Credit Crunches and Credit Allocation in a Model of Entrepreneurship*

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Abstract

We study the effects of credit shocks in a model with heterogeneous entrepreneurs, financing constraints, and a realistic firm size distribution. As entrepreneurial firms can grow only slowly in this set-up, we show that, by reducing entrepreneurial firm size, negative shocks have a very persistent effect on real activity.

1 Introduction

The recent turmoil in financial markets has had deep consequences for the allocation of credit within the economy. Access to credit is particularly important for nascent and growing firms, for which it is much more difficult to only rely on retained earnings as a source of financing.

In this paper, we study the effect of various types of financial shocks in a model with two non-financial sectors: a corporate sector, primarily composed of mature firms, and an entrepreneurial

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sector, whose leverage is limited by their inability to fully commit to repay their debts. The constraints generate a large, and realistic, dispersion in firm size, and limit the rate at which entrepreneurial firms can grow. We build on the entrepreneurship model of Cagetti and De Nardi [12, 13] and introduce a financial intermediation sector and three types of financial shocks:

- An intermediation shock that makes it more costly to channel funds from savers to borrowers; this shock could be either a “black-box” TFP shock, or the destruction of capital specific to the intermediation sector (e.g., the loss in value of mortgage-backed securities).
- A collateral shock, that makes it harder for entrepreneurs to pledge future repayment of debt, similar to Jermann and Quadrini [26];
- Government targeted intervention in the financial markets, that drives a wedge in the cost of funds across different classes of borrowers. Examples are the U.S. Treasury’s guarantee of money market mutual funds (and implicitly the underlying commercial paper), the subsidies implicit in the TALF program, the recapitalization of banks and automakers under TARP, additional programs undertaken by the Small Business Administration, and asset purchases by the Federal Reserve System.

We show that in our set-up all these types of negative credit shocks have a very persistent effect on real activity. While the corporate sector recovers fairly quickly after the financial shock is over, the wealth accumulation of the entrepreneurs is affected in almost a permanent way. Negative credit shocks reduce firm size, and, because entrepreneurial firms can grow only slowly, limit the speed at which firms return to their previous scale when the shocks subside. This slow transition is characterized by more capital misallocation and hence lower output than in steady state.

We also find that the fiscal implications of the recession induced by financial shocks are important. The recession and its associated drop in tax revenues generates a public deficit. If the government increases income taxes (on labor and capital jointly) after the financial shock is over, it effectively increases again the wedge between the rate of return earned by savers and that paid by borrowers; this has similar implications to the original financial shock, and it slows
down recovery even further.

2 Related Works

Many works incorporate credit-market frictions in macroeconomic models and study how these frictions affect aggregate investment and help generate and amplify business fluctuations. Among the earlier and most influential contributions, Bernanke and Gertler [7] introduce agency problems such as costly state verification in a dynamic general equilibrium set-up, and Kiyotaki and Moore [29] further illustrate the impact of collateral constraints and their interaction with asset prices and firms net worth. In both papers, credit imperfections link investment decisions to the firms’ balance sheets and generate a “financial accelerator” that amplifies and propagates shocks to the macroeconomy.

The recent financial crisis has given further impetus to this literature, highlighting both the many channels through which credit market imperfections can affect real activity, and the possible effects of government interventions to improve the functioning of credit markets and the flow of funds between borrowers and lenders. For a review of this literature, see Bernanke, Gertler and Gilchrist[8] for earlier contributions and Gertler and Kiyotaki [19], Brunnermeier and Sannikov [9] and Krishnamurthy [31] for more recent ones. Here, we will only mention a few of the papers most related to our work.

We model several types of financial frictions. Financial intermediation (and more in general frictions in credit markets) introduce a wedge between the returns to lender and the cost of capital to borrowers, a wedge related to the spread between liquid and easily intermediated securities such as Treasuries and corporate bonds. These credit spreads vary over time and their level and variation have been shown to be empirically correlated to and potentially key to understand output fluctuations (for instance, Gilchrist, Sim and Zakrajsek[20], Christiano, Motto and Rostagno [15], Adrian and Shin [1]). Their role has been highlighted, among others, by Hall [22], who show that in a simple representative-agent economy credit spreads (including those for households) are powerful determinants of economic activity and can generate fluctuations of the magnitude of those seen in the recent crisis, and by Curdia and Woodford [18], who study
how monetary policy rules should respond to shocks to credit spreads. We also find that spreads have a significant impact on aggregate output during a credit crisis; by themselves, spreads have a fairly short-lived effect in our model economy. It is a different source of frictions that propagates the effect of spreads and generates a very persistent drop in output.

Among borrowers, we explicitly distinguish corporate and entrepreneurial firms; the latter potentially face different constraints and have reduced access to financial markets (see e.g. Quadrini [37]). We model credit frictions to entrepreneurs as endogenous borrowing constraints arising from imperfect enforceability of debt contracts (as in Kehoe and Levine [27] and Alvarez and Jermann [4]). In this set-up, credit availability to entrepreneurs depends on their balance sheet and their available collateral. This class of models has been shown useful to explain, for instance, firm-size distribution (Akyol and Athreya [5], Monge [36]), firm dynamics (Albuquerque and Hopenhayn [2]), macroeconomic fluctuations (Cooley, Marimon, and Quadrini [16], Jermann and Quadrini [25]), and growth (Buera and Shin [11]). The presence of limited commitment slows the growth of nascent firms and links it to the entrepreneurs’ cash flow. It is this channel that propagates the initial financial shock in our model and is responsible for our main results.

