pi}{r - \pi}\right) + \pi(1 - \pi) \log\left(\frac{1}{r(r - \pi) - \pi(1 - \pi)}\right) \right]
\]

\[
= \left[ \pi + \pi(r - \pi) + r(r - \pi) \right] \frac{w}{\kappa} - w\pi \log\left[\left(\frac{1 - \pi}{r - \pi}\right) \frac{1}{\left(\frac{r}{r(r - \pi) - \pi(1 - \pi)}\right)^{1-\pi}}\right].
\]

\[
rK_1^* = K_2^* + \frac{w}{\kappa} - w\pi \log\left(\frac{1 - \pi}{r - \pi}\right)
\]

\[
w y_0^* = K_1^* + \frac{w}{\kappa} - rK_0.
\]

For the log utility case, we have the following optimal tax code:

\[
\tau_c^* = 1 - \frac{c_t^*}{rc_0^*} \quad \text{and} \quad \tau_h^* E = 1 - \frac{c_h^*}{c_t^*}
\]

(4.25)

Putting (4.22) and (4.23) into (4.25), we have

\[
1 - \tau_c^* = \frac{u'(c_0^*)}{ru'(c_t^*)} = \frac{r - \pi}{r(1 - \pi)}.
\]

Then, simple algebra gives the following proposition.

**Proposition 4.** Suppose the agent has the log utility and the linear disutility functions of (4.21). Moreover, assume (4.24) is satisfied. Other things being equal, the following comparative statics analysis holds.

(i) The corporate tax rate \( \tau_c^* \) increases in the measure (population) of high skill agents, \( \pi \). In other words, \( \frac{d\tau_c^*}{d\pi} > 0 \).
Proposition 4 can be interpreted as follows. Assume there are two closed economies: 
(i) The corporate tax rate may be higher in the economy populated with more skilled workers. 
(ii) The corporate tax rate may be higher in the economy having higher return on investment. 
(iii) The corporate tax rate may be higher in the economy having higher labor productivity. Notice that statement (i) should be understood under the assumption of the law of large numbers. It is also generally true if the production technology is given by a constant returns to scale production function.

5 Aggregate Leverage and Some Comparative Statics Analysis

In this section we investigate how the taxes, in particular, the individual labor taxes affect the leverage ratio. We first identify the explicit solution for $(B'_{1}, E'_{1})$ in Section 5.1 and characterize its properties. It turns out that the debt and equity holding depends on the labor taxes. Therefore, the labor income taxes affect the leverage ratio. It implies that not only the capital income tax code (including the corporate tax) but also the labor income tax code are important, when we investigate the effect of a tax reform on the leverage ratio. However, the labor tax codes have been often ignored in capital structure theories.

In particular, we perform some comparative statics analysis on the aggregate leverage with respect to the change of labor tax codes. Recall that the labor taxes are indeterminate by the Ricardian equivalence. Section 5.2 deals with the effect of change in the labor tax. Suppose there is no period-by-period resource transfer. If the tax authority provides more (less) insurance against, then the leverage ratio increases (decreases). A similar result holds for the case when the intertemporal resource transfer is allowed.

5.1 Endogenous Leverage

Recall that we have two budget constraints of high and low skill agent in period 1 and the initial investment decision $B_{1} + E_{1} = K'_{1}$. As described in Section 4.4.3 of Kocherlakota (2009), the timing and the amount of labor taxation is arbitrary as
long as (4.13) and (4.14) is satisfied. Then, in fact, the individual optimal investment 
\((B_1^*, E_1^*)\) in period 0 and \((k_{2h}^*, k_{2l}^*)\) in period 1 depend on how the government, period-
by-period, transfers labor taxes (or subsidies). The following proposition provide the 
analytic form of the debt and equity holding. In order for simpler exposition, we 
introduce some positive number \(\hat{k}_2\) which is equal to the period 1 aggregate investment, 
\(\pi k_{2h}^* + (1 - \pi)k_{2l}^* = \hat{k}_2\).

**Proposition 5.** Let \(\pi k_{2h}^* + (1 - \pi)k_{2l}^* = \hat{k}_2\). Let \((\tau_c^*, \tau_h^B, \tau_h^E, \tau_m^B, \tau_m^E, \tau_l^B, \tau_l^E)\) be an 
optimal capital tax system given by (4.12). Then, given the labor tax code, \((\alpha_h, \alpha_l)\), the 
optimal portfolio of debt and equity \((B_1^*, E_1^*)\) is given by

\[
\begin{align*}
B_1^* &= \frac{-X(\alpha_h, \alpha_l) - K_2^* + (\pi \tau_h^E + \tau_c^* - \pi \tau_h^E \tau_c^*)r K_1^*}{-r (\pi \tau_h^E \tau_c^* + \pi \tau_h^B - \pi \tau_h^E + \tau_c^* - (1 - \pi) \tau_l^B)} \quad \text{(5.1)} \\
E_1^* &= \frac{X(\alpha_h, \alpha_l) + K_2^* - (\pi \tau_h^B + (1 - \pi) \tau_l^B)r K_1^*}{-r (\pi \tau_h^E \tau_c^* + \pi \tau_h^B - \pi \tau_h^E + \tau_c^* - (1 - \pi) \tau_l^B)} \quad \text{(5.2)}
\end{align*}
\]

where \(X(\alpha_h, \alpha_l) := (\pi \alpha_h + (1 - \pi) \alpha_l) - \hat{k}_2\).

**Proof.** See the appendix.

First notice that the denominator in (5.1) and (5.2) are positive, which is sum-
marized in Lemma 5 in the appendix. Before describing the meaning of the above 
proposition, we first narrow down the case where there is no period-by-period transfer 
by the government.

**Corollary 4.** Let \((\tau_c^*, \tau_h^B, \tau_h^E, \tau_m^B, \tau_m^E, \tau_l^B, \tau_l^E)\) be an optimal capital tax system given 
by (4.12). Suppose the period-by-period government budget is balanced. More precisely, 
suppose that we take some positive numbers \(\hat{k}_{1b}, \hat{k}_{1e}, \hat{k}_{2h}, \text{ and } \hat{k}_{2l}\) to have \((\alpha_h, \alpha_l)\) such 
that

\[
\begin{align*}
\alpha_h &= c_h^* - r(1 - \tau_h^B)\hat{k}_{1b} - r(1 - \tau_h^E)(1 - \tau_c^*)\hat{k}_{1e} + \hat{k}_{2h} - wy_h^* \\
\alpha_l &= c_l^* - r(1 - \tau_l^B)\hat{k}_{1b} - r(1 - \tau_c^*)\hat{k}_{1e} + \hat{k}_{2l}
\end{align*}
\]

where

\(\hat{k}_{1b} + \hat{k}_{1e} = K_1^* \quad \text{and} \quad \pi \hat{k}_{2h} + (1 - \pi) \hat{k}_{2l} = K_2^*\).

Then, we have \(B_1^* = \hat{k}_{1b}, E_1^* = \hat{k}_{1e}, k_{2h}^* = \hat{k}_{2h}, \text{ and } k_{2l}^* = \hat{k}_{2l}\).

**Proof.** See the appendix.

Proposition 5 shows that the aggregate capital structure is determined by \(X(\alpha_h, \alpha_l)\) 
as well as the capital tax code. Therefore, we present comparative statics results with
respect to change of the labor income tax code and change of the corporate income tax code in the next two subsections.

Notice \( B_1^* + E_1^* = K_1^* \) is fixed. Therefore, we only need to see the change of \( B_1^* \) in order to see the change of leverage ratio \( \frac{B_1^*}{B_1^* + E_1^*} \).

5.2 Comparative Statics: Labor Taxation

For the comparative statics analysis on labor taxation, we should notice that the labor tax code must satisfy the Ricardian equivalence: \( (4.13) \) and \( (4.14) \). For example, if \( \alpha_l \) goes up, either or all of \( \alpha_l, \alpha_{hh}, \) or \( \alpha_{hl} \) must go down as in \( (4.13) \). Although the tax authority cannot arbitrarily change the labor taxes, they have enough degree of freedom. Due to this indeterminacy property, we face too many cases. Hence, we focus on simple reasonable examples. Fixing the optimal allocation, we divide the analysis into two cases: \( (i) \) when only period 1 labor taxes \( (\alpha_h, \alpha_l) \) is changed (without intertemporal resource transfer) and \( (ii) \) when the expected value of labor taxes will be changed (with the intertemporal resource transfer).

5.2.1 Comparative Statics: Period 1 Labor Taxes \( (\alpha_h, \alpha_l) \)

Suppose, in this subsection, the labor taxes in the third period, \( (\alpha_{hh}, \alpha_{hl}, \alpha_{lh}, \alpha_{ll}) \), is unchanged. \( (4.13) \) implies that \( \alpha_l \) is increased if and only if \( \alpha_h \) is decreased. This observation and proposition 5 give the following proposition.

**Proposition 6.** Suppose that given the optimal allocation, the tax authority only changes the period 1 labor taxes whereas the period 2 labor taxes are fixed, i.e., \( \alpha_{hh}, \alpha_{hl}, \alpha_{lh}, \) and \( \alpha_{ll} \) are fixed. Then, we have

\[
\frac{dB_1^*}{d\alpha_l} > 0 \quad \text{and} \quad \frac{dE_1^*}{d\alpha_l} < 0.
\]

In other words, if the tax system provide more (less) insurance against low skill shocks, then the leverage ratio goes up (down).