The extent of borrowing constraints depends crucially on characteristics of the borrower such as firm size, balance sheet, and personal wealth (Buera [10])). For this reason, we build a model that quantitatively reproduces the high level of dispersion in these variables observed in the data. Our work is thus related to the literature on wealth inequality and its determinants (such as Quadrini and Ríos-Rull [38] and Castaneda, Diaz-Gimenez and Ríos-Rull [14]), and especially to the literature that identifies entrepreneurial wealth as a key force generating inequality (Quadrini[37], Cagetti and De Nardi [12]). The interaction between frictions, entrepreneurship, and inequality is crucial to understand the response to macroeconomic shocks (Jermann and Quadrini[26]), the effect of certain government policies (Cagetti and De Nardi [13], Meh[35], Kitao [28]), and asset pricing (Heaton and Lucas [23], Roussanov [39], Covas and Fujita [17]).

3 The Model

The model described here is based on Cagetti and De Nardi [13].
3.1 Demographics

A young person faces a constant probability of aging during each period \((1 - \pi_y)\), and an old person faces a constant probability of dying during each period \((1 - \pi_o)\). When an old person dies, his offspring enters the model, carrying the assets bequeathed to him by the parent.

3.2 Preferences

The household’s flow of utility from consumption is given by \(c^{1-\sigma} t^{1-\sigma}\). The households discount the future at rate \(\beta\) and are perfectly altruistic toward their descendants.

3.3 Technology

Each person possesses two types of ability, which we take to be exogenous, stochastic, positively autocorrelated, and stochastically independent of each other. Entrepreneurial ability \((\theta_t)\) is the capacity to invest capital and labor more or less productively using one’s own production function. Working ability \((y_t)\) is the capacity to produce income out of labor by working for others.

The entrepreneurs can borrow, invest capital, hire labor, and run a technology whose return depends on their own entrepreneurial ability: those with higher ability levels have higher average and marginal returns from capital and labor. When the entrepreneur invests \(k_t\) production is given by

\[
 f(k_t, n_t) = \theta_t (k_t^\gamma (1 + n_t)^{(1-\gamma)})^\nu
\]

where \(\nu, \gamma \in [0, 1]\), and \(n\) is hired labor \((n \geq 0)\). We normalize the labor of the entrepreneur to 1. Entrepreneurs thus face decreasing returns from investment, as their managerial skills become gradually stretched over larger and larger projects (as in Lucas[34]). While entrepreneurial ability is exogenously given, the entrepreneurial rate of return from investing in capital is endogenous and is a function of the size of the project that the entrepreneur implements.

There is no within-period uncertainty regarding the returns of the entrepreneurial project. The ability \(\theta_t\) is observable and known by all at the beginning of the period. We therefore abstract
from problems arising from partial observability, costly state verification, and from diversification of entrepreneurial risk.

In addition to entrepreneurs, there is also a non-entrepreneurial sector, represented by a standard Cobb-Douglas production function:

\[ F(K^c_t, L^c_t) = A(K^c_t)^\alpha(L^c_t)^{1-\alpha} \]  

where \( K^c_t \) and \( L^c_t \) are the total capital and labor inputs in the non-entrepreneurial sector and \( A \) is a constant. In both sectors, capital depreciates at a rate \( \delta \).

### 3.4 Credit

External financing to both entrepreneurs and non-entrepreneurial firms is provided by competitive financial intermediaries. The intermediaries borrow funds from workers (and possibly entrepreneurs, though in equilibrium almost all entrepreneurs will be credit constrained and will invest all their wealth in their own firm).

Intermediation is costly. For each unit of capital, it requires \( \phi_t \) units of the consumption good as an intermediate input.

Financial intermediaries operate competitively. At any time \( t \), they take as given the interest rate required by savers (\( i_t \)) and the interest rate paid by borrowers (\( r_t \)). Given the technology, an equilibrium with a positive and finite supply of intermediation requires

\[ r_t = i_t + \phi_t. \]  

For the non-entrepreneurial sector, we start by assuming that it must finance a given fraction \( \xi_t \) of its capital through external borrowing (we calibrate this to data).\(^1\)

The entrepreneurial demand for borrowed funds arises endogenously in the model. As in Kehoe and Levine [27], entrepreneurs are subject to borrowing constraints that are endogenously determined in equilibrium and stem from the assumptions that contracts are imperfectly enforceable.

\(^1\)We will endogenize this in the future, modeling an agency conflict between shareholders and managers.
In particular, as in Cagetti and De Nardi [12], we assume that the entrepreneurs who borrow either can invest the money and repay their debt at the end of the period or can run away without investing it and be workers for one period. In the latter case, they retain a fraction $f$ of their working capital $k_t$ (which includes own assets and borrowed money) and their creditors seize the rest. We assume that labor services are paid at the end of the period, hence entrepreneurs are not constrained in the amount of labor that they hire.

3.5 Government and taxation

The government is infinitely lived. It levies taxes, pays a pension $p_t$ to each retiree, provides a certain level $g_t$ of public purchases (which do not enter the households’ utility function), and pays interest on the accumulated debt. During every period, tax revenues from income, consumption, and estate taxes are equal to government purchases, pension payments, and interest payments on the debt.

We model progressive taxation of total income as in Cagetti and De Nardi [13], and use their parameter estimates.

Total income taxes paid by each household are given by

$$T^i_t(Y_t) = \tau^i_t(Y_t) Y_t + \tau^s_t Y_t,$$

where $\tau^s_t$ captures state and other income taxes (other than federal). The government also levies a sales tax on consumption, at rate $\tau^c$. Estates larger than a given value $e$ are taxed at rate $\tau^b$ on the amount in excess of $e$. In the experiments below, the tax rate $\tau^s_t$ will be allowed to adjust to meet the government budget constraint.

As a first pass, we abstract from the tax implications of corporate finance decisions by assuming that corporate income taxes are zero and that capital gains are taxed as regular income.\footnote{These two assumptions tend to offset each other.}
3.6 The corporate firms’ problem

In each period \( t \), a corporate firm starts with resources \( A_t^C \), which include undepreciated capital from last period, retained earnings, and last period’s equity issuance. The firm uses \( A_t^C \) and new debt (external) financing \( B_t \) to purchase capital for operation in period \( t \) (\( K_t^C \)), subject to the minimum external finance constraint

\[
B_t \geq \xi K_t^C. \tag{3}
\]

Since corporate firms will always be owned by savers (workers), their objective function is to maximize the discounted sum of profits, using the interest rate \( i_t \) as a discount factor.

Formally, the problem a firm faces as of period \( t \) is described recursively as follows:

\[
J_t(A_t^C) = \max_{K_t^C, L_t^C, B_t, A_{t+1}^C} F(K_t^C, L_t^C) + A_t^C - w_t L_t^C - r_t B_t - A_{t+1}^C - \delta K_t^C + \frac{1}{1 + i_{t+1}} J_{t+1}(A_{t+1}^C), \tag{4}
\]

subject to

\[
K_t^C \leq A_t^C + B_t \tag{5}
\]

and (3). In equation (4), \( J \) represents the value of the firm’s equity. In period \( t \), the firm’s profits are given by \( F(K_t^C, L_t^C) - w_t L_t^C - r_t B_t - \delta K_t^C \). Of these profits, the firm retains \( A_{t+1}^C \) to finance future operations, and it pays out the rest as dividends (with negative dividends corresponding to new equity issuance).\(^3\)

It is straightforward to verify that the firms’ problem is homogeneous of degree 1 in \( A_t^C \). This implies that the size distribution of corporate firms is irrelevant, and we can work with one representative (competitive) firm. It also implies that the firm’s value is proportional to its initial internal funds: \( J_t(A_t^C) \equiv \hat{J}_t A_t^C \). Using \( \hat{\cdot} \) to denote the optimal choice rescaled by \( A_t^C \) and denoting by \( \omega_{1t} \) and \( \omega_{2t} \) the Lagrange multipliers on (5) and (3) respectively, the first-order conditions that will hold if the corporate sector is active yield:

\[
F_K(\hat{K}_t^C, \hat{L}_t^C) - \delta = \omega_{1t} + \xi \omega_{2t},
\]

\[
F_L(\hat{K}_t^C, \hat{L}_t^C) = w_t, \tag{6}
\]

\(^3\)In our calibration, dividends from the corporate sector will always be positive, so the introduction of costs of equity issuance would be moot.
\[ r_t = \omega_{1t} + \omega_{2t}, \]

and

\[ 1 = \frac{\dot{J}_{t+1}}{1 + i_{t+1}}. \] (7)

For \( t > 0 \), the envelope condition yields

\[ \dot{J}_t = 1 + \omega_{1t}. \]

From these equations, we obtain

\[ F_K(\hat{K}_t^C, \hat{L}_t^C) = \delta + (1 - \xi)i_t + \xi r_t, \quad t > 0 \] (8)

In the initial period, the internal funds of the corporate sector \((A_0^C)\) are exogenously given. Depending on its value and factor prices, the corporate firms’ optimization problem yields\(^4\)

\[ F_K(\hat{K}_0^C, \hat{L}_0^C) \in [\delta, \delta + r_0], \quad \hat{B}_0 > \xi \hat{K}_0^C \implies F_K(\hat{K}_0^C, \hat{L}_0^C) = \delta + r_0. \] (9)

Finally, using the properties of homogeneous functions and the first-order conditions, we obtain

\[
\begin{align*}
\dot{J}_0 &= F(\hat{K}_0, \hat{L}_0) - w_0 \hat{L}_0 - r_0 \hat{B}_0 - \delta \hat{K}_0 + 1 = \\
&= F_K(\hat{K}_0, \hat{L}_0) \hat{K}_0 - r_0 \hat{B}_0 - \delta \hat{K}_0 + 1 = \\
&= F_K(\hat{K}_0, \hat{L}_0) \hat{K}_0 - r_0(\hat{K}_0 - 1) - \delta \hat{K}_0 + 1 = \\
&= 1 + r_0 + \hat{K}_0 \left[ F_K(\hat{K}_0, \hat{L}_0) - \delta - r_0 \right] = \\
&= \begin{cases} 
1 + r_0 & \text{if } \hat{B}_0 > \xi \hat{K}_0 \\
1 + r_0 + \frac{1}{1-\xi} \left[ F_K(\frac{1}{1-\xi}, \hat{L}_0) - \delta - r_0 \right] & \text{otherwise.}
\end{cases}
\end{align*} \] (10)