**Proof.** See the appendix.

The intuition for Proposition 6 is as follows. Recall that in period 1 the ex-post low skill agents will prefer to hold more equities than debts. If the tax authority insures more against the low skill shocks, then the agent in period 0 generally wants to choose more debts. This effect pushes the leverage ratio up.
5.2.2 Comparative Statics: Expected Labor Taxes

Even if the period-by-period resource transfer is allowed, the basic idea of proposition 6 still holds. The leverage ratio increases if the discounted expected subsidy on being a low skill agent onward increases. From the optimal tax code (4.12), we can have the relationship between the labor income tax and the optimal investment:

\[
\begin{align*}
    k_{2h}^* &= \frac{u'(c_{ih}^*)}{u'(c_{ih})} (c_{ih}^* - wy_{ih} - \alpha_{hh}) = \frac{u'(c_{ih}^*)}{u'(c_{ih})} (c_{il}^* - \alpha_{hl}) \\
    k_{2l}^* &= \frac{u'(c_{il}^*)}{u'(c_{il})} (c_{il}^* - wy_{il} - \alpha_{ll}) = \frac{u'(c_{il}^*)}{u'(c_{il})} (c_{il}^* - \alpha_{ll})
\end{align*}
\]  

(5.4)

Using (5.4), we can rewrite \(X(\alpha_h, \alpha_l)\) as

\[
X(\alpha_h, \alpha_l) = (\pi \alpha_h + (1 - \pi) \alpha_l) - (\pi k_{2h}^* + (1 - \pi) k_{2l}^*)
\]

\[
= \frac{\pi}{u'(c_{ih}^*)} \left[ u'(c_{ih}^*) \alpha_h + u'(c_{ih}) \alpha_{hh} \right] + \frac{1 - \pi}{u'(c_{il}^*)} \left[ u'(c_{il}^*) \alpha_l + u'(c_{il}) \alpha_{ll} \right] + C_1
\]

\[
= \frac{\pi}{u'(c_{ih}^*)} \left[ u'(c_{ih}^*) \alpha_h + \pi u'(c_{ih}) \alpha_{hh} + (1 - \pi) u'(c_{il}) \alpha_{hl} \right]
\]

\[
+ \frac{1 - \pi}{u'(c_{il}^*)} \left[ u'(c_{il}^*) \alpha_l + \pi u'(c_{il}) \alpha_{ll} + (1 - \pi) u'(c_{il}) \alpha_{ll} \right] + C_2,
\]

(5.5)

where \(C_1\) and \(C_2\) are some constants consisting of optimal values \((c^*, y^*)\) independent of \(\alpha\)'s. Then, using the above expression (5.5) and the labor income budget constraints (4.13) and (4.14), we have the following proposition about how the change in labor taxes affects the debt and equity choice given the optimal allocation. To be more specific, we need to define the expected present value of labor taxes conditional on being a high skill agent:

\[
A := u'(c_{ih}) \alpha_h + \pi u(c_{ih}) \alpha_{hh} + (1 - \pi) u(c_{il}) \alpha_{hl}.
\]

Proposition 7. Given the optimal allocation \((c^*, k^*, y^*)\), suppose the government changes the labor income tax codes \((\alpha_h, \alpha_l)\) and \((\alpha_{hh}, \alpha_{hl}, \alpha_{ll}, \alpha_{ll})\) that satisfies (4.13) and (4.14). Other things being equal, we have

\[
\frac{\partial B_1^*}{\partial A} < 0 \quad \text{and} \quad \frac{\partial E_1^*}{\partial A} > 0.
\]

Proof. See the appendix. \(\square\)

The intuition for Proposition 7 is quite similar to that of Proposition 6. Notice \(A\) is negative. Therefore, \(A\) goes up if and only if the expected present value of labor taxes conditional on being the high type goes down since the high skill agents in equilibrium should pay the labor taxes and the low skill agents receive the subsidy. In other
words, \( A \) goes up if and only if the government provide less insurance against being low skilled. Thus, agents choose more amount of equity (therefore less amount of debt) for self-insuring her against the low skill shock. The ratio of debt holding is negatively correlated with the expected present value of labor taxes conditional on being the high type. On the other hands, the leverage goes up if the expected discounted value of being low skill agent in period 1 and being whoever in period 2 increases, i.e., basically the tax authority provides more insurance against low skill shocks.

6 More Than Two Types

In this section, we extend the model of previous sections into the case for more than two types of agents. The fundamental idea is exactly same as before. We can explicitly derive the tax system and the optimal market portfolio of debt and equity that turn out to be easy extension of the previous results of the case for two types. However, there is one crucial difference, which is the reason why we write this section. The corporate tax rate when there are more than two types of agents is indeterminate while the uniqueness does hold when there are only two types.

We first summarize the tax code in Section 6.1 and the optimal portfolio of debt and equity in Section 6.2, which are analogues of previous results. Then, we continue to investigate the other properties. Section 6.3 shows the indeterminacy of the corporate tax level. In fact, it turns out to be that \( \tau_c \) suggested in Section 6.1 is the minimal level and the government can choose the corporate income tax rates greater than or equal to \( \tau_c \) by properly adjusting the other individual capital taxes according to the change of the corporate tax. This indeterminacy can explain why it is possible to have the historically fairly high corporate income tax rates levied in many countries, in particular, during the last centuries in U. S..

The indeterminacy raises an immediate question: Given the current rate is high enough, what if we increase or decrease the corporate tax rate? Section 6.4 deals with the effect of the change of the corporate tax on the firm’s leverage ratio. Surprisingly, unlike the classical capital structure theories, the change of the corporate tax rate does not have impact on the leverage ratio. This result is quite consistent with the long run stationarity of capital structure in U.S. while the corporate tax rates have been significantly declined. Finally, due to the existence of the corporate tax it is never surprising that the aggregate capital income tax is nonzero, which is different from the classical result of the Ramsey taxation (Section 6.5).
6.1 Basic Results: A Simple Extension

The previous analysis should also work for any finite number of agents. Since the basic intuition will be the same, here we show how to pin down the corporate tax and how to set up the individual capital taxes when there are three types of agents. It is straightforward to derive the general result for the case of \( n \) types of agents. Suppose that there are three skill types \( \theta_h, \theta_m, \theta_l \) with \( \theta_h > \theta_m > \theta_l \).

Let \( Pr(\theta = \theta_i) = \pi_i \) with \( i = h, m, l \). So, \( \pi_h + \pi_m + \pi_l = 1 \). \( \theta_i \) is private information. Shocks are i.i.d. over time across agents as well. Everybody can work. Their utility functions are assumed to be the same as before:

\[
\sum_{t=0}^{2} u(c_t) - v(e_t)
\]

with \( y_t = e_t \theta_t \), where \( e_t \) is the effort level at time \( t \) and \( y_t \) is the labor provided by the agent. \( e_t \) is private information. The production function is the same as before:

\[
f(K,Y) = rK + wY.
\]

All the setup and the analysis are very similar as before. It is tedious to write down the planner’s problem again. Thus, we skip it. The first order conditions are similarly obtained. Assume that we have already characterized \((c^*, y^*, k^*)\), the constrained optimal allocation in this case. The most important key is the following inverse Euler equation in period 1:

\[
u'(c^*_0) = \frac{\pi_h r(1 - \tau_h)}{u'(c^*_h)} + \frac{\pi_m r(1 - \tau_m)}{u'(c^*_m)} + \frac{\pi_l r(1 - \tau_l)}{u'(c^*_l)}.
\]

Each agent is indexed by subscripts \( h, m, \) and \( l \), respectively. Then, the corporate tax rate \( \tau_c \) and the optimal individual capital tax code \((\tau^B_h, \tau^E_h, \tau^B_m, \tau^E_m, \tau^B_l, \tau^E_l)\) in period 1 are given by

\[
\begin{cases}
    r(1 - \tau_c)u'(c^*_0) = u'(c^*_0) \\
    r(1 - \tau_c)(1 - \tau^E_m)u'(c^*_m) = u'(c^*_0) \\
    r(1 - \tau_c)(1 - \tau^E_h)u'(c^*_h) = u'(c^*_0) \\
    \pi_h r(1 - \tau^B_h)u'(c^*_h) + \pi_m r(1 - \tau^B_m)u'(c^*_m) + \pi_l r(1 - \tau^B_l)u'(c^*_l) = u'(c^*_0), \\
    (1 - \tau^B_h) > (1 - \tau^E_h)(1 - \tau_c) \\
    (1 - \tau^B_m) > (1 - \tau^E_m)(1 - \tau_c) \\
    (1 - \tau^B_l) < (1 - \tau_c) \\
    \tau^E > 0
\end{cases}
\]
The first three equations in (6.1) is derived by setting the capital tax rates equal to the ex-post wedges, each of which is the component of the Euler equation with respect to $E_1$. The forth equation is the Euler-equation derived from the first order condition with respect to $B_1$. The next four inequalities are the conditions where the high and middle skill agents will prefer debt while the lowest skill agents will prefer equity in the next period, which in turn remove the tax timing options of the lowest skill agents. Technically, we first pin down $\tau_c, \tau_E^h$, and $\tau_E^l$, and then choose $\tau_B^h, \tau_B^m, \tau_B^l$, and $\tau_E^l$ flexibly through the inequalities.