### 3.7 Households

Since our model does not feature any uncertainty within the period, each household is indifferent on how to allocate funds that are not invested in its own enterprise; all of these options will

\(^4\)If we let the firm invest in government bonds or deposit resources with the financial intermediary in period \( t \), then the lower bound would be \( \delta + i_0 \). This lower bound is irrelevant in our numerical computations, since the equilibrium rate of return on capital always exceeds it.
have to yield the same return $i_t$. We thus only need to keep track of total assets held by the household.

At the beginning of each period the current ability levels are known with certainty, while next period’s levels are uncertain.

Each young individual starts the period with assets $a_t$, entrepreneurial ability $\theta_t$, and worker ability $y_t$, and chooses whether to be an entrepreneur or a worker during the current period.

An old entrepreneur can decide to keep the activity going or retire, while a retiree cannot start a new entrepreneurial activity.

**The young’s problem**

The value function of a young person is

$$V_t(a_t, y_t, \theta_t) = \max \{V^e_t(a_t, y_t, \theta_t), V^w_t(a_t, y_t, \theta_t)\}$$

subject to

$$Y^e_t = \theta(k^\gamma_t(1 + n_t)^{(1-\gamma)})^\nu - \delta k_t - (k_t - a_t)(r_t I_{k_t > a_t} + i_t I_{k_t < a_t}) - w_t n_t$$

$$a_{t+1} = Y^e_t - T^e_t(Y^e_t) + a_t - (1 + \tau^e_t) c_t$$

$$u(c_t) + \beta \pi_y E_t V_{t+1}(a_{t+1}, y_{t+1}, \theta_{t+1}) + \beta (1 - \pi_y) E_t W_{t+1}(a_{t+1}, \theta_{t+1}) \geq V^w_t(f \cdot k_t, y_t, \theta_t)$$

$$a_t \geq 0$$

$$n_t \geq 0$$

$$k_t \geq 0.$$
The term $Y_t$ represents the entrepreneur’s total profits. The expected value of the value function is taken with respect to $(y_{t+1}, \theta_{t+1})$, conditional on $(y_t, \theta_t)$. Eq. (15) determines the maximum amount that an entrepreneur with given state variables can borrow. The term $W_t(a_{t+1}, \theta_{t+1})$ is the value function of the old entrepreneur at the beginning of the period, before deciding whether to stay in business or retire. We have

$$V_t^w(a_t, y_t, \theta_t) = \max_{c_t, a_{t+1}} \left\{ u(c_t) + \beta \pi_y E_t V_{t+1}(a_{t+1}, y_{t+1}, \theta_{t+1}) + \beta (1 - \pi_y) W_{t+1}^r(a_{t+1}) \right\}$$  \hspace{1cm} (19)

subject to eq. (16) and

$$Y_t^w = w_t y_t + i_t a_t$$  \hspace{1cm} (20)

$$a_{t+1} = (1 + i_t) a_t - T_t^w(Y_t^w) - (1 + \tau_t) c_t,$$  \hspace{1cm} (21)

where $w_t$ is the equilibrium wage rate.

**The old’s problem**

Since the old entrepreneur can choose to continue the entrepreneurial activity or retire, his state variables are his current assets $a_t$ and his entrepreneurial ability level $\theta_t$. His value function is given by

$$W_t(a_t, \theta_t) = \max\{W_t^e(a_t, \theta_t), W_t^r(a_t)\},$$  \hspace{1cm} (22)

where $W_t^e(a_t, \theta_t)$ is the value function for the old entrepreneur who stays in business, and $W_t^r(a_t)$ is the value function of the old retired person. Define the inherited assets, net of estate taxes, as $a_{n+1} = a_{t+1} - \tau_{t+1}^b \cdot \max(0, a_{t+1} - e_{t+1})$. We have

$$W_t^e(a_t, \theta_t) = \max_{c_t, k_t, n_t, a_{t+1}} \left\{ u(c_t) + \beta \pi_o E_t W_{t+1}(a_{t+1}, \theta_{t+1}) + \beta (1 - \pi_o) E_t V_{t+1}(a_{n+1}^{t+1}, y_{t+1}, \theta_{t+1}) \right\}$$  \hspace{1cm} (23)

subject to eq. (13), eq. (14), eq. (16), eq. (17), eq. (18) and

$$u(c_t) + \beta \pi_o E_t W_{t+1}(a_{t+1}, \theta_{t+1}) + \beta (1 - \pi_o) E_t V_{t+1}(a_{n+1}^{t+1}, y_{t+1}, \theta_{t+1}) \geq W_t^r(f, k_t).$$  \hspace{1cm} (24)

The child of an entrepreneur is born with ability level $(\theta_{t+1}, y_{t+1})$. The expected value of the child’s value function with respect to $y_{t+1}$ is computed using the invariant distribution of $y_t$, while the one with respect to $\theta_{t+1}$ is conditional on the parent’s $\theta_t$ and evolves according to the
same Markov process that each person faces for $\theta_t$ while alive. This is justified by the assumption that the child of an entrepreneur inherits the parent’s firm.