The crucial condition is (6.2). This condition is designed to make firms indifferent to choosing between debt and equity. (6.2) was not necessary for the case where there are two only types of agents. In that case, the last equation is automatically satisfied (See the proof of Theorem 1). However, for the case where there are more than two types of agents, we should impose this condition when setting up the capital tax rates. This is because the number of equity tax rates (equal to the number of types) to determine is more than the number of assets (debt and equity) in the market. If the last equation of (6.1) is not satisfied, then the firm will provide either 100% debt or 100% equity financing while every agent chooses both debt and equity with positive amount, which in turn fails to meet the market clearing condition.

This idea to set (6.1) is also easier to understand if we look at the following budget constraint of each type agent. In period 0,

$$c_0 = rk_0 - (B_1 + E_1) + wy_0 \quad \text{with} \quad B_1 + E_1 = K_1^*,$$

In period 1,

$$c_h = r(1 - \tau_B^h)B_1 + \max_{\text{realize, not}} ((1 - \tau_E^h)(1 - \tau_c), 1 - \tau_c)rE_1 - k_{2h} + wy_h + \alpha_h,$$

$$c_m = r(1 - \tau_B^m)B_1 + \max_{\text{realize, not}} ((1 - \tau_E^m)(1 - \tau_c), 1 - \tau_c)rE_1 - k_{2m} + wy_m + \alpha_m,$$

$$c_l = r(1 - \tau_B^l)B_1 + \max_{\text{realize, not}} ((1 - \tau_c)(1 - \tau_E^l), 1 - \tau_c)rE_1 - k_{2l} + wy_l + \alpha_l$$

Now, it is easy to show the following lemma which is an extension of Lemma 2.

**Lemma 4.** The tax system (6.1) satisfies

$$\tau_E^h < \tau_E^m < 0 < \tau_c < \tau_E^l.$$

**Proof.** See the appendix.
Similarly to Lemma 2, Lemma 4 tells that this tax system makes the ex-post lowest skill agents prefer equity and all the other types prefer bonds. The only lowest skill agents need to pay individual capital income taxes in period 1. This is still true if we have more and more types. Only the lowest types of agents face a positive capital tax rates. However, in a model with more than 3 periods, it is no more true that the currently lowest type’s capital tax rates is the highest. In Intuitively it would be usually true that the one who becomes very low skilled in the current period relative to the previous skill status pays the highest tax rates (See Section 7.1).

6.2 Endogenous Leverage for More than Two Types

The next proposition is analogous to Proposition 5. It provides the analytic form of the debt and equity holding. In order for simpler exposition, we introduce some positive number $\hat{k}_2$ which is equal to the period 1 aggregate investment, $\pi_h k_{2h}^* + \pi_m k_{2m}^* + \pi_l k_{2l}^* = \hat{k}_2$.

**Proposition 8.** Let $\pi_h k_{2h}^* + \pi_m k_{2m}^* + \pi_l k_{2l}^* = \hat{k}_2$. Let $\tau_c, \tau_B^h, \tau_E^h, \tau_B^m, \tau_E^m, \tau_B^l, \tau_E^l$ be the optimal tax system given in Proposition (6.1). Then, given the labor tax code, $(\alpha_h, \alpha_m, \alpha_l)$, the optimal portfolio of debt and equity $(B_1^*, E_1^*)$ is given by

$$B_1^* = \frac{-X(\alpha_h, \alpha_m, \alpha_l) - K_2^* + [\pi_h \tau_h^E + \pi_m \tau_m^E + \pi_l \tau_l^E - (\pi_h \tau_h^E + \pi_m \tau_m^E) \tau_c]}{r D_3} r K_1^*$$

$$E_1^* = \frac{X(\alpha_h, \alpha_m, \alpha_l) + K_2^* - [\pi_h \tau_h^B + \pi_m \tau_m^B + \pi_l \tau_l^B] r K_1^*}{r D_3}$$

where $X(\alpha_h, \alpha_m, \alpha_l) := (\pi_h \alpha_h + \pi_m \alpha_m + \pi_l \alpha_l) - \hat{k}_2$ and

$$D_3 = \pi_h [(1-\tau_h^B)(1-\tau_h^E)(1-\tau_c)] + \pi_m [(1-\tau_m^B)(1-\tau_m^E)(1-\tau_c)] + \pi_l [(1-\tau_l^B)(1-\tau_l^E)(1-\tau_c)].$$

**Proof.** See the appendix.

First notice that the denominator in (6.3) and (6.4) are positive, which is summarized in Lemma 6 (easy extension of Lemma 5) in the appendix. One remark is that the comparative statics analysis with respect to the change in labor taxation is exactly same as shown in Proposition 6 and 7. The intuition is also the same, thus we skip this analysis.

6.3 Indeterminacy

The new result in this section is the indeterminacy of the capital income tax code. Notice that if the tax authority take the corporate tax level less than $\tau_c$ in (6.1), then
the low skill agents still have incentives to defer the realization of capital income. Then, what if the corporate tax level is higher than $\tau_c$? The next proposition provides an answer to this question.

**Proposition 9.** Let $(\tau_c, \tau^B_h, \tau^E_h, \tau^B_m, \tau^E_m, \tau^B_l, \tau^E_l)$ be the optimal tax system given by (6.1). Let $\tilde{\tau}_c = \tau_c + \epsilon$ for some $\epsilon > 0$. Then, there exist $\delta_h > 0$ and $\delta_m > 0$ such that $\tilde{\tau}_c$, $\tilde{\tau}^B_h$, $\tilde{\tau}^E_h$, $\tilde{\tau}^B_m$, $\tilde{\tau}^E_m$, $\tilde{\tau}^B_l$, $\tilde{\tau}^E_l$ is also an optimal tax system. In addition, the other tax rates can be properly adjusted as long as the following inequalities are satisfied.

\[
(1 - \tilde{\tau}^B_h) > (1 - \tau^E_h + \delta_h)(1 - \tau_c - \epsilon) \\
(1 - \tilde{\tau}^B_m) > (1 - \tau^E_m + \delta_m)(1 - \tau_c - \epsilon) \\
(1 - \tilde{\tau}^B_l) < (1 - \tau_c - \epsilon) \\
\tilde{\tau}^E_l > 0
\]

*Proof.* See the appendix. \qed

The proof of Proposition 9 is constructive, which means that we obtain $\delta_h$ and $\delta_m$ explicitly in the proof. Proposition 9 also tells that the corporate tax rate $\tau_c$ in the tax system (4.12) is the minimal level to support the socially optimal allocation. The tax authority can take $\tilde{\tau}_c$ greater than this minimal value $\tau_c$. However, if the corporate tax rate increases by $\epsilon$, then the other individual capital taxes should be properly adjusted as well. In particular, the tax on equity of the higher skilled agents decreases by $\delta_h$ and $\delta_m$, respectively. The other tax rates must satisfy the four inequalities and the Euler equation with respect to debt holding. In other words, these tax rates can be either increased or decreased.

Although the model has three periods, one can infer from this result that the corporate tax rates time series data of U.S. and many other OECD countries may not be puzzling at all. As mentioned before, in U.S. the corporate tax rates were over 50% before 1960s and constantly decreased down to 35%, which is around 30% change. The corporate tax rate must be initially too high. Therefore, it has been possible for the IRS to keep decreasing the rates during the last 60 years, in particular, in accordance with the constant requests for decreasing the rate from general investors.
6.4 Comparative Statics: Corporate Taxation

As shown in Proposition 9, the corporate tax is indeterminate as long as rate, $\tilde{\tau}_c$ is greater than or equal to the minimal level $\tau^*_c$ of (6.1). In other words, the tax authority is free to change the rates. Therefore, given the sufficiently high level of corporate tax rates, we can consider how the change in the rate affects the leverage ratio (or cross-country comparison). More precisely we rewrite (6.3) and (6.4) using the tax code $(\tilde{\tau}_c, \tilde{\tau}_B, \tilde{\tau}_E)$ suggested in Proposition 9. Thus, we introduce the following definition.

**Definition 1.** Let $(\tilde{B}_1^*, \tilde{E}_1^*)$ be the debt and equity holding when the capital tax code is given by $(\tilde{\tau}_c, \tilde{\tau}_B, \tilde{\tau}_E)$. Classical capital structure literature often predicts the positive correlation between the leverage ratio and the corporate tax rates, namely,

$$\frac{d\tilde{B}_1^*}{d\tilde{\tau}_c} > 0.$$  \hspace{1cm} (6.5)

In particular, the leverage ratio decreases if the corporate tax rate decreases because the use of debt becomes less advantageous. Surprisingly, however, our paper predict that the leverage ratio is independent of the change in corporate tax rates. The change of the corporate tax need not affect the firm’s leverage ratio in the optimal tax framework.