A retired person (who is not an entrepreneur) receives pensions and social security payments ($p_t$) and consumes his assets. His value function is

$$W_r^t(a_t) = \max_{c_t,a_{t+1}} \{u(c_t) + \beta \pi_o W_r^{t+1}(a_{t+1}) + \beta(1 - \pi_o)E_t V_{t+1}(a_{t+1}, y_{t+1}, \theta_{t+1})\} \quad (25)$$

subject to eq. (16) and

$$a_{t+1} = (1 + i_t)a_t + p_t - T^w_t(p_t + i_t a_t) - (1 + \tau^c_t)c_t. \quad (26)$$

The expected value of the child’s value function is taken with respect to the invariant distribution of $y_t$ and $\theta_t$.

3.8 Equilibrium definition

Let $x_t = (a_t, y_t, \theta_t, z_t)$ be the state vector, where $z$ distinguishes young workers, young entrepreneurs, old entrepreneurs, and old retired. From the decision rules that solve the maximization problem and the exogenous Markov process for income and entrepreneurial ability, we can derive a transition function $M_t(x_t, \cdot)$, which provides the probability distribution of $x_{t+1}$ (the state next period) conditional on the current state $x_t$.

At any time $t$ an equilibrium is given by the following functions

\[
\begin{align*}
\{ & \text{interest rates } r_t, i_t, \text{ a wage rate } w_t, \\
& \text{taxes } (T^w_t(\cdot), T^c_t(\cdot), \tau^c_t, \tau^b_t, e_t) \text{ and social security payments } p_t, \\
& \text{allocations } c_t(x), \text{ and } a_t(x), \text{ occupational choices,} \\
& \text{entrepreneurial labor hiring } n_t(x), \text{ and investments } k_t(x), \\
& \text{and a distribution of people over the state variables } x_t: m_t(x),
\end{align*}
\]

such that, given $i_t$, $r_t$, $w_t$, and government taxes and transfer schedules:

- The functions $c_t$, $a_t$, $n_t$ and $k_t$ solve the maximization problems described above.
- The amounts of labor and capital employed by the corporate sector satisfy (6), (8), and (9).
• Financial intermediaries break even, that is, equation (2) holds.

• The value of corporate firms is given by (7) from period 1 onwards and (10) for the initial period. Equation (5) holds as an equality, i.e., firms do not leave capital idle.

• The labor market clears, that is, the total labor supplied by the workers equals the total labor employed in the non-entrepreneurial sector and total labor hired by the entrepreneurs.

• The capital markets clear. Total household savings are equal to the sum of the value of the corporate sector’s assets, government debt, corporate and noncorporate debt, and the capital used by financial intermediaries as an intermediate input.\footnote{Note that the value of a firm’s assets at the beginning of the period is \( \hat{J}_t/(1 + i_t) \), where \( \hat{J}_t \) is given by (7) from period 1 onwards, and by (10) in period 0. This is because dividends are paid at the end of the period, at the same time as interest on debt.}

• The government budget constraint balances: total taxes collected equal government purchases, transfers, and interest payments on government debt,

\[
\int \left( T^x_t(Y_x) + \tau^x_t c(x) + I_o(x) \tau^h_t (1 - \pi_o) \cdot \max(0, a_{t+1}(x_t) - e_t) \right) dm_t(x) = p_t \pi_r + g_t + i_tD_t.
\]

The integral is over all of the population, \( I_o \) is an indicator function that is equal to one if the person is old and zero otherwise, and \( \pi_r \) is the fraction of retired people in the population. In steady state \( D_t = \bar{D} \).

• The distribution of people \( m_t \) is induced by the transition matrix of the system as follows

\[
m'_{t+1} = M_t(x_t, \cdot)' m(t)'.
\]

In steady state \( m_t = m^* \) is the invariant distribution for the economy and debt, prices, and government policies are constant and the individual’s decision rules are time-independent.

4 Calibration

We take some parameters as given, while we use the others to match moments of the data. Regarding the first set of parameters, we take the coefficient of relative risk aversion to be 1.5,
a depreciation rate $\delta$ of 6%, and a capital share in the non-entrepreneurial production function of .33.

We set the steady-state financial intermediation cost to obtain a 1.5% spread between the interest rate paid by borrowers and that received by lenders. This is calibrated to the historical average of the spread between Baa-rated companies and Treasuries. In our model, both public and private debt is risk free, and the spread is entirely due to the special liquidity role of Treasuries, that are assumed not to require any intermediation. For this reason, we choose to match our private borrowing rate to an empirical counterpart that features low default risk but is also unlikely to carry any liquidity premium (see Krishnamurthy-Vissing Jorgensen[32] for more discussion).

The probability of aging and of death are such that the average length of the working life is 45 years and the average length of the retirement period is 11 years.

The logarithm of the income $y$ process for working people is assumed to follow an AR(1) with a persistence of .95 and variance chosen to match the Gini coefficient for earnings of .38. We assume that the income process and the entrepreneurial ability processes evolve independently. The social security replacement rate is 40% of average gross income (see Kotlikoff, Smetters and Walliser [30]). The steady-state ratio of government spending to GDP is set to 18.7%, and the tax rate on consumption is 11%. All of these parameter choices are discussed in Cagetti and De Nardi [13]. We also use estimates of the parameters of the tax function from that paper.

We pick the level of government debt (as a fraction of output) so that, given the equilibrium interest rate, every period the total interest payments on government debt equal 3% of output (as in Altig et al. [3]).

In previous work, Cagetti and De Nardi [13] have discussed the relevant empirical counterpart to the entrepreneur in the model we adopt. Our entrepreneurs are the self-employed business owners that actively manage their own firm(s). We identify them in the Survey of Consumer Finances (SCF) with those that declare that they are self-employed, that they own a business, and that they actively manage it.

We consider only two values of entrepreneurial ability: zero (no entrepreneurial ability) and
<table>
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<tr>
<th>Parameter</th>
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<td>Preferences, technology, and demographics</td>
<td></td>
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<tr>
<td>$\sigma$</td>
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<td>Attanasio et al. [6]</td>
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<tr>
<td>$\delta$</td>
<td>.06</td>
<td>Stokey and Rebelo [40]</td>
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<td>$\alpha$</td>
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<td>normalization</td>
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<td>$\phi$</td>
<td>.015</td>
<td>Baa-Treasury spread</td>
</tr>
<tr>
<td>$\xi$</td>
<td>.33</td>
<td>Flow of funds</td>
</tr>
<tr>
<td>$\pi_y$</td>
<td>.98</td>
<td>average working life: 45 years</td>
</tr>
<tr>
<td>$\pi_o$</td>
<td>.91</td>
<td>average retirement life: 11 years</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor income process and social security payments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y, P_y$</td>
<td>see appendix in Cagetti and De Nardi [13]</td>
<td>Huggett [24], Lillard et al. [33]</td>
</tr>
<tr>
<td>$p$</td>
<td>40% average yearly income</td>
<td>Kotlikoff et al. [30]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public expenditure, government debt, and taxes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>18.7% GDP</td>
<td>NIPA</td>
</tr>
<tr>
<td>$D$</td>
<td>see text</td>
<td>Altig et al. [3]</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>11%</td>
<td>Altig et al. [3]</td>
</tr>
<tr>
<td>$b_w$</td>
<td>.32</td>
<td>Cagetti and De Nardi [13]</td>
</tr>
<tr>
<td>$b_e$</td>
<td>.26</td>
<td>Cagetti and De Nardi [13]</td>
</tr>
<tr>
<td>$s_w$</td>
<td>.22</td>
<td>Cagetti and De Nardi [13]</td>
</tr>
<tr>
<td>$p_w$</td>
<td>.76</td>
<td>Cagetti and De Nardi [13]</td>
</tr>
<tr>
<td>$p_e$</td>
<td>1.4</td>
<td>Cagetti and De Nardi [13]</td>
</tr>
<tr>
<td>$s_e$</td>
<td>.42</td>
<td>Cagetti and De Nardi [13]</td>
</tr>
</tbody>
</table>

Table 1: Fixed parameters and their sources.
a positive number. This implies that $P_\theta$ is a two-by-two matrix. Since its rows have to sum to one, this gives us two parameters to calibrate. We also have to choose values for $\nu$, the degree of decreasing returns to scale to entrepreneurial ability; $\gamma$, the share of income going to entrepreneurial working capital; $f$, the fraction of working capital the entrepreneur can keep in case he defaults; the estate tax rate, and its corresponding exemption level.

In total, these are nine parameters to be used to match nine moments of the data. We use the first seven to target the following moments generated by the model: the capital-output ratio, the fraction of entrepreneurs in the population, the fraction of entrepreneurs exiting entrepreneurship during each period, the fraction of workers becoming entrepreneurs during each period, the ratio of median net worth of entrepreneurs to that of workers, the fraction of people with zero wealth, and the fraction of entrepreneurs hiring workers on the labor market. We choose the other two parameters to match the revenue from estate and gift taxes and the fraction of the estates that pay estate taxes. Table 2 reports values for all of these targets and the corresponding value generated by our current calibration. Table 3 reports the parameter values that we are currently using to produce our results.

5 Some Preliminary Experiments

Throughout the experiments below, a financial shock hits the economy unexpectedly in year 2 and lasts for 2 years. All the agents in the economy are assumed to have perfect foresight from year 2 onwards. After year 3, the financial parameters return to their steady state level.