**Proposition 10.** Assume there is no period-by-period resource transfer and $(\alpha_h, \alpha_m, \alpha_l)$ are fixed. Let the current tax system be given by $(\tilde{\tau}_c, \tilde{\tau}_h, \tilde{\tau}_m, \tilde{\tau}_l, \tilde{\tau}_B, \tilde{\tau}_E)$ and $\tilde{\tau}_c$ is sufficiently higher than the minimal level $\tau_c$ defined by (6.1). Let the debt and equity holding be given by $(\tilde{B}_1^*, \tilde{E}_1^*)$ corresponding to the current tax system. If there no change in $(\tilde{\tau}_h, \tilde{\tau}_m, \tilde{\tau}_l, \tilde{\tau}_B, \tilde{\tau}_E)$, then

$$\frac{d\tilde{B}_1^*}{d\tilde{\tau}_c} = \frac{d\tilde{E}_1^*}{d\tilde{\tau}_c} = 0.$$

**Proof.** See the appendix.

Notice that from Proposition 9, if $\tilde{\tau}_c$ changes, then $\tilde{\tau}_h$ and $\tilde{\tau}_l$ do change as well. However, the other tax rates, $(\tilde{\tau}_h, \tilde{\tau}_m, \tilde{\tau}_l, \tilde{\tau}_B, \tilde{\tau}_E)$, do not necessarily change. If these tax rates are constant, then the leverage ratio is unchanged although the corporate tax rate is changing. Therefore, Proposition 10 tells that the changes in the other individual tax rates are much more important rather than that of the corporate tax rates when we examine the impact of tax reforms on the leverage ratio. Notice that the aggregate leverage ratio in U.S. is around 0.4, which has been quite stationary during the last 5-60 years (See Frank and Goyal (2010)). Notice that the results in this section is only
a comparative static analysis. However, this result should probably be consistent with
the time-series data as well.

6.5 Non-zero Aggregate Capital Taxes

In the classical Ramsey models, the optimal capital tax rates should be zero if the
agents have constant relative risk aversion utility function or should converge to zero
as time goes by if they have general utility functions. It is still true in Kocherlakota
(2005) that the aggregate capital income taxes are zero (therefore capital income taxes
are purely redistributed) although individual capital taxes are never zero. In this paper,
even the aggregate capital taxes are never zero since the corporate tax exists.

Proposition 11. Suppose the capital income tax code is given by (4.12). In period 0,
the aggregate (expected) optimal capital tax of period 1 is negative.

Proof. See the appendix.

In the proof of Proposition 11, the aggregate total capital income tax of period 1
is given by

\[
\begin{align*}
\text{(a)} & \quad r(\pi_h \tilde{\tau}_h^B + \pi_m \tilde{\tau}_m^B + \pi_l \tilde{\tau}_l^B) B_1^* \\
\text{(b)} & \quad + \left[ r \left( \pi_l (1 - \tilde{\tau}_c) + \pi_m \pi_l (1 - \tilde{\tau}_m) (1 - \tilde{\tau}_E^m) + \pi_m (1 - \tilde{\tau}_c) (1 - \tilde{\tau}_E^h) \right) \right] E_1^*. 
\end{align*}
\]  

(6.6)

Component (a) of equation (6.6) is negative and component (b) of equation (6.6) is
0. This means that the capital taxes from equity are purely redistributive while the
capital taxes from debt are not. (Why? Can explain this further?)

7 Other Generalization

7.1 More than Three Periods

The model also can be easily extended to a multiperiods’ model even incorporating
many types of agents suggested in the previous section. The crucial thing is to how
to take the corporate tax in each period. Recall that the corporate tax is designed to
remove the tax timing option of the lowest skill agents in the three period model. The
lowest skill agent is the one who should pay the maximum capital income taxes in the
second best world. Then, in the multi-period model, we should remove the tax timing
option of the agent who faces the largest capital income tax in each period. That is, the corporate tax in period \( t + 1 \) (contingent on \( t + 1 \) history) is set to be

\[
1 - \tau_{t+1,c} = \inf \frac{u'(c^*_t)}{\beta r u'(c^*_t+1)},
\]

where \( c^*_t \) is the socially optimal allocation in period \( t \) and \( \beta \) is the discount factor. Then, the other individual capital taxes should be adjusted according to the corporate tax rates \( \tau_{t+1,c}^* \).

### 7.2 Aggregate Uncertainty: Production Shock

Suppose that the production function in period 2 is given by \( f(k, y) = \tilde{r}k + wy \), where \( \tilde{r} \) is a random variable independent of \( \theta \),

\[
\tilde{r} = \begin{cases} r_1, & \text{with probability } p \\ r_2, & \text{with probability } 1 - p \end{cases}
\]

with \( r_1 < r < r_2 \). Note that \( \tilde{r} = r \) in period 0 and 2. Let \( c^*_i(\tilde{r}), i = l, h \) denote the optimal consumption under the aggregate shock. Then, the optimal allocation should satisfy the inverse Euler equation with \( \lambda(r_i) > 0, i = 1, 2 \):

\[
\lambda(r_i)u'(c^*_0) = \frac{1}{E \left[ \frac{1}{u'(c^*_0)} \mid \tilde{r} = r_i \right]} = \frac{1}{\frac{1}{u'(c^*_h(r_i))} + \frac{1}{u'(c^*_l(r_i))}} \quad i = 1, 2
\]

\[
p\lambda(r_1)r_1 + (1 - p)\lambda(r_2)r_2 = 1.
\]

Now the corporation raises funds by equities and debts. Let \( R_1 \) and \( R(\tilde{r}) \) be the period 1 return on one unit of debt and equity in period 0. Then, their relation is given by

\[
R(\tilde{r}) = \frac{\tilde{r}(B_1 + E_1) - R_1B_1}{E_1}. \tag{7.1}
\]

Then, each period budget constraint is rewritten as follows. In period 0,

\[
c_0 = k_0 - (B_1 + E_1) + wy_0 \quad \text{with} \quad B_1 + E_1 = k_1^* \tag{7.2}
\]

In period 1,

\[
c_h(\tilde{r}) = (1 - \tau_h(\tilde{r}))R_1B_1 + \max_{\text{realize}, \not= h} \left\{ (1 - \tau_h(\tilde{r}))(1 - \tau_c(\tilde{r})), 1 - \tau_c(\tilde{r}) \right\} R(\tilde{r})E_1
\]

\[
- k_{2h}(\tilde{r}) + wy_h + \alpha_h(\tilde{r}), \tag{7.3}
\]

\[
c_l(\tilde{r}) = (1 - \tau_l(\tilde{r}))R_1B_1 + \max_{\text{realize}, \not= l} \left\{ (1 - \tau_l(\tilde{r}))(1 - \tau_c(\tilde{r})), 1 - \tau_c(\tilde{r}) \right\} R(\tilde{r})E_1
\]

\[
- k_{2l}(\tilde{r}) + \alpha_l(\tilde{r}), \tag{7.4}
\]

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where each variable is contingent on $\tilde{r}$. The optimal tax system shows the state-contingency: \( \{ \tau_c(\tilde{r}), \tau_B^h(\tilde{r}), \tau_E^h(\tilde{r}), \tau_B^l(\tilde{r}), \tau_E^l(\tilde{r}) \} \) with $\tilde{r} = r_1, r_2$ satisfying

\[
\begin{align*}
R(\tilde{r})(1 - \tau_c(\tilde{r}))u'(c_t^*(\tilde{r})) &= \lambda(\tilde{r})\tilde{r}u'(c_0^*) \hfill \\
R(\tilde{r})(1 - \tau_e(\tilde{r}))(1 - \tau_E^h(\tilde{r}))u'(c_h^*(\tilde{r})) &= \lambda(\tilde{r})\tilde{r}u'(c_0^*) \\
\pi R_1(1 - \tau_B^h(\tilde{r}))u'(c_h^*(\tilde{r})) + (1 - \pi) R_1(1 - \tau_B^l(\tilde{r}))u'(c_l^*(\tilde{r})) &= \lambda(\tilde{r})\tilde{r}u'(c_0^*)
\end{align*}
\]

In sum, there are two equations from (7.1), 4 equations from (7.3) and (7.4), and the following three equations:

\[
\pi k_{2h}(\tilde{r}) + (1 - \pi) k_{2l}(\tilde{r}) = K_2^*, \quad (\tilde{r} = r_1, r_2)
\]

\[
B_1 + E_1 = K_1^*
\]

Then, we can get 9 unknowns: \( R_1, (B_1, E_1) \) and \( (k_{2h}(\tilde{r}), k_{2l}(\tilde{r}))_{\tilde{r}=r_1, r_2}, \{ R(\tilde{r}) \}_{\tilde{r}=r_1, r_2} \). It is not hard to see that there is an interior solution of \((B_1, E_1)\). Therefore, the aggregate shock affects the capital structure in the quantitative sense.