For each experiment, we isolate the effects of taxes and interest-rate changes by proceeding as follows. First, we keep taxes and lending rates fixed at the initial steady state level (blue lines in the pictures). Second, we still keep lending rates fixed, but we let taxes vary so that govern-
## Table 2: Target values.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital-output ratio</td>
<td>3.0</td>
<td>2.9</td>
</tr>
<tr>
<td>Percentage of Entrepreneurs</td>
<td>7.6</td>
<td>7.6</td>
</tr>
<tr>
<td>Percentage of Exiting Entrepreneurs</td>
<td>22-24</td>
<td>22.5</td>
</tr>
<tr>
<td>Percentage of Workers Entering Entrepreneurship</td>
<td>2-3</td>
<td>2.3</td>
</tr>
<tr>
<td>Median Net Worth of Entrepreneurs to Workers</td>
<td>7</td>
<td>6.2</td>
</tr>
<tr>
<td>Percentage of People at Zero Wealth</td>
<td>7-13</td>
<td>11.2</td>
</tr>
<tr>
<td>Percentage of Entrepreneurs Hiring on the Labor Market</td>
<td>60</td>
<td>49</td>
</tr>
<tr>
<td>Revenue from Estate and Gift Taxes (as % of output)</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Percentage of Estates Paying Estate Taxes</td>
<td>2</td>
<td>1.8</td>
</tr>
</tbody>
</table>

## Table 3: Calibrated parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>.9</td>
</tr>
<tr>
<td>$\theta$</td>
<td>{0, 0.7}</td>
</tr>
<tr>
<td>$P_\theta$</td>
<td>see text</td>
</tr>
<tr>
<td>$\nu$</td>
<td>.88</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>.84</td>
</tr>
<tr>
<td>$f$</td>
<td>75%</td>
</tr>
<tr>
<td>$\tau_b$</td>
<td>16%</td>
</tr>
<tr>
<td>$e$</td>
<td>120</td>
</tr>
</tbody>
</table>

Table 2: Target values.

Table 3: Calibrated parameters.
ment debt converges back to the original steady state (green lines). Specifically, an additional proportional income tax rate is levied after the end of the financial shock, in years 4 through 11; the tax rate increases linearly from year 4 to 7, reaches a peak at 7, and decreases back linearly to its steady-state level by year 12. This case, represented by green lines in the pictures, corresponds to a small open economy, where the rest of the world is not affected by the shock. Finally, we consider the full general equilibrium (closed-economy) experiment, where we let both the tax rate vary as above and the interest rate adjust so that capital markets clear (red lines).

5.1 Negative technology shock in the intermediation sector.

We consider the effect of a shock that unexpectedly increases $\phi$ from 1.5% to 3.5% for two years.8 This is a crude way of capturing either of two alternative shocks:

- More monitoring is necessary to ensure loan performance due to the financial turmoil.

- $\phi$ stands in as payments to a factor that is fixed in the short run and that is temporarily depleted. As an example, suppose that banks face capital requirements and that some initial losses wipe some of the capital out, constraining the banks’ ability to offer additional intermediation services. In this case, the increase in $\phi$ would reflect the additional reward for the scarcer banking capital.9

Figures 1-6 show our main results. For a fixed interest rate, the shock causes a rapid drop in domestic output:10 the negative shock to domestic intermediation implies a significant capital outflow. In this case, domestic output (which excludes payments from capital invested abroad)

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7We view this as mostly an intermediate step to illustrate the economic forces at work. If we were to truly model an open economy, we would want to include costs of reallocating capital across countries, to dampen the flows and generate persistence, in line with the data.

8For a comparison, the spread between Baa corporate bonds and Treasuries jumped to more than 5% after the recent crisis, and decreased only gradually over the course of 2009.

9To spell out completely this story, we should explain what prevents capital from immediately flowing back into the banking sector.

10National output declines much less, since the capital invested abroad continues to earn a rate of return.
overstates the consequences of the shock. However, even considering consumption, we see a nontrivial drop of almost 1% from the relatively minor increase in the interest rate spread.

The recession causes a budget deficit, which requires an increase in the income tax rate. Figure 6 shows the increase required to restore balance with fixed lending rates; at the peak, the tax rate is increased by about 1%.\footnote{The steady-state value is the amount of additional income taxes that are needed in the calibration to maintain debt constant, compared to the tax rates that we estimate in the data. For the calibration that we choose, the estimated taxes are insufficient, and approximately an extra 3% needs to be levied to keep budget balance.} The government imbalance does not have a large impact on the depth of the initial recession, but it prolongs further the time that the economy takes to recover. This is mostly driven by the fact that the increase in the income tax hits capital as well as labor income, and it thus acts as a wedge that is very similar to the original financial-market shock.

When we consider a closed economy and we let the interest rate adjust, we obtain a very small output response on impact. This is not surprising: in the current version of the model, the labor supply of workers is fixed, and so is the amount of capital on impact, which means that any drop in output must be generated only by the misallocation of resources across entrepreneurs and between the entrepreneurial and corporate sector.\footnote{We are currently introducing an endogenous labor supply decision in the problem to better match the short-run cyclical effects of the shocks.}

A striking feature of the pictures is that the model generates a very long-lasting drop. With a fixed interest rate, output stages a big recovery once the financial shock is over, due to the repatriation of capital, but it then takes a long time to close the gap. Consumption takes even longer. The general-equilibrium picture remains the same: 20 years after the shock, output is still 0.5% below its steady state value. Figure 2 breaks the GDP down into its separate components, and shows that the long-lasting effect is entirely due to the entrepreneurial sector;\footnote{This figure only presents the case where the interest rate and the tax rate are fixed; the conclusion holds across the three experiments.} as soon as the shock is over, the corporate sector’s output jumps actually above steady state. Figures 4 and 5 break down the persistent response of the entrepreneurs along two dimensions: the number of entrepreneurs and their average firm size (by capital employed). Our model generates a limited...
amount of endogenous entry and exit in response to such a short-lived shock, since it affects relatively few potential entrepreneurs that are right at the wealth margin in those two years. The bigger response stems from firm size: the shock slows down the wealth accumulation of entrepreneurs. Since both the wealth distribution and the distribution of assets across firms that we match is very spread out, our model implies a very gradual growth of firms, with almost no entrepreneur attaining sufficient wealth that borrowing constraints cease to bind. It follows that any negative shock has almost a permanent effect on each entrepreneur, and its aggregate impact vanishes fully only when each entrepreneur loses his ability and closes the firm.