8 Conclusion

We clarify the role the corporate tax in order to achieve the constrained optimal allocation under the Mirrleesian taxation framework with an additional realistic assumption. In addition, the existence of the corporate tax requires the individual taxation properly adjusted. This sophisticated tax system affects an individual agent’s portfolio holdings of debt and equity, in turn, it determines the aggregate leverage ratio. Along this line, this paper investigates the endogenous characteristics between the optimal tax system and the capital structure. The optimal tax mechanism in this paper is designed to prevent the agents from using tax timing options. Understanding the capital structure in optimal taxation framework may seem somewhat unusual because taxation is often regarded as a normative theory. However, we hope this approach can potentially shed light in designing a workhorse model in understanding capital structure issues better.
Appendix

A Appendix for Section 2

A.1 Proof of Lemma 1

Proof. Recall the inverse Euler equation.

\[
\frac{r}{u'(c_0^*)} = \frac{\pi}{u'(c_h^*)} + \frac{1 - \pi}{u'(c_l^*)}.
\]

Then, by the Jensen inequality we have

\[
u'(c_0^*) < r\pi u'(c_h^*) + r(1 - \pi)u'(c_l^*) < r\pi u'(c_l^*) + r(1 - \pi)u'(c_l^*) = ru'(c_l^*),
\]

which completes the proof. \(\square\)

A.2 Proof of Corollary 1

Proof. From the inverse Euler equation, we have

\[
\frac{u'(c_0^*)}{ru'(c_h^*)} = \frac{1}{\pi} - \frac{(1 - \pi)u'(c_0^*)}{\pi ru'(c_l^*)} > \frac{1}{\pi} - \frac{(1 - \pi)}{\pi} = 1,
\]

where the inequality follows by Lemma 1. \(\square\)

B Appendix for Section 3

B.1 Proof of Proposition 1

Proof. In fact, this proposition can be regarded as a special case of the general theorem shown in Kocherlakota (2005). Hence, Readers who are interested in the general set-up and its proof should refer Kocherlakota (2005). Under the tax system (3.4) and (3.5) we rewrite the agent’s budget constraint as following:

\[
\begin{align*}
c_l &= c_l^* + r(1 - \tau_l)(k_1 - k_1^*) \\
c_h &= c_h^* + r(1 - \tau_h)(k_1 - k_1^*) + w(y_h - y_h^*) \\
c_{hh} &= c_{hh}^* + r(1 - \tau_{hh})(k_{2h} - k_{2h}^*) + w(y_{hh} - y_{hh}^*) \\
c_{hl} &= c_{hl}^* + r(1 - \tau_{hl})(k_{2l} - k_{2l}^*) \\
c_{lh} &= c_{lh}^* + r(1 - \tau_{lh})(k_{2l} - k_{2l}^*) + w(y_{lh} - y_{lh}^*) \\
c_{hl} &= c_{hl}^* + r(1 - \tau_{hl})(k_{2l} - k_{2l}^*)
\end{align*}
\]
Then, the first order conditions are given by

\[
\begin{align*}
    u'(c_h) &= \pi r (1 - \tau_h) u'(c^*_h) + (1 - \pi) r (1 - \tau_l) u'(c^*_l), \\
    u'(c_h) &= \pi r (1 - \tau_{hh}) u'(c_{hh}) + (1 - \pi) r (1 - \tau_{hl}) u'(c_{hl}), \\
    u'(c_l) &= \pi r (1 - \tau_l) u'(c_h) + (1 - \pi) r (1 - \tau_l) u'(c_l), \\
    w u'(c_0) &= v'(y_0), \quad w u'(c_h) = v'(y_h), \quad w u'(c_{hh}) = v'(y_{hh}), \quad w u'(c_l) = v'(y_l)
\end{align*}
\]

Then, it is not hard to see that the solution to the above system coincides with the constrained optimal solution. In fact, we need to check whether the individual agent will optimally choose the corresponding planner’s allocation in each of following cases: (i) \(y_h > 0, y_{hh} > 0\), (ii) \(y_h = 0, y_{hh} > 0\), (iii) \(y_h > 0, y_{hh} = 0\), (iv) \(y_h = 0, y_{hh} = 0\). Since the agent’ derived utility is strict concave with respect to \((y, k)\), each pair of allocation \((c, k, y)\) corresponding to all cases from (i) to (iv) is the unique solution coinciding with the socially optimal allocation by using the above first order conditions. We omit the tedious algebra.

\[\square\]

**B.2 Proof of Proposition 2**

*Proof.* Notice the following 3 equations for the first equality of (a):

\[
r(1 - \tau_h) u'(c^*_h) = u'(c^*_0), \quad r(1 - \tau_l) u'(c^*_l) = u'(c^*_0), \quad u'(c^*_0) = \frac{r}{\frac{\pi}{u'(c^*_h)} + \frac{1 - \pi}{u'(c^*_l)}}.
\]

Then,

\[
\begin{align*}
    \pi \tau_h + (1 - \pi) \tau_l &= \pi \left(1 - \frac{u'(c^*_0)}{ru'(c^*_h)}\right) + (1 - \pi) \left(1 - \frac{u'(c^*_0)}{ru'(c^*_l)}\right) \\
    &= 1 - \frac{u'(c^*_0)}{r} \left(\frac{\pi}{u'(c^*_h)} + \frac{1 - \pi}{u'(c^*_l)}\right) = 0.
\end{align*}
\]

Then, since \(c^*_h > c^*_l\), we have the second property of (a). The proof for (b) is similar.

For the proof of footnote 22, if there is no intertemporal transfer of resources through the government, we have \(\pi \alpha_h + (1 - \pi) \alpha_l = r (\pi \tau_{kh} + (1 - \pi) \tau_{kl}) k_1 = 0\).

\[\square\]
B.3 Proof of Proposition 3

Before we start the proof of Proposition 3, there are two comments for easier understanding. First, the proof focuses only on the behavior of the low skill agents in period 1. The high skill agents already do not have incentives to deviate under the second best world tax scheme. Second, although in the second best world we only investigated the case where there is no intertemporal transfer of resources, one should notice that, in general, the labor taxation is indeterminate. More precisely, \((k^*_{2h}, k^*_{2l})\) in the tax system (3.4) can be assigned arbitrarily as long as the sum of optimal capital accumulation of all the agents is equal to the capital investment of the constrained optimum, in other words, as long as \(\pi k^*_{2h} + (1 - \pi)k^*_{2l} = K^*_{2l}\) is satisfied. Therefore, the agent’s investment (or saving) strategy depends on how much labor taxes will be assigned in period 1, in particular, how big \((\alpha_h, \alpha_l)\) in (3.4) are. Due to this indeterminacy the proof of proposition 3 is divided into 2 cases. Therefore, the proof is valid regardless of whether the government period-by-period transfers resources.

**Proof.** Consider an agent who exclusively owns a firm in period 0 become a low skill agent in period 1. If she gets the capital income \(r k^*_1\), consume \(c^*_l\), and invest \(k^*_{2l}\) as in Section 3.2, her remaining expected utility \(X\) at period 1 is

\[
X := u(c^*_l) + \pi u(c^*_h) - \pi v(y^*_h) + (1 - \pi) u(c^*_l). \tag{8.1}
\]

Now we investigate the two cases. In each case, we suggest a strategy to deviate from the socially optimal allocation and show that the this allocation gives the low skill agent better off, which completes the proof.

First suppose

\[
k^*_{2l} \geq r(1 - \tau_l)k^*_1,
\]

which means that the low skill agent get enough labor subsidy. Consider the strategy that the firm does not distribute the capital rent \(r k^*_1\) and she additionally invest \(k^*_1\) into her firm. In this case her consumption in period 1 is \(\alpha_l - k^*_l\) since she does not pay the capital tax and gets the subsidy \(\alpha_l\). Then, her remaining expected utility \(Y\) is now

\[
Y := u(\alpha_l - k^*_l) + \pi u[r(1 - \tau_l)(r k^*_1 + k^*_1) + w y_h + \alpha_l] - \pi v(y^*_h)
\]

\[
+ (1 - \pi) u[r(1 - \tau_l)(r k^*_1 + k^*_1) + \alpha_l]
\]

\[
= u(c^*_l + k^*_{2l} - r(1 - \tau_l)k^*_1 - k^*_1) + \pi u \left( c^*_l + \frac{c^*_h}{c^*_l} (r k^*_1 + k^*_1 - k^*_{2l}) \right) - \pi v(y^*_h)
\]

\[
+ (1 - \pi) u \left( c^*_l + \frac{c^*_h}{c^*_l} (r k^*_1 + k^*_1 - k^*_{2l}) \right) \tag{8.2}
\]
In this case, we have $X < Y$ as long as we can pick any $k'_1$ satisfying

$$k^*_2 - r(1 - \tau)k^*_1 \geq k'_1 \geq k^*_2 - rk^*_1.$$  

This is possible since $\tau_l > 0$ and $k^*_2 \geq r(1 - \tau)k^*_1$. Note that $k'_1 = 0$ can be allowed.