5.2 Negative technology shock in the intermediation sector, only for entrepreneurs.

In this section, we consider the same shock to $\phi$ as in the previous section, but we neutralize its impact on the corporate sector by varying $\xi_t$ to hold $\xi_t \phi_t$ constant throughout. One possible interpretation of this experiment is a government guarantee of the commercial paper of corporate industrial firms, under the assumption that this guarantee is not used ex post and it does not add to the government deficit.\footnote{We could easily add a cost to this guarantee, in which case taxes would have to go up more during the transition, and would exacerbate the persistence of the drop in output.} The results from this experiment are shown in figures 8-10.

For the small open-economy case, the shock implies a much smaller drop in domestic output. This is because more capital is reallocated to the corporate sector, rather than being driven abroad, as shown by figure 9. As time passes, the beneficial effect of excluding corporate firms from the shock becomes less important, and this experiment becomes closer to the previous one. This happens because the key mechanism that generates persistence in our environment remains: wealth accumulation by the entrepreneurs is hampered by the shock. For them, changes in $\xi_t$ only provide a second-order effect.

In the general-equilibrium closed-economy case, the difference between this experiment and the previous one is extremely small. When the shock hits both sectors, the interest rate for savers has to drop more to restore equilibrium. In both cases, the output drop is entirely due
to the misallocation of resources across sectors and among entrepreneurs, and guaranteeing the corporate sector’s debt does nothing to improve along this dimension (see figure 7).

5.3 A shock to required collateral.

Here, we consider a shock that increases the collateral that the entrepreneurs need to secure their loans. Specifically, we raise the fraction of capital than can be absconded ($f$) from 75% to 80%. While on impact this shock has bigger consequences for output than the two previous ones, particularly in the entrepreneurial sector, the lingering impact after $f$ returns to its long-term value is similar to the other two experiments. Since this shock has such drastic implications on firm size, it might seem surprising that it does not bear bigger implications for the entrepreneurs wealth in the long run. The reason for this result is that a shock to $f$ hits only the marginal profits of the firm: it forces entrepreneurs to shrink their scale, but it has no effect on their profits for a given scale of operations. In contrast, an increase in $\phi_t$ raises the rental rate of capital paid by entrepreneurs; this effect applies to all of the capital that they rent, and has a negative effect on their profits even conditioning on their scale of operations. This experiment is illustrated in figures 11-13.

References


Figure 1: Real GDP (excluding financial services), fixed $\tau$ and $i$ (blue), fixed $i$ (green), and general equilibrium (red). SS=100; shock to $\phi$. 
Figure 2: Real GDP in the corporate sector (blue) and entrepreneurial sector (green): fixed \( \tau \) and \( i \), and general equilibrium (red). SS=100; shock to \( \phi \).
Figure 3: Real consumption (excluding financial services), fixed $\tau$ and $i$ (blue), fixed $i$ (green), and general equilibrium (red). SS=100; shock to $\phi$. 
Figure 4: Total measure of entrepreneurs (excluding financial services), fixed $\tau$ and $i$ (blue), fixed $i$ (green), and general equilibrium (red). SS=100; shock to $\phi$. 
Figure 5: Average capital used by entrepreneurial firms (excluding financial services), fixed $\tau$ and $i$ (blue), fixed $i$ (green), and general equilibrium (red). SS=100; shock to $\phi$. 
Figure 6: Proportional component of the income tax rate, fixed $i$: shock to $\phi$ (blue) and to both $\phi$ and $\xi$ (green).
Figure 7: Rate of return earned by savers (general equilibrium): shock to $\phi$ (blue) and to both $\phi$ and $\xi$ (green).
Figure 8: Real GDP (excluding financial services), fixed $\tau$ and $i$ (blue), fixed $i$ (green), SS=100; shock to $\phi$ and $\xi$. 
Figure 9: Real GDP in the corporate sector (blue) and entrepreneurial sector (green): fixed $\tau$ and $i$, SS=100; shock to $\phi$ and $\xi$. 
Figure 10: Average capital used by entrepreneurial firms (excluding financial services), fixed $\tau$ and $i$ (blue), fixed $i$ (green), SS=100; shock to $\phi$ and $\xi$. 
Figure 11: Real GDP (excluding financial services), fixed $\tau$ and $i$ (blue), fixed $i$ (green), SS=100; shock to $f$. 
Figure 12: Real GDP in the corporate sector (blue) and entrepreneurial sector (green): fixed $\tau$ and $i$, SS=100; shock to $f$. 
Figure 13: Average capital used by entrepreneurial firms (excluding financial services), fixed $\tau$ and $i$ (blue), fixed $i$ (green), SS=100; shock to $f$. 