Secondly, suppose

$$r(1 - \tau_l)k^*_1 > k^*_2,$$

which means that the labor subsidy is not enough, so the agent cannot afford to invest more. Consider the strategy that the firm distributes only $r\tilde{k}_1 < rk^*_1$ amount of capital rent to the owner (the disable agent). In this case, she pays $r\tau_l\tilde{k}_1$ as a capital income tax and has $\alpha_l + r(1 - \tau_l)\tilde{k}_1$ as net consumption in period 1. The rest of capital rent ($rk^*_1 - r\tilde{k}$) is just remained (therefore reinvested) in the firm without being taxed. Then, her remaining expected utility $Y$ is

$$Y := u(\alpha_l + r(1 - \tau_l)\tilde{k}_1) + \pi u[r(1 - \tau_l)(rk_1 - r\tilde{k}_1) + wy_{lh} + \alpha_{lh}] - \pi v(y^*_{lh})$$

$$+ (1 - \pi)u[r(1 - \tau_l)(rk_1 - r\tilde{k}_1) + \alpha_{lh}]$$

$$= u(c^*_l + k^*_2 - r(1 - \tau_l)(k^*_1 + \tilde{k}_1)) + \pi u\left(c^*_h + \frac{c^*_h}{c^*_l}(rk^*_1 - r\tilde{k}_1 - k^*_2)\right) - \pi v(y^*_{lh})$$

$$+ (1 - \pi)u\left(c^*_l + \frac{c^*_h}{c^*_l}(rk^*_1 - r\tilde{k}_1 - k^*_2)\right) \quad (8.3)$$

We compare (8.1) with (8.3). Notice that $r(1 - \tau_l)k^*_1 - k^*_2 < rk^*_1 - k^*_2$. Then, if we take $\tilde{k}_1 > 0$ such that

$$r(1 - \tau_l)\tilde{k}_1 \approx r(1 - \tau_l)k^*_1 - k^*_2,$$

then $Y - X > 0$. This completes the proof. \qed

C Appendix for Section 4

C.1 Proof of Lemma 2

Proof. By simple algebra, showing $0 < \tau^*_c$ and $\tau^*_h < \tau_h$ is equivalent to showing

$$u'(c^*_0) < ru'(c^*_1),$$

which is result of Lemma 1. On the other hand, from (4.15) and (4.19), $\tau^*_c < \tau^*_l$ is equivalent to $\tau^*_l < 1 - \frac{u'(c^*_0)}{ru'(c^*_0)}$, which is exactly (4.18). \qed
C.2 Proof of Theorem 1

Proof. Only the period 1 tax codes are different between the second and the third best world. The period 2 tax codes are the same. The optimal choice of the agent between period 1 and 2 is same as the constrained optimal allocation, i.e., the agent’s consumption in $t = 2$ and investment in $t = 1$ are the same as the constrained optimal allocation (This is simply the result of Proposition 1. Readers can refer Kocherlakota (2005) for more general proof). Therefore, we focus on the allocation between $t = 0$ and $t = 1$ given that

$$(k_{2h}, k_{2l}, c_{hh}, c_{hl}, c_{lh}, y_{hh}, y_{lh}) = (k^*_2, k^*_2, c^*_h, c^*_l, c^*_l, c^*_h, y^*_h, y^*_l)$$

Without loss of generality we also assume that there is no period-by-period transfer of resources. The result can be easily generalized for the case of resource transfer although the individual investment $\{k_1(=B_1 + E_1), k_{2h}k_{2l}\}$ will be different from the constrained optimal allocation for this case.

First, consider the individual agent’s problem. Notice that after choosing between realizing and not-realizing their capital income, the budget constraints of the agent are

$$
c_0 = rk_0 + wy_0 - (B_1 + E_1)$$
$$
c_h = r(1 - \tau^B_h)B_1 + (1 - \tau^*_c) (1 - \tau^E_h)rE_1 - k_{2h} + wy_h + \alpha_h, \text{ if } y_h > 0$$
$$
c_h = r(1 - \tau^B_h)B_1 + (1 - \tau^*_c) rE_1 - k_{2l} + \alpha_l, \text{ if } y_h = 0$$
$$
c_l = r(1 - \tau^B_l)B_1 + (1 - \tau^*_c) rE_1 - k_{2l} + \alpha_l.$$

We only need to consider two strategies of a high skill agent since a low skill agent cannot tell a lie. Suppose the agent works if she becomes a high skill agent in period 1. Substituting $(c_h, c_l)$ into the objective function, we get the first order conditions with respect to $B_1$ and $E_1$ as follows.

$$u'(c_0) = \pi r (1 - \tau^B_h) u'(c_h) + (1 - \pi) r (1 - \tau^B_l) u'(c_l)$$
$$u'(c_0) = \pi r (1 - \tau^E_h)(1 - \tau^*_c) u'(c_h) + (1 - \pi) r (1 - \tau^*_c) u'(c_l)$$
$$v'(y_0) = w u'(c_0), \quad v'(y_h) = w u'(c_h)$$

$$c_0 = rk_0 + wy_0 - (B_1 + E_1)$$
$$c_h = r(1 - \tau^B_h)B_1 + (1 - \tau^*_c)(1 - \tau^E_h)rE_1 - k_{2h} + wy_h + \alpha_h$$
$$c_l = r(1 - \tau^B_l)B_1 + (1 - \tau^*_c) rE_1 - k_{2l} + \alpha_l.$$
Notice the objective function is strictly concave. Given (8.4), \((c_0, c_l, c_h, y_h) = (c_0^*, c_l^*, c_h^*, y_h^*)\) is satisfied since the above first order conditions are the same as those first order conditions for the constrained optimal allocation in (2.7), (2.8), and (2.9) in Section 3.2.

The similar argument also applies for \(y = 0\). Suppose a high skill agent does not work in \(t = 1\), i.e. \(y_h = 0\). Then, the under the given tax system, he will choose 100% equity investment since \(\tau_c^a < \tau_l^B\). The first order conditions in this case are

\[
\begin{align*}
   u'(c_0) &= r(1 - \tau_c^*)u'(c_h) = r(1 - \tau_c^*)u'(c_l) \\
   v'(y_0) &= wu'(c_0),
\end{align*}
\]
\[
\begin{align*}
   c_0 &= r k_0 + w y_0 - E_1 \\
   c_h &= c_l = (1 - \tau_c^*)r E_1 - k_2 + \alpha_l.
\end{align*}
\]

Given (8.4), setting \((c_0, c_h, c_l, y_0, B_1, E_1)\) equal to \((c_0^*, c_l^*, c_h^*, y_0^*, 0, k_1^*)\) satisfies the above first-order conditions by comparing these with (2.7), (2.8), and (2.9). Hence, the agent is indifferent between working \(y_h > 0\) in period 1 (when becoming high skilled) and not working in period 1.

Second, we consider the firm’s problem. Again we only focus on the firm’s decision for period 0 capital structure to install capital and period 1 labor employment, assuming period 1 investment and period 2 labor employment optimally take place. In fact, in period 1, the market becomes the classical second best world, that is the Modigliani-Miller theorem world. Thus, we can, without loss of generality, assume that the firm only the spot market to rent capital in period 1 as in classical macroeconomic models. Define \(f\) by any general constant-returns-to-scale production function (Thus, this proof is for general CRS production functions).

Let \((r_e, r_b)\) denotes by the return on equity and debt and \(w'\) denotes by the price of labor. Here we first show that \(r_b = r_e\) in equilibrium. Given the next period investment plan \(K_2\), the firm’s problem is to raise debt \(B_1\) and equity \(E_1\) to install capital \(K_1\) in period 0 and rent labor \(Y_1\) in period 1 to maximize

\[
\begin{align*}
   r_e E_1 &:= \max_{(K_1, B_1, Y_1)} \left(1 - \tau_c^*\right)E[f(K_1, Y_1) - w' Y_1 - r_b B_1] \\
   \text{subject to } B_1 + E_1 &\geq K_1
\end{align*}
\]

Notice that \(K_2 = K_2^*\) and this does not affect the value of equity in period 0. Then, putting \(B_1 + E_1 = K_1\), we write the expectation operator in detail as follows.

\[
\begin{align*}
   r_e E_1 &= \max_{B_1, Y_1} \left(1 - \tau_c^*\right)\left\{\pi(1 - \tau_h^E) + (1 - \pi)\right\}[f(E_1 + B_1, Y_1) - w' Y_1 - r_b B_1] \\
   &= \max_{B_1, Y_1} f(E_1 + B_1, Y_1) - w' Y_1 - r_b B_1.
\end{align*}
\]
since the tax code satisfies
\[\pi(1 - \tau_h^E)(1 - \tau_c^E) + (1 - \pi)(1 - \tau_c^E) = \frac{\pi u'(c_0^e)}{ru'(c_h^e)} + \frac{(1 - \pi)u'(c_0^e)}{ru'(c_l^e)} = 1. \tag{8.5}\]

by the inverse Euler equation. Suppose there is an interior solution \(B_1 \in (0, K_1^*)\). First order conditions with market clearing provide

\[r_b = f_1(K_1^*, Y_1^*) \quad \text{and} \quad w' = f_2(K_1^*, Y_1^*)\]

Since \(f\) is CRS, we also obtain \(r_e = r_b = f_1(K_1^*, Y_1^*)\). (This also justifies why we have used \(r_e = r_b = r\) in the main context without special comment when \(f(k, y) = rk + wk\).

It is also clear to have \(w' = w\) for this case.) On the other hand, no arbitrage argument also can be applied: If \(r_e > r_b\), then an agent will buy a stock using a money from selling a bond with interest rate \(r' \in (r_b, r_e)\), which gives arbitrage. If \(r_b > r_e\), then one will establish his own firm with no debt financing to get \(r\) return, instead of investing into a firm with return \(r_e\).

Now consider equation (8.5). This is the expected effective after tax net return on equity, which is one. Thus, in aggregation, the representative shareholder does not pay the corporate tax. Since there is no bankruptcy, the firm is indifferent to choosing between debt and equity. In addition, the firm value is indifferent to capital structure. More precisely, suppose that there is a general equilibrium that the firm has a particular value of debt and equity \((B_1^c, E_1^c)\). Then, we have

\[rE_1^c = f(K_1^*, Y_1^*) - w'Y_1^* - rB_1^c.\]

or

\[E_1^c + B_1^c = \frac{f(K_1^*, Y_1^*) - w'Y_1^*}{r}.\]

Thus, the firm value depends on the aggregate variable, which is determined by the market supply of capital and labor. The idea is quite similar to Stiglitz (1969). This completes the proof.

\[\square\]

D Appendix for Section 5

D.1 Proof of Proposition 5

Proof. Given the tax system, we already know that the constrained optimal solution of consumption and labor vectors \((c^*, y^*)\) coincide with the solution to the competitive
Proof. Lemma 5. The following lemma is useful to figure out the sign of aggregate debt and equity holding in Proposition 5.

Let \( D_2 = -(\pi \tau_h^E \tau_c^* + \pi \tau_h^B - \pi \tau_h^E + \tau_c^* - (1 - \pi) \tau_t^B) \). Then, we have \( D_2 > 0 \).

**Proof.**

\[
D_2 = \pi[(1 - \pi_h^B) - (1 - \pi_h^E)(1 - \tau_c^*)] + (1 - \pi)[(1 - \tau_t^B) - (1 - \tau_c^*)]
\]

\[
= \pi(1 - \tau_h^B) - \frac{\pi u'(c_h^0)}{ru'(c_h^*)} + \frac{u'(c_h^0)}{ru'(c_h^*)} - \pi(1 - \tau_h^B) \frac{u'(c_h^*)}{ru'(c_h^*)} - \frac{(1 - \pi)u'(c_h^*)}{ru'(c_h^*)}
\]

\[
= \pi(1 - \tau_h^B) \left(1 - \frac{u'(c_h^*)}{u'(c_h^*)}\right) - 1 + \frac{u'(c_h^*)}{ru'(c_h^*)},
\]

\[
> \frac{\pi u'(c_h^0)}{ru'(c_h^*)} \left(1 - \frac{u'(c_h^*)}{u'(c_h^*)}\right) + \frac{u'(c_h^*)}{ru'(c_h^*)} - 1 = \frac{\pi u'(c_h^0)}{ru'(c_h^*)} + \frac{(1 - \pi)u'(c_h^*)}{ru'(c_h^*)} - 1 = 0.
\]

where the first inequality is a rewriting of \( D_2 \), the second equality is by using (4.12), the third and the last equality are by the inverse Euler equation, and the third inequality is by (4.17).

D.2 Sign of Denominators of (5.1) and (5.1)

The following lemma is useful to figure out the sign of aggregate debt and equity holding in Proposition 5.

**Lemma 5.** Let \( D_2 = -(\pi \tau_h^E \tau_c^* + \pi \tau_h^B - \pi \tau_h^E + \tau_c^* - (1 - \pi) \tau_t^B) \). Then, we have \( D_2 > 0 \).

**Proof.**

\[
D_2 = \pi[(1 - \pi) - (1 - \pi)(1 - \tau_c^*)] + (1 - \pi)[(1 - \tau_t^B) - (1 - \tau_c^*)]
\]

\[
= \pi(1 - \tau_h^B) - \frac{\pi u'(c_h^0)}{ru'(c_h^*)} + \frac{u'(c_h^0)}{ru'(c_h^*)} - \pi(1 - \tau_h^B) \frac{u'(c_h^*)}{ru'(c_h^*)} - \frac{(1 - \pi)u'(c_h^*)}{ru'(c_h^*)}
\]

\[
= \pi(1 - \tau_h^B) \left(1 - \frac{u'(c_h^*)}{u'(c_h^*)}\right) - 1 + \frac{u'(c_h^*)}{ru'(c_h^*)},
\]

\[
> \frac{\pi u'(c_h^0)}{ru'(c_h^*)} \left(1 - \frac{u'(c_h^*)}{u'(c_h^*)}\right) + \frac{u'(c_h^*)}{ru'(c_h^*)} - 1 = \frac{\pi u'(c_h^0)}{ru'(c_h^*)} + \frac{(1 - \pi)u'(c_h^*)}{ru'(c_h^*)} - 1 = 0.
\]

where the first inequality is a rewriting of \( D_2 \), the second equality is by using (4.12), the third and the last equality are by the inverse Euler equation, and the third inequality is by (4.17).

D.3 The Proof of Corollary 4

**Proof.** Given \((\alpha_h, \alpha_l)\), the aggregate transfer of labor income subsidy is given by

\[
X(\alpha_h, \alpha_l) = \pi \alpha_h + (1 - \pi)\alpha_l = r(\pi \tau_h^* + (1 - \pi) \tau_t^*) \hat{k}_1 + r(\pi \tau_h^* + \tau_c^*) \hat{k}_{1e}.
\]

Since there is no governmental transfer, \( \hat{k}_1 = k_1^* \). Plugging the above equation and \( \hat{k}_1 = k_1^* \) into (5.1) and (5.2), we have the required result.
D.4 Proof of Proposition 6

Proof. If \( \alpha_l \) goes up by \( \epsilon \), then \( \alpha_h \) should be decreased by \( \frac{(1-\pi)u'(c_l^*)}{\pi u'(c_h^*)} \epsilon \) from (4.13). Therefore, the change in \( X(\alpha_h, \alpha_l) \) is

\[
\Delta X(\alpha_h, \alpha_l) = \pi \left( -\frac{(1-\pi)u'(c_l^*)}{\pi u'(c_h^*)} \epsilon \right) + (1-\pi)\epsilon
\]

\[
= \epsilon(1-\pi)u'(c_h^*) - u'(c_l^*) < 0.
\]

In this case, (5.1) and (5.2) tell that change in debt will be positive and the change in equity is negative, which shows that the leverage ratio goes up. On the other hand, if \( \alpha_l \) goes down, then the opposite implication holds, which means the leverage ratio goes down. This completes the proof. \( \square \)

D.5 Proof of Proposition 7

Proof. Using (4.14), we can rewrite (4.13) as

\[
\pi[u'(c_h^*)\alpha_h + \pi u(c_{hh}^*)\alpha_{hh} + (1-\pi)u(c_{hl}^*)\alpha_{hl}]
\]

\[
+ (1-\pi)[u'(c_l^*)\alpha_h + \pi u(c_{lh}^*)\alpha_{lh} + (1-\pi)u(c_{ll}^*)\alpha_{ll}] = D_1,
\]

where \( D_1 \) is some constant consisting of optimal values \( (c^*, y^*) \) independent of \( \alpha \)'s. Then, plugging this into (5.5) and rearranging the equation to get

\[
X(\alpha) = \pi \left( \frac{1}{u'(c_h^*)} - \frac{1}{u'(c_l^*)} \right)[u'(c_h^*)\alpha_h + \pi u(c_{hh}^*)\alpha_{hh} + (1-\pi)u(c_{hl}^*)\alpha_{hl}] + D_2,
\]

for some constant \( D_2 \) consisting of optimal values \( (c^*, y^*) \) independent of \( \alpha \)'s. Notice that \( c_h^* > c_l^* \). Then, \( X(\alpha) \) has the same sign with the expected present value of labor subsidies conditional on being the high type, \( A \),

\[
A := u'(c_h^*)\alpha_h + \pi u(c_{hh}^*)\alpha_{hh} + (1-\pi)u(c_{hl}^*)\alpha_{hl}.
\]

This shows that \( \frac{\partial k_1^*}{\partial A} < 0 \) and \( \frac{\partial k_2^*}{\partial A} > 0 \) since \( \pi > \frac{\tau_l^* - \tau_l^*}{\tau_l^*} \). This completes the proof. \( \square \)

E. Appendix for Section 6

E.1 Proof of Lemma 4

Proof. First two inequalities result from \( c_l^* < c_m^* \). Showing the third inequality is equivalent to showing

\[
u'(c_0^*) < ru'(c_l^*).
\] (8.7)
Recall the inverse Euler equation.

\[ \frac{r}{u'(c_0^*)} = \frac{\pi_h}{u'(c_h^*)} + \frac{\pi_m}{u'(c_m^*)} + \frac{\pi_l}{u'(c_l^*)}. \]

Then, inequality (8.7) comes from the Jensen’s inequality:

\[ u'(c_0^*) < r\pi_h u'(c_h^*) + r\pi_m u'(c_m^*) + r\pi_l u'(c_l^*) \]
\[ < r\pi_l u'(c_l^*) + r\pi_m u'(c_m^*) + r\pi_l u'(c_l^*) = ru'(c_l^*). \]

This completes the proof. \[ \square \]

### E.2 Proof of Proposition 8

**Proof.** The proof is basically the extension of the proof of Proposition 5. Given the tax system, we already know that the constrained optimal solution of consumption and labor vectors \((c^*, y^*)\) coincide with the solution to the competitive equilibrium. Now, \((k_{2h}^*, k_{2m}^*, B_1^*, E_1^*)\) are obtained by solving the following system of equations:

\[
\begin{bmatrix}
\pi_h & \pi_m & \pi_l & 0 & 0 \\
0 & 0 & 1 & -r(1 - \tau_l^E) & -r(1 - \tau_c) \\
0 & 1 & 0 & -r(1 - \tau_m^E) & -r(1 - \tau_m)(1 - \tau_c) \\
1 & 0 & 0 & -r(1 - \tau_h^E) & -r(1 - \tau_h^E)(1 - \tau_c) \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
k_{2h} \\
k_{2m} \\
k_{2l} \\
B_1 \\
E_1
\end{bmatrix}
= \begin{bmatrix}
k_2 \\
\alpha_l - c_l^* + wy_l^* \\
\alpha_m - c_m^* + wy_m^* \\
\alpha_h - c_h^* + wy_h^* \\
K_1^*
\end{bmatrix} \tag{8.8}
\]

Solving the above matrix equation (8.8), we have (6.3) and (6.4). \[ \square \]

### E.3 Sign of Denominators of (6.3) and (6.4) in Proposition 8

The following lemma is useful to characterize the sign of aggregate debt and equity holding. This lemma is also used later.

**Lemma 6.** We have \(D_3 > 0\).
Proof.

\[ D_3 = \pi_h[(1 - \tau_h^B)(1 - \tau_h^E)(1 - \tau_c)] + \pi_m[(1 - \tau_m^B)(1 - \tau_m^E)(1 - \tau_c)] + \pi_l[(1 - \tau_l^B)(1 - \tau_l^E)] \]

\[ = \pi_h(1 - \tau_h^B) + \pi_m(1 - \tau_m^B) + \pi_l(1 - \tau_l^B) \]

\[ = \pi_h(1 - \tau_h^B) - \frac{u'(c_0^*)}{ru'(c_h^*)} + \pi_m(1 - \tau_m^B) - \frac{u'(c_m^*)}{ru'(c_m^*)} + \pi_l(1 - \tau_l^B) - \frac{u'(c_l^*)}{ru'(c_l^*)} \]

\[ = \pi_h(1 - \tau_h^B) - \frac{u'(c_0^*)}{ru'(c_h^*)} + \pi_m(1 - \tau_m^B) - \frac{u'(c_m^*)}{ru'(c_m^*)} + \pi_l(1 - \tau_l^B) - \frac{u'(c_l^*)}{ru'(c_l^*)} \]

where the second equality is by using (6.1), the third and the last equality are by the inverse Euler equation, and the third inequality is by (6.1). \(\square\)

E.4 Proof of Proposition 9

Proof. We will find \((\delta_h, \delta_m, \delta_l)\) explicitly. The first order conditions in the individual agent problem under the tax system \((\tilde{\tau}_c, \tilde{\tau}_h^B, \tilde{\tau}_h^E, \tilde{\tau}_m^B, \tilde{\tau}_m^E, \tilde{\tau}_l^B, \tilde{\tau}_l^E)\) are given by

\[ u'(c_0) = \pi_l r [1 - (\tau_c + \epsilon)]u'(c_l) + \pi_m r [1 - (\tau_c + \epsilon)]u'(c_m) + \pi_h r [1 - (\tau_h + \epsilon)]u'(c_h) \]  

\[ (8.9) \]

\[ u'(c_0) = \pi_l r [1 - (\tilde{\tau}_h^B)]u'(c_l) + \pi_m r [1 - (\tilde{\tau}_m^B)]u'(c_m) + \pi_h r [1 - (\tilde{\tau}_h^E)]u'(c_h) \]  

\[ (8.10) \]

In order to make the firm indifferent to issuing between debt and equity, we have the following condition

\[ \pi_l(1 - \tilde{\tau}_c) + \pi_m(1 - \tilde{\tau}_c)(1 - \tilde{\tau}_m^E) + \pi_m(1 - \tilde{\tau}_c)(1 - \tilde{\tau}_h^E) = 1. \]  

\[ (8.11) \]

for any optimal tax system. In this case,

\[ \pi_l[1 - (\tau_c + \epsilon)] + \pi_m[1 - (\tau_c + \epsilon)][1 - (\tau_m^E - \delta_m)] + \pi_h[1 - (\tau_c + \epsilon)][1 - (\tau_h^E - \delta_m)] = 1. \]

Let us define (8.9) and (8.10) by resulting equations after putting the optimal solution \((c_l^*, c_m^*, c_h^*)\) into (8.9) and (8.10). Solving (8.9) and (8.11), we have

\[ \delta_m = \frac{(1 - \tau_m^E)}{\tau_c - \epsilon} + \frac{\pi_l \epsilon}{\pi_m(1 - \tau_c - \epsilon)} \left( \frac{u'(c_l^*) - u'(c_l^*)}{u'(c_m^*) - u'(c_h^*)} \right) \]  

\[ (8.12) \]

\[ \delta_h = \frac{(1 - \tau_h^E)}{\tau_c - \epsilon} + \frac{\pi_l \epsilon}{\pi_h(1 - \tau_c - \epsilon)} \left( \frac{u'(c_l^*) - u'(c_l^*)}{u'(c_m^*) - u'(c_h^*)} \right) \]  

\[ (8.13) \]
Finally, the other tax rates, $\tilde{\tau}_B^h, \tilde{\tau}_B^m, \tilde{\tau}_E^l$, and $\tilde{\tau}_E^l$ can be arbitrarily determined by (8.10) and the following four inequalities

\[
(1 - \tilde{\tau}_B^h) > (1 - \tilde{\tau}_E^h + \delta_h)(1 - \tau_c - \epsilon)
\]
\[
(1 - \tilde{\tau}_B^m) > (1 - \tilde{\tau}_E^m + \delta_m)(1 - \tau_c - \epsilon)
\]
\[
(1 - \tilde{\tau}_E^l) < (1 - \tau_c - \epsilon)
\]
\[
\tilde{\tau}_E^l > 0
\]

Now, finally if we take the tax system $(\tilde{\tau}_c, \tilde{\tau}_B^h, \tilde{\tau}_E^h, \tilde{\tau}_B^m, \tilde{\tau}_E^m, \tilde{\tau}_B^l, \tilde{\tau}_E^l)$, then it is easy to see that $(c^*_0, c^*_h, c^*_m, c^*_l)$ is the solution to the agent’s problem since $(c^*_h, c^*_m, c^*_l)$ is the solution to the Euler equation (8.9) and (8.10) and the concavity is still preserved under this transform with $(\delta_h, \delta_m, \delta_l)$.

\[\square\]

### E.5 Proof of Proposition 10

**Proof.** Suppose $\tilde{\tau}_c$ increases by $\epsilon$. Let operator $\Delta$ denote by the change in any variable corresponding to $\epsilon$ amount increase in $\tilde{\tau}_c$. For example, $\Delta \tilde{\tau}_B^h = 0$ for all $i = h, m, l$ by the condition of the Proposition, we have $\Delta D_3 = 0$. Recall that in order to make the firm indifferent to issuing between debt and equity, for any optimal tax system $(\tilde{\tau}_c, \tilde{\tau}_B^h, \tilde{\tau}_E^h, \tilde{\tau}_B^m, \tilde{\tau}_E^m, \tilde{\tau}_B^l, \tilde{\tau}_E^l)$, the following equation should be satisfied.

\[
\pi_l(1 - \tilde{\tau}_c) + \pi_m(1 - \tilde{\tau}_c)(1 - \tilde{\tau}_m) + \pi_m(1 - \tilde{\tau}_c)(1 - \tilde{\tau}_E^m) = 1.
\]

Using the above equation, we can rewrite $D_3$ as

\[
D_3 = \pi_h(1 - \tilde{\tau}_B^h) + \pi_m(1 - \tilde{\tau}_B^m) + \pi_l(1 - \tilde{\tau}_B^l) - 1.
\]

Since $\Delta \tilde{\tau}_i^B = 0$ for all $i = h, m, l$ by the condition of the Proposition, we have $\Delta D_3 = 0$.

Note that $\Delta X(\alpha_h, \alpha_m, \alpha_l) = 0$ since $(\alpha_h, \alpha_m, \alpha_l)$ is fixed. Then, By using the similar analysis, the numerators of $\tilde{B}_1^*$ and $\tilde{E}_1^*$ are unchanged. In sum, there is no change in the numerators and the denominators in $\tilde{B}_1^*$ and $\tilde{E}_1^*$, which completes the proof. \[\square\]

### E.6 proof of Proposition 11

**Proof.** The expected capital taxes (as the income of the government) are given as

\[
\underbrace{r\left(\pi_h \tilde{\tau}_B^h + \pi_m \tilde{\tau}_B^m + \pi_l \tilde{\tau}_l^B\right)}_{:= (a)} B_1^*
\]
\[
+ r\left(1 - \{\pi_l(1 - \tilde{\tau}_c) + \pi_m(1 - \tilde{\tau}_c)(1 - \tilde{\tau}_m) + \pi_m(1 - \tilde{\tau}_c)(1 - \tilde{\tau}_E^m)\}\right) E_1^*
\]

\[50\]
Notice that \((b)\) is zero (due to the condition that firms are indifferent between issuing debt and equity), i.e.,

\[
\pi_l(1 - \tilde{\tau}_c) + \pi_m(1 - \tilde{\tau}_c)(1 - \tilde{\tau}_E^m) + \pi_m(1 - \tilde{\tau}_c)(1 - \tilde{\tau}_E^h) = 1
\]

Now we will show that part \((a)\) is negative, which completes the proof as follows.

\[
\pi_h \tilde{\tau}_h^B + \pi_m \tilde{\tau}_m^B + \pi_l \tilde{\tau}_l^B = 1 - \{\pi_h (1 - \tilde{\tau}_h^B) + \pi_m (1 - \tilde{\tau}_m^B) + \pi_l (1 - \tilde{\tau}_l^B)\} < 0. \tag{8.14}
\]

since we have

\[
\pi_h (1 - \tilde{\tau}_h^B) + \pi_m (1 - \tilde{\tau}_m^B) + \pi_l (1 - \tilde{\tau}_l^B) = D_3 + (b) = D_3 > 0,
\]

by Lemma 6.

\[
\square
\]

References